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On Repeated Myopic Use of the Inverse Elasticity Pricing Rule

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Abstract

We examine the effects of repeated myopic use of the inverse elasticity pricing rule. By myopic, we mean ignoring that elasticity and marginal cost both may vary with output and thus indirectly with price. It has been shown that myopic use of the rule will typically lead to price changes which are too large relative to the optimal price change (Fjell, 2003). While some mainstream microeconomics textbooks suggest that the rule may be used repeatedly to reach optimal price, they are vague about the conditions for when this works. We show that repeated myopic use will lead to convergence only if demand is sufficiently convex, and specify the exact condition for this.

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1. Introduction

In academics, a well-known rule for marking up marginal cost to ensure optimal or profit maximizing price, is the so called inverse elasticity rule (e.g. Browning and Zupan, 2002; Mansfield, 1994; Nicholson and Snyder, 1985; Pindyck and Rubinfeld, 2005):

$$p = \left(\frac{\varepsilon}{1+\varepsilon}\right) mc \tag{1}$$

where price elasticity is $\varepsilon \equiv \frac{\partial q}{\partial p}\frac{p}{q}$ and marginal cost is *mc*.

We examine the effects of a repeated myopic use of the inverse elasticity pricing rule in (1). By myopic, we mean ignoring that elasticity and marginal cost both may vary with output and thus with price. Although some mainstream microeconomics textbooks suggest that the rule may be used repeatedly to set profit maximizing price (e.g. Browning and Zupan, 2002; Pindyck and Rubinfeld, 2005), the conditions for when repeated use will converge on the optimal price are only vaguely described. The focus of this paper is to determine the specific conditions for when repeated myopic use will reach the optimal price.

All literature we have encountered agrees that the relation (1) holds exactly at the optimal price. Thus the relation can confirm or disconfirm the optimality of the current price. Some literature also explicitly states that the rule can be used to determine if current price is too low or too high. Fjell (2003) proposes that the rule mainly provides the direction of the optimal price (up or down from the current price), but does not go beyond this. Similarly, Besanko et al. (2017) suggest that the rule can be used to determine whether price should be raised or lowered based on local estimates, though not by how much.

However, some texts go beyond this and suggest that the rule may be used repeatedly to reach optimal price. For instance, Browning and Zupan (2002) state that (p. 302, our emphasis): “If you know your demand elasticity and marginal cost, this expression can be used to calculate the profit-maximizing price. … This formula has one difficulty: it holds exactly only at the point of profit maximization, and because marginal cost and elasticity may vary with output, **you may need to use this expression repeatedly** to locate the profit-maximizing price. However, if cost and elasticity **vary only a little** over the range of output you are considering, this formula can approximate the profit-maximizing price quite closely.” In another influential microeconomics textbook, by Pindyck and Rubinfeld (2005), the rule in (1) is referred to as “A Rule of Thumb for Pricing.” (p. 344). Similarly to Browning and Zupan (2002) they suggest that the rule can be used repeatedly to reach optimal price. Pindyck and Rubinfeld (2005) also vaguely indicate under what circumstances this process may converge on the optimal price (p. 345, our emphasis): “Remember that this markup equation applies at the point of profit maximum. If both elasticity of demand and marginal cost **vary considerably** over the range of outputs under consideration, you **may** have to know the entire demand and marginal cost curves to determine the optimal output level.” From this, one may infer that if elasticity of demand and marginal cost vary only a little (i.e. less than “considerably”) the rule can be used repeatedly to reach optimal price. We ask for what demand curves will such repeated myopic use of the rule converge on the optimal price?

The problem analyzed by Fjell (2003) is related to the above question, but he only examines the consequences in a static setting, i.e. for a single, myopic use of the rule. He concludes that:

1) if marginal cost and elasticity are both constant, then optimal price follows directly from the rule
2) If only elasticity varies, then myopic use of the rule will always bypass optimal price (too large price change)
3) If both elasticity and marginal cost vary, price will bypass the optimal unless marginal cost is declining rapidly

We go beyond this and examine repeated myopic use for non-constant elasticity, i.e. a dynamic setting.

There is a large literature on empirical estimation of demand (and cost) which is beside our focus. We do not make any attempts to estimate demand or cost functions.

More related to our problem is the use of heuristics based on a few observations to make conjectures about demand and costs. For instance, one could make assumptions about functional form of demand and cost, and then fit this to recent observations to make estimates of the entire functions, and subsequently maximize profit based on this (Baumol and Quandt, 1964; Laitinen, 2009). Similar to this literature, we use local (point) information of demand and cost. However, unlike this literature, we do not make conjectures about the functional form of demand to pseudo maximize profit. Rather, we explore the conditions for which repeated myopic (conjecture free) use of the inverse elasticity pricing rule may succeed in determining optimal price.

Our approach is more analogous to that of Ray and Gramlich (2015) who show that a full-cost pricing heuristic can converge on the optimal price when the firm can estimate its equilibrium profit.[1] They make no conjectures on functional form, but simply use knowledge of the marginal-

[1] They use the term "equilibrium income" (p. 27).

demand curve. This knowledge, they argue, can be easily estimated through arbitrarily small price movements around the current price. Similar to us, the focus of Ray and Gramlich (2015) is thus to use marginal (or local) information of demand repeatedly – not to estimate or conjecture about the entire demand (or cost) function. Indeed, the usefulness or significance of the rule in (1) can, at least partly, be interpreted to hinge on whether repeated, myopic use converges on optimal price. If not, we may need to know the entire demand and marginal cost curves to determine the optimal output level and price (Pindyck and Rubinfeld, 2005).

Section 2 analyses under what conditions on the demand curve will repeated use of the rule in (1) converge on the optimal price? Section 3 concludes the paper.

2. Analysis

We are interested in the conditions on demand for which the equation in (1), when applied repeatedly in a myopic manner, will converge on the optimal price. In other words, we are interested in the time path where new price, p_{i+1}, is given by the point elasticity, $\varepsilon_i(p_i)$, and marginal cost, *mc*, at the previous price, p_i, such that $p_{i+1} = g(p_i)$ where *i* is the number of iterations $i \in [1, \infty]$. Based on (1), we get more specifically:

$$p_{i+1} = \left(\frac{\varepsilon_i(p_i)}{\varepsilon_i(p_i)+1}\right) mc \tag{2}$$

We assume that marginal cost, *mc*, is positive and constant, and further that $\varepsilon_i(p_i) = \frac{\partial q_i}{\partial p_i} \frac{p_i}{q_i(p_i)}$, as elasticity is a function of price (and, of course also quantity were $\frac{\partial q}{\partial p} < 0$). Also, we only consider prices in the elastic range of demand, i.e. $\varepsilon < -1$ since the rule in (1) yields negative prices otherwise.

The right hand (rhs) side of (2) may be interpreted as a phase line for which a sufficient condition for convergence is that the absolute value of its slope is less than one, i.e. that $|g'(p_i)| < 1$ (see e.g. Chiang, 1984). Hence, repeated myopic use of the rule will converge on the optimal price, $p_{i+1} = p_i$, if the absolute value of the derivative of the rhs of (2) is less than one, i.e.:

$$\left|\frac{1}{(\varepsilon_i+1)^2} mc \frac{\partial \varepsilon_i}{\partial p_i}\right| < 1 \tag{3}$$

From the straight bracket in (3) we see that the relationship between current and myopically prescribed price (i.e. $\frac{\partial p_{i+1}}{\partial p_i}$) is negative if $\frac{\partial \varepsilon_i}{\partial p_i} < 0$. This is true by assumption, i.e. that demand becomes more elastic the higher the price. For negatively sloped phase lines, the iteration path will be one of oscillation (Chiang, 1984). Thus, repeated myopic use will yield prices which oscillate around (or bypass) the optimal price, and converge on it if (3) holds.

For constant elasticity demand functions, for which $\frac{\partial \varepsilon_i}{\partial p_i} = 0$, the left side of (3) reduces to zero and we see that the condition in (3) always holds; we always get convergence. Further, since we then also have that $\frac{\partial p_{i+1}}{\partial p_i} = 0$, we get convergence in one iteration since new price does not depend on the old price, as pointed out by Fjell (2003).

If elasticity is not constant, we see from (3) that the smaller the impact of a price change is on price elasticity, i.e. the lower $\left|\frac{\partial \varepsilon_i}{\partial p_i}\right|$ is, the more likely it is that we get convergence, and vice versa. This is in line with what Browning and Zupan (2002) and Pindyck and Rubinfeld (2005) suggest.

Note that if (3) holds with equality, then we get an infinite price loop around the optimal price. In other words, price iterates from the initial to the new price and back again indefinitely.[2] If the left hand side (lhs) of (3) is greater than one, we get divergence. In other words, repeated myopic use leads to a price which oscillates further and further away from the optimal.

Since $\varepsilon_{i(p_i)} = \frac{\partial q_i}{\partial p_i}\frac{p_i}{q_i(p_i)}$, we can expand on the condition in (3). First, we have that:

$$\frac{\partial \varepsilon_i}{\partial p_i} = \frac{\partial^2 q_i}{\partial p_i^2}\frac{p_i}{q_i} + \frac{\partial q_i}{\partial p_i}\left(\frac{q_i - p_i\frac{\partial q_i}{\partial p_i}}{q_i^2}\right) \tag{4}$$

$$= \frac{\partial^2 q_i}{\partial p_i^2}\frac{p_i}{q_i} + \frac{\varepsilon_i}{p_i}(1-\varepsilon_i)$$

Substituting (4) into (3) yields the following, general convergence condition:

$$\left|\frac{1}{(\varepsilon_i+1)^2} mc\left[\frac{\partial^2 q_i}{\partial p_i^2}\frac{p_i}{q_i} + \frac{\varepsilon_i}{p_i}(1-\varepsilon_i)\right]\right| < 1 \tag{5}$$

Since we know that the expression inside the straight bracket is always negative, we can rewrite the condition in (5) as the following range for tractability:

$$-1 < \frac{1}{(\varepsilon_i+1)^2} mc\left[\frac{\partial^2 q_i}{\partial p_i^2}\frac{p_i}{q_i} + \frac{\varepsilon_i}{p_i}(1-\varepsilon_i)\right] \leq 0 \tag{6}$$

The expression in (6) can in turn be expressed as the following condition on convexity of the demand curve:

$$\varepsilon_i(\varepsilon_i - 1)\frac{q_i}{p_i^2} - \frac{(\varepsilon_i+1)^2}{mc}\frac{q_i}{p_i} < \frac{\partial^2 q_i}{\partial p_i^2} \leq \varepsilon_i(\varepsilon_i - 1)\frac{q_i}{p_i^2} \tag{7}$$

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[2] The condition for this is $\frac{p_{i+1}}{p_i} = \frac{\frac{\varepsilon_i}{1+\varepsilon_i}}{\frac{\varepsilon_{i+1}}{1+\varepsilon_{i+1}}}$, that is, the ratio between the new price and the initial price equals the ratio between the markup (of a constant marginal cost) at the initial price and the markup at the new myopic price.

The condition in (7) must hold for all relevant prices if repeated myopic use of (1) is to converge on the optimal price. Note that the right hand side is always positive and equivalent to the convexity of constant elasticity demand functions.[3] Hence, we have that the condition for convergence is always satisfied for constant elasticity demand functions, in line with what Fjell (2003) and Browning and Zupan (2002) state.

The lhs is simply the same convexity of constant elasticity demand functions less a positive term. Note that the lhs is also positive as long as price is not too high. Specifically:

$$p < \frac{\varepsilon(\varepsilon-1)}{(\varepsilon+1)^2} mc \tag{8}$$

Note that this upper price limit in (8) exceeds the optimal price given by (1). Hence, the lhs of (7) will also always be positive at the optimal price, and lower prices. This means that demand must be strictly convex, which among other excludes linear demands. An illustration of divergence for linear demand is provided in the Appendix.

In sum, we thus have that as long as the demand curve is sufficiently convex, i.e. that its second derivative fulfills the condition in (7), and price is sufficiently low as given by (8), then repeated myopic use of the inverse elasticity pricing rule will converge on the optimal price.

3. Conclusion

Some mainstream microeconomics textbooks suggest that the inverse elasticity pricing rule may be used repeatedly to reach optimal price (Browning and Zupan, 2002; Pindyck and Rubinfeld, 2005). However, they are vague about the conditions for when this works. We show that repeated myopic

[3] These demand functions are generally on the form $q = Ap^{\varepsilon}$ where A is a positive constant and elasticity is constant and less than -1. Thus we have that $\frac{\partial^2 q_i}{\partial p_i^2} = \varepsilon_i(\varepsilon_i - 1)\frac{q_i}{p_i^2}$.

use will converge only if demand is sufficiently convex, i.e. where the second derivative is in the following, positive range: $\varepsilon_i(\varepsilon_i - 1)\frac{q_i}{p_i^2} - \frac{(\varepsilon_i+1)^2}{mc}\frac{q_i}{p_i} < \frac{\partial^2 q_i}{\partial p_i^2} \leq \varepsilon_i(\varepsilon_i - 1)\frac{q_i}{p_i^2}$. This means we will not have convergence, among other, for linear demands.

4. References

Baumol, W. J., & Quandt, R. E. (1964). Rules of thumb and optimally imperfect decisions. *The American economic review*, *54*(2), 23-46.

Besanko, D. A., Dranove, D. S., Shanley, M., & Schaefer, M. (2017). *Economics of Strategy*. Wiley.

Browning, E. K., & Zupan, M. A. (2009). *Microeconomics: Theory & applications.* Wiley.

Chiang, A. C. (1984). *Fundamental methods of mathematical economics*. McGraw-Hill.

Fjell, K. (2003). Elasticity based pricing rules: a cautionary note. *Applied Economics Letters*, *10*(12), 787-791.

Laitinen, E. K. (2009). From complexities to the rules of thumb: towards optimisation in pricing decisions. *International Journal of Applied Management Science*, *1*(4), 340-366.

Mansfield, E. (1994). *Applied microeconomics*. WW Norton & Company.

Nicholson, W., & Snyder, C. (1985). *Microeconomic Theory: Basic Principles and Extensions*, Dryden Press.

Pindyck, R. S., & Rubinfeld, D. L. (2005). *Microeconomics* (6th edn). Upper Saddle River, NJ: Pearson Prentice Hall.

Ray, K., & Gramlich, J. (2015). Reconciling Full-Cost and Marginal-Cost Pricing. *Journal of Management Accounting Research*, *28*(1), 27-37.

5. Appendix – linear demand example

Let the demand be $q = A - Bp$ and the marginal cost $mc = c$. Therefore, the profit is: $\pi = pq - cq = (p-c)(A-Bp)$. The corresponding profit maximizing price is $p^* = \frac{A+Bc}{2B}$.

The elasticity for the demand is: $\epsilon = \frac{dq}{dp}\frac{p}{q} = -B\frac{p}{q} = -B\frac{p}{A-Bp}$.[4] Suppose the initial price $p_0 \neq p^*$. Myopic use of the rule changes that price from p_0 to p_1, where $p_1 = \frac{\epsilon}{1+\epsilon} * \mathrm{c}$. Here the elasticity ϵ is computed at price p_0. Hence,

$$p_1 = \frac{\varepsilon_0}{1+\varepsilon_0} * c = \left(\frac{\frac{-Bp_0}{A-Bp_0}}{1-\frac{Bp_0}{A-Bp_0}}\right) c = \left(\frac{-Bp_0}{A-2Bp_0}\right) c\ .$$

If $p_1 \neq p^*$, then the manager updates the price from p_1 to p_2, where $p_2 = \left(\frac{-Bp_1}{A-Bp_1}\right) c$. Repeated updating of the price is given by $p_i = \left(\frac{-Bp_{i-1}}{A-Bp_{i-1}}\right) c$. Let A = 10, B = 1, c = 2. Then profit maximizing price is $p^* = 6$ and the corresponding point elasticity is $\varepsilon^* = -1.5$. Assume that initial price is $p_0 = 5.99$. As we see, from Table 1, even though the initial price, p_0, is only marginally below the optimal price, we still get a diverging, oscillating price path around the optimal price with inelastic demand at the fourth iteration and a negative price at the fifth iteration.

Price	**Elasticity**
$p^* = 6$	$\varepsilon^* = -1.5$
$p_0 = 5.99$	$\varepsilon_0 = -1.49377$

[4] The slope of the demand curve is implicitly known from the point elasticity at each price. Price and quantity data from repeated use could be used to statistically estimate demand or at least make increasingly accurate conjectures (also for marginal cost). However, our focus is on myopic application of the rule, i.e. ignoring such information and instead acting as if local estimates are valid for all output ranges.

$p_1 = 6.05051$	$\varepsilon_1 = -1.53197$
$p_2 = 5.75962$	$\varepsilon_2 = -1.35828$
$p_3 = 7.58228$	$\varepsilon_3 = -3.13613$
$p_4 = 2.93627$	$\varepsilon_4 = -0.415684$
$p_5 = -1.4228$	$\varepsilon_5 = 0.124558$

Table 1. A = 10, B = 1, c = 2

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