Energy Economics

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November 27, 2019

Contents

Chapter 1

Electricity markets. Overview

The objective of this chapter is to provide a general **overview** of the restructuring process of electricity markets that has taken place in the last years. Electricity markets have been liberalized in the last two decades. In this block we study four papers that analyse that process in detail. Those papers have been written for prominent researchers that has been involve in the liberalization process of electricity markets in Europe and United States.

Based on those papers, we could identify the main problems in the restructuring process, and how we could use those lessons to analyse the current challenges in electricity markets.

The authors analyse the restructuring process using different approaches, however there are many ideas that appear recursively in the papers. Try to identify those key aspects and try to think how those main aspects affect other utilities rather than electricity.

1.1 Borenstein, S., 2002, "The Trouble with Electricity Markets: Understanding California's Restructuring Disaster," *Journal of Economic Perspectives,* 16, 1, 191-211.

Question 1: Which are the **characteristics of electricity markets** that facilitate the exercise of market power?

- Demand is difficult to **forecast**.
- Demand is **insensitive** to price fluctuations.
- Supply faces **binding constraints** at peak times.
- **Storage** is prohibitively costly.
- Demand and supply have to **match** all the time.

Question 2: Which are the **market designs** proposed by Borenstein to mitigate market power in electricity markets?

- **Long-term contracts** between wholesale buyers and sellers.
- **Real-time retail pricing** of electricity, which indicates to the final consumer on an hourly basis when electricity is more or less costly to consume.

Question 3: Which is the role of long-term contracting in electricity markets? How **long-term contracting** could contribute to mitigate market power?

- When a firm has sold some output in advance, it has **less incentive to restrict its output** in the spot market in an attempt to push up prices in that market, since it does not receive the higher spot price on the output it has already sold through a forward contract.
- The incentive of a generating company to exercise market power will depend on its **net purchasing position** in the market at a given point in time.
  - If a firm were a large **net seller**, it would likely have an incentive to restrict output to raise price.
  - If it had sold much of its output under forward contracts,then it would have much less incentive to restrict its output to increase the spot price.

Question 4: How **real time pricing** could contribute to mitigate market power?

- Real time pricing would prevent **extreme price spikes**.
- It would also reduce the financial incentive of sellers to **exercise market power**, since one firm's reduction of output would have a smaller effect on price than it does when demand is completely price-inelastic.
- The effect of real-time pricing also has very important implications for the **negotiation of long-term contracts**. If sellers, at the time of negotiation, believe that real-time pricing is likely, then they will reduce their forecasts of the average spot prices they would be able to earn if they did not sell through a long-term contract. As a result, the sellers will be willing to accept a lower long-term contract price than they otherwise would.

1.2 Joskow, P., 2008, "Lessons Learned from Electricity Market Liberalization," *The Energy Journal, Special Issue on the Future of Electricity,* 9-42.

Question 1: Which are the **characteristics of electricity markets** that facilitate the exercise of market power?

- Generator market power arises as a consequence of **transmission constraints** that limit the geographic expanse of competition.
- **Generation ownership concentration** within constrained import areas.
- The **non-storability** of electricity.
- The very **low elasticity of demand** for electricity

Question 2: Which are the basic features that guarantees a proper **market design** in electricity markets?

- Transparent organized **spot** markets for energy and **ancillary** services (day-ahead and real time balancing).
- **Locational pricing** of energy reflecting the marginal cost of congestion and losses at each location.
- The integration of spot wholesale markets for energy with the efficient allocation of scarce transmission capacity.
- Auctioning of **transmission rights** to hedge congestion, serve as a basis for incentives for good performance by system operators and transmission owners, and partially to support new transmission investment.

- An **active demand side** that can respond to spot market price signals.
- **Forward contracts** to mitigate market power.

Question 3: Could you identify bad practises that induce a **lack of investment in transmission capacity**? Could an only market mechanism induce the correct investment decision in transmission capacity

- Fragmented transmission **ownership**.
- Separation of **system operations** from **transmission maintenance** and **investment**.
- Poorly designed incentive regulation mechanisms (Joskow, 2005).
- Relying primarily on **market-based "merchant transmission" investment**, that is where new transmission investments must be fully supported by congestion rents (the difference in locational prices times the capacity of a new link) is likely to lead to inefficient investment in transmission capacity (Joskow and Tirole, 2005).

1.3 Newbery, D., 2005, "Electricity Liberalization in Britain: The Quest for a Satisfactory Wholesale Market Design,", *The Energy Journal,* 26, 43-70.

Question 1: Can you enumerate some of the ideas proposed by Newbery to **increase competition in electricity markets**?

Newbery proposed a market design similar to the one proposed in the previous papers. His paper is relevant because he also analyses the role of **market structure** to guarantee that the electricity markets work properly.

1.4 Wilson, R., 2002, "Architecture of Power Markets," *Econometrica,* 70, 4, 1299-1340.

1.4.1 Integrated vs unbundled market

Question 1: Wilson define three main points to define the **role of the System Operator (SO)**, and to determine which **market design** is the best (integrated systems vs. unbundled systems). Could you identify them?

- **Allocate** multiple scarce resources and to account for other constraints that are not priced explicitly,
- Enabling market participants to **contest the prices** derived from this optimization by offering better terms, and
- Taking advantage of participants' superior **information** about local factors affecting scheduling and operations of their own plants.

Question 2: Can you identify the main **weaknesses of integrated systems**?

- In some cases prices are related vaguely to optimized shadow prices on scarce resources. $\longrightarrow$ The difference between the injection prices at two locations can be interpreted as the implied scarcity value of transmission between these locations, but it is only by solving a large set of equations that one might infer the implicit shadow prices on the transmission constraints enforced by the engineers. **In contrast**, unbundled systems are more explicit, and more important, every price can be contested by competing offers.

- Pricing is especially vulnerable to **incentive effects**. ⟶ Forward contracts can be difficult to be modelled and those could cause incomplete markets.
- Pricing is distorted whenever **optimization is imperfect**. ⟶ Lack of information on suppliers' costs could make prices uninformative.

Question 3: Model comparison. Under which circumstances **integrated systems perform better than unbundled systems**?

- When optimization to meet system constraints is more important than participants' flexibility, and
- Shadow prices on system constraints are more accurate measures of opportunity costs than clearing prices in markets.

1.4.2 Market Microstructure

Question 1: When the transmission line is congested, the dispatch in the market could follow two approaches. Could you explain the approach proposed in the **NordPool** and the approach proposed in **California**? Which are the main differences between both approaches? In the California market design, the suppliers play a game called the **dec game**; could you explain the logic behind that game and its economic implications?

- In the **NordPool** the SO raises the price charged in the importing zone for withdrawing power, and to reduce the price paid in the exporting zone for injecting power, until the net flow matches the available capacity; the difference between these two zonal energy prices is then the usage fee charged for flows from the exporting zone to the importing zone— and equal credit is given for counterflows. In effect, NordPool uses the inframarginal bids in the supply and demand functions submitted in each zone as offers to increment or decrement energy output.
- Zonal pricing in an unbundled system like **California**'s enables strategies like the following — called the dec game. A supplier who anticipates intrazonal congestion affecting his injection node can sell a quantity $3Q$ in the day-ahead energy market at its clearing price P when he knows that in real time the SO will be forced to invoke the dec he offers for the quantity $2Q$ at the spot price p^*, which is typically lower than P when decs are invoked, or at his bid price p, which is even lower (even negative) when his dec is invoked out of merit order.

  The net result is that the supplier collects a profit $[P - p^*]2Q$ or even $[P - p]2Q$ on the extra quantity $2Q$ that he knew initially he would not produce.

  The adverse consequences could be long-term if anticipated profits from the dec game induce an entrant to build a new plant in the most congested area, the opposite of what is required for efficiency.

Bibliography

Borenstein, S., 2002, "The Trouble with Electricity Markets: Understanding California's Restructuring Disaster," *Journal of Economic Perspectives,* 16, 1, 191-211.

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Wilson, R., 2002, "Architecture of Power Markets," *Econometrica,* 70, 4, 1299-1340.

Chapter 2

Electricity auction designs: Uniform and discriminatory price auctions

The aim of this chapter is to study the impact that the **structural parameters of the model** (production capacity and demand; but also, production costs, transmission capacity and transmission costs), and the **market design** (uniform and discriminatory price auction) have on the equilibrium in electricity auctions.

2.1 Theory

2.1.1 Set up and timing

Set up: There are two players with production capacity k_1 and k_2, where $k_1 > k_2$. The level of demand, θ is independent of market price, i.e., perfectly inelastic. Moreover, $\theta \in [k_2, k_1+k_2]$, i.e., the demand is large enough to guarantee that at least one player faces a positive residual demand.

Timing: Having observed the realization of demand θ, each player simultaneously and independently submits a bid specifying the minimum price at which it is willing to supply up to its capacity, $b_i \in [b_{min}, P]$, $i = 1, 2$, where b_{min} and P are determined by the auctioneer.[1] Let $b \equiv (b_1, b_2)$ denote a bid profile. On basis of this profile, the auctioneer calls players into operation. If players submit different bids, the capacity of the lower-bidding player is dispatched first. If the capacity of the lower-bidding player is not sufficient to satisfy total demand, the higher-bidding player's capacity, is then dispatched to serve residual demand. If the two players submit equal bids, they are dispatched in proportion to their production capacities.

The output allocated to supplier $i, i = 1, 2$, denoted by $q_i(b; \theta, k)$, is given by

$$q_i(b;\theta,k) = \begin{cases} \min\{\theta, k_i\} & \text{if } b_i < b_j \\ \dfrac{k_i}{k_i + k_j}\theta & \text{if } b_i = b_j \\ \max\{0, \theta - k_j\} & \text{if } b_i > b_j \end{cases} \tag{2.1}$$

Finally, the payments are worked out by the auctioneer. When the auctioneer runs a uniform price auction, the price received by a player for any positive quantity dispatched by the auctioneer is equal to the higher offer price accepted in the auction. Hence, for a given realization of demand θ and a bid profile $b \equiv (b_1, b_2)$, player i's profits, $i = 1, 2$, can be expressed as

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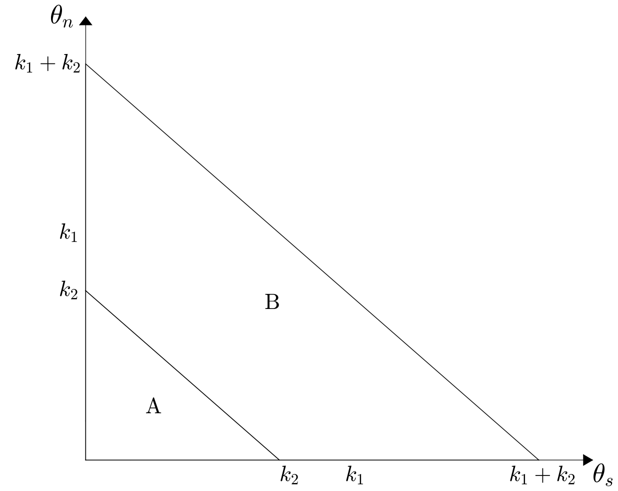
[1] In this section, we set $b_{min} = 0$, but it is important to notice that the setting the minimum bid has important consequences determining the equilibrium.

Figure 2.1: *Zero* production costs. Equilibrium areas



$$\pi_i^u(b;\theta,k) = \begin{cases} b_j \min\{\theta, k_i\} & \text{if } b_i < b_j \text{ and } \theta > k_i \\ b_i \dfrac{k_i}{k_i + k_j}\theta & \text{if } b_i = b_j \\ b_i \max\{0, \theta - k_j\} & \text{otherwise} \end{cases} \tag{2.2}$$

When the auctioneer runs a discriminatory price auction, the price received by a player for any positive quantity dispatched by the auctioneer is equal to its own offer price. Hence, for a given realization of demand θ and a bid profile $b \equiv (b_1, b_2)$, player i's profits, $i = 1, 2$, can be expressed as

$$\pi_i^d(b;\theta,k) = \begin{cases} b_i \min\{\theta, k_i\} & \text{if } b_i < b_j \\ b_i \dfrac{k_i}{k_i + k_j}\theta & \text{if } b_i = b_j \\ b_i \max\{0, \theta - k_j\} & \text{otherwise} \end{cases} \tag{2.3}$$

2.1.2 Uniform price auction. Equilibrium

Proposition 1. In the presence of production capacity constraints and *zero* production costs, the characterization of the equilibrium falls into one of the next two categories.

i Low demand (area A). An unique pure strategies exists where the suppliers submit a bid equal to their production costs.

ii High demand (area B). Multiplicity of pure strategies equilibrium exist where one of the suppliers submit the maximum bid allowed by the auctioneer and the other supplier submits a bid that make undercutting unprofitable.

Proof. When the demand is low (area A), both suppliers have enough production capacity to satisfy the demand. Therefore, they compete fiercely to be dispatched first in the auction by submitting a bid equal to their production cost ($b_i = b_j = c = 0\ \forall i, j$).

When the demand is high (area B), at least one of the suppliers face a positive residual demand, and it has incentives to satisfy the residual demand by submitting the maximum price allowed by the auctioneer. In that case, supplier i can guarantees for itself a profit equal to

$P(\theta_n - k_j)$, where P is the maximum price allowed by the auctioneer and $(\theta - k_j)$ is supplier i's residual demand. To make undercutting unprofitable, supplier j can submit a bid $\underline{b}_i$ such that $P(\theta_n - k_j) = \underline{b}_i k_i$. In that case supplier i don't have incentives to deviate, and supplier j neither, since it sells its total production capacity k_j at the maximum price allowed by the auctioneer. Therefore, $b_i = P; b_j \in [0, \underline{b}_i] = \left[0, \frac{P(\theta_n - k_j)}{k_i}\right] \forall i, j$ defines a Nash Equilibrium □

2.1.3 Discriminatory price auction. Equilibrium

Lemma 1. In a discriminatory price auction with production capacity constrained and zero production costs suppliers, when the demand is low (area A), the equilibrium is in pure strategies, when the demand is high (area B), a pure strategies equilibrium does not exist (figure 2.1).

Proof. When the demand is low (area A), both suppliers have enough capacity to satisfy total demand in both markets. Therefore, they compete fiercely to be dispatched first in the auction. Hence, the equilibrium is the typical Bertrand equilibrium where both suppliers submit bids equal to their marginal cost.

When the demand is high (area B), at least one of the suppliers faces a positive residual demand. Therefore, a pure strategies equilibrium does not exist. First, an equilibrium such that $b_i = b_j = c$ does not exist because at least one supplier has the incentive to increase its bid and satisfy the residual demand. Second, an equilibrium such that $b_i = b_j > c$ does not exist because at least one supplier has the incentive to undercut the other to be dispatched first. Finally, an equilibrium such that $b_j > b_i > c$ does not exist because supplier i has the incentive to shade the bid submitted by supplier j. □

A pure strategies equilibrium does not exist when the demand is intermediate or high. However, the model satisfies the properties established by Dasgupta and Maskin (1986) which guarantee that a mixed strategies equilibrium exists.

Lemma 2. In a discriminatory price auction with production capacity constrained and zero production costs suppliers, in a mixed strategies equilibrium, no supplier submits a bid lower than bid $(\underline{b}_i)$ such that $\underline{b}_i k_i = P(\theta - k_j)$. Moreover, the support of the mixed strategies equilibrium for both suppliers is $S = [\max\{\underline{b}_1, \underline{b}_2\}, P]$.

Proof. Given that the demand is inelastic, the supplier's profit is maximized when it sets the reservation price. Therefore, the reservation price is the upper-bound of the support.

Each supplier can guarantee for itself the payoff $P(\theta - k_j)$, since each supplier can always submit the highest bid and satisfy the residual demand. Therefore, in a mixed strategy equilibrium, no supplier submits a bid that generates a payoff equilibrium lower than $P(\theta - k_j)$. Hence, no supplier submits a bid lower than $\underline{b}_i$, where $\underline{b}_i$ solves $\underline{b}_i k_i = P(\theta - k_j)$.

No supplier can rationalize submitting a bid lower than $\underline{b}_i, i = 1, 2$. In the case when $\underline{b}_i = \underline{b}_j$, the support is symmetric. In the case when $\underline{b}_i < \underline{b}_j$, supplier i knows that supplier j never submits a bid lower than $\underline{b}_j$. Therefore, in a mixed strategy equilibrium, supplier i never submits a bid b_i such that $b_i \in (\underline{b}_i, \underline{b}_j)$, because supplier i can increase its expected payoff choosing a bid b_i such that $b_i \in [\underline{b}_j, P]$. Hence, the equilibrium strategy support for both suppliers is $S = [\max\{\underline{b}_1, \underline{b}_2\}, P]$ □

Using Lemmas one and two, I characterize the equilibrium.

Proposition 2. In a discriminatory price auction with production capacity constrained and zero production costs suppliers, the characterization of the equilibrium falls into one of the next two categories.

i Low demand (area A). The equilibrium strategies pair is in pure strategies.

ii High demand (area B). The equilibrium strategies pair is in mixed strategies.

Proof. When the demand is low (area A), suppliers compete fiercely to be dispatched first in the auction and the equilibrium is the typical Bertrand equilibrium in which both suppliers submit bids equal to their marginal cost.

When the demand is high (area B), the equilibrium is in mixed strategies. The support of the mixed strategies equilibrium is defined by lemma 2. The cumulative distribution function is worked out following the next steps:

First, the profit function for a given bid b is defined by

$$\pi_1(b) \quad = \quad b\,[F_2(b)(\theta - k_2) + (1 - F_2(b))(k_1)] \tag{2.4}$$

where, $F_2(b)$ is the probability that supplier 2's bid is lower than supplier 1's bid. Therefore, with probability $F_2(b)$ supplier 1 is dispatched last and it satisfies the residual demand. Therefore, in that case, supplier 1's profits are $\pi_1(b) = F_2(b)b(\theta - k_2)$. With probability $(1 - F_2(b))$ supplier 2 is dispatched first, and it satisfies the total demand. Therefore, in that case, supplier 1's profits are $\pi_1(b) = (1 - F_2(b))b(\theta - k_2)$.

By doing some algebra in equation 2.4, we obtain:

$$\begin{aligned} \pi_1(b) &= -bF_2(b)\,[(k_1) - (\theta - k_2)] + b(k_1) \Rightarrow \\ F_2(b) &= \frac{b(k_1) - \pi_1(b)}{b((k_1) + (\theta - k_2)} \end{aligned} \tag{2.5}$$

Second, in the lower bound the support, the value of the cumulative distribution function is zero. Otherwise, one supplier can undercut the other and it increases its profit. Therefore, in the lower bound of the support equation 2.5 becomes $\pi_1(\underline{b}) = \underline{b}(k_1)$.

Third, the profit for any bid in the support is the same. Otherwise, the suppliers increases its profit by reassigning probabilities. Therefore, $\pi_1(b) = \pi_1(\underline{b}) \forall b \in [\underline{b}, P]$. Therefore, equation 2.5 can be rewritten as:

$$F_2(b) \quad = \quad \frac{b(k_1) - \underline{b}(k_1)}{b((k_1) + (\theta - k_2)} = \frac{(k_1)}{(k_1) - (\theta - k_2)} \frac{(b - \underline{b})}{b} \tag{2.6}$$

Following the same approach it easy to derive the cumulative distribution function for supplier 2.

2.2 Exercises

2.2.1 Exercise 1. Nash equilibrium in uniform and discriminatory price auctions

In an electricity market, there are two suppliers, where supplier 1's production capacity is $k_1 = 8.7$, and supplier 2's production capacity is $k_2 = 6.5$. The demand is $\theta = 10$ and the maximum price allowed by the auctioneer is $\overline{P} = 10$.

1. Uniform price auction I

Question 1. Is the pair of strategies $(b_1 = 9, b_2 = 1)$ a pure strategies Nash equilibrium?

A pair of strategies is a Nash equilibrium if none of the players has incentives to deviate unilaterally. Therefore, it is necessary to study suppliers' profit functions and analyze if some of them has incentive to deviate.

$$\begin{aligned} \pi_1(b_1 = 9, b_2 = 1) &= b_1(\theta - k_2) = 9(10 - 6.5) = 31.5 \\ \pi_2(b_1 = 9, b_2 = 1) &= b_1 k_2 = 9(6.5) = 58.5 \end{aligned}$$

Supplier 1 has two possible options in case that it deviates: First, it can undercut its rival, and sell her entire production capacity. Second, it can raise her bid to the maximum possible bid allowed by the auctioneer.

If supplier 1 undercuts supplier 2, her profits are defined by:

$$\pi_1(b_1 = 1, b_2 = 1) = b_1(k_1) = 1(8.7) < \pi_1(b_1 = 9, b_2 = 1) = 31.5.$$

Therefore, supplier 1 does not want to undercut supplier 2. Hence, we could have a possible candidate to be a Nash Equilibrium. However, we still have to verify that supplier 1 does not want to raises her bid. If supplier 1 raises her bid, her profits are defined by:

$$\pi_1(b_1 = 10, b_2 = 1) = b_1(\theta - k_2) = 10(10 - 6.5) = 35 > \pi_1(b_1 = 9, b_2 = 1) = 31.5.$$

Therefore, supplier 1 has incentives to deviate. Hence, the pair of strategies $(b_1 = 9, b_2 = 1)$ cannot be a pure strategies Nash equilibrium.

Check that supplier 2 does not have incentives to deviate.

Question 2. Can you modify one of the bids in the pair $(b_1 = 9, b_2 = 1)$ to obtain a pure strategies Nash equilibrium?

If supplier 1 raises her bid from 9 to 10, it increases her profits from 31.5 to 35. Moreover, supplier 1 does not have incentives to undercut supplier 2, since by doing that her profits decrease from 31.5 to 8.7. Therefore, given that supplier 2 has submitted a bid equal to 1, the best strategy of supplier 1 is to submit a bid equal to 10. Therefore, the pair of strategies $(b_1 = 10, b_2 = 1)$ could be a potential pure strategies Nash equilibrium.

To guarantee that it is a pure strategies Nash equilibrium, we have to check that supplier 2 has no incentives to deviate. Supplier 2's profits are defined by:

$$\pi_2(b_1 = 10, b_2 = 1) = b_1(k_2) = 10(6.5) = 65.$$

Supplier 2 cannot increases her profits, since it is selling her entire production capacity at the maximum price allowed by the auctioneer.

Therefore, we have found a Nash equilibrium in which supplier 1 submits a bid equal to 10, and supplier 2 submits a bid equal to 1.

Question 3. Which is the minimum bid that player 2 can submit to sustain the equilibrium in which player 1 submits the maximum bid allowed by the auctioneer?

In the two previous questions, we have analyzed if a pair of strategies is a Nash equilibrium. In this question, we try to find a general formula that guarantees the existence of a Nash equilibrium.

If supplier 1 submits the maximum bid allowed by the auctioneer, it only can deviate by undercutting supplier 2. Which is the minimum bid that supplier 2 has to submit to make undercutting unprofitable? This is the key question to find a pure strategies equilibrium in which supplier 1 submits the maximum bid allowed by the auctioneer. When supplier 1 submits the maximum bid allowed by the auctioneer, her profits are defined by: $P(\theta - k_2)$. Therefore, supplier 2 has to submit a bid that makes undercutting unprofitable. That bid is defined implicitly by:

$$P(\theta - k_2) = \underline{b}_1 k_1.$$

And implicitly by:

$$\underline{b}_1 = \frac{P(\theta - k_2)}{k_1} = \frac{10(10 - 6.5)}{8.7} = 4.$$

Question 4. Define the set of strategies for which supplier 1 submits the maximum bid allowed by the auctioneer.

The pair of strategies that define that equilibrium is:

$$b_1 = P;\; b_2 \in [0, \underline{b}_1]$$

Question 5. Find graphically that equilibrium, and draw supplier 1's profits and supplier 2' profits.

A pair of strategies that sustains that equilibrium, and suppliers' equilibrium profits are in figure 2.2.

2. Uniform price auction II

In the previous subsection, we studied step by step the way to find the set of pure strategies Nash equilibria in which the supplier with higher production capacity (supplier 1) submits the maximum bid allowed by the auctioneer. In this subsection, we proceed in the same way to find the set of pure strategies Nash equilbria in which the supplier with lower production capacity (supplier 2) submits the maximum bid allowed by the auctioneer.

Question 1. Is the pair of strategies ($b_1 = 1, b_2 = 8$) a pure strategies Nash equilibrium?

A pair of strategies is a Nash equilibrium if none of the players has incentives to deviate unilaterally. Therefore, it is necessary to study suppliers' profit functions and analyze if some of them has incentives to deviate.

$$\begin{aligned}
\pi_1(b_1 = 1, b_2 = 8) &= b_2 k_1 = 8(8.7) = 69.6 \\
\pi_2(b_1 = 1, b_2 = 8) &= b_2(\theta - k_2) = 8(10 - 8.7) = 10.4
\end{aligned}$$

Supplier 2 has two possible options in case that it deviates: First, it can undercut supplier 1to sell her entire production capacity. Second, it can raise her bid to the maximum possible bid allowed by the auctioneer.

If supplier 2 undercuts supplier 1, her profits are defined by:

$$\pi_2(b_1 = 1, b_2 = 1) = b_1(k_2) = 1(6.5) < \pi_2(b_1 = 1, b_2 = 8) = 10.4.$$

Therefore, supplier 2 does not want to undercut supplier 1, and we could have a possible candidate for a Nash equilibrium. However, we still have to verify that supplier 2 does not want to raises her bid. If supplier 2 raises her bid, her profits are defined by:

$$\pi_2(b_1 = 1, b_2 = 10) = b_2(\theta - k_1) = 10(10 - 8.7) = 13 > \pi_1(b_1 = 1, b_2 = 8) = 10.4.$$

Therefore, supplier 2 has incentives to deviate, and the pair of strategies $(b_1 = 1, b_2 = 8)$ cannot be a pure strategies Nash equilibrium.

Check that supplier 1 does not have incentives to deviate.

Question 2. Can you modify one of the bids in the pair $(b_1 = 1, b_2 = 8)$ to obtain a pure strategies Nash equilibrium?

If supplier 2 raises her bid from 8 to 10, it increases her profits from 10.4 to 13. Moreover, supplier 2 does not have incentives to undercut supplier 1, since by doing that her profits decrease from 10.4 to 6.5. Therefore, given that supplier 1 has submitted a bid equal to 1, the best strategy for supplier 2 is to submit a bid equal to 10. Therefore, the pair of strategies $(b_1 = 1, b_2 = 10)$ could be a potential pure strategies Nash equilibrium.

To guarantee that it is a pure strategies Nash equilibrium, we have to check that supplier 1 has no incentives to deviate. Supplier 1' profits are defined by:

$$\pi_1(b_1 = 1, b_2 = 10) = b_2(k_1) = 10(8.7) = 87.$$

Supplier 1 cannot increase her profits, since it is selling her entire production capacity at the maximum price allowed by the auctioneer.

Therefore, we have found a Nash equilibrium in which supplier 1 submit a bid equal to 1, and supplier 2 submits a bid equal to 10.

Question 3. Which is the minimum bid that player 1 can submit to sustain the equilibrium in which player 2 submits the maximum bid allowed by the auctioneer?

In the two previous questions, we have analyzed when a pair of strategies is or it is not a pure strategies Nash equilibrium. In this question, we try to find a general formula that guarantees the existence of a Nash equilibrium.

If supplier 2 submits the maximum bid allowed by the auctioneer, it only can deviate by undercutting supplier 1. Which is the minimum bid that supplier 1 has to submit to make undercutting unprofitable? This is the key question to find a pure strategies equilibrium in which supplier 2 submits the maximum bid allowed by the auctioneer. When supplier 2 submits the maximum bid allowed by the auctioneer, her profits are defined by: $P(\theta - k_1)$. Therefore, supplier 1 has to submit a bid that makes undercutting unprofitable. That bid is defined implicitly by:

$$P(\theta - k_1) = \underline{b}_2 k_2.$$

And implicitly by:

$$\underline{b}_2 = \frac{P(\theta - k_1)}{k_2} = \frac{10(10 - 8.7)}{6.5} = 2.$$

Question 4. Define the set of strategies for which supplier 2 submits the maximum bid allowed by the auctioneer.

The pair of strategies that define that equilibrium is:

$$b_1 \in [0, \underline{b}_2];\ b_2 = P.$$

Question 5. Find graphically that equilibrium, and draw supplier 1's profits and supplier 2' profits.

A pair of strategies that sustains that equilibrium, and suppliers' equilibrium profits are in figure 2.2.

3. Discriminatory price auction

In the previous two sections, we have studied the uniform price auction, and we have found two interesting thresholds: $\underline{b}_1$, and $\underline{b}_2$, where $\underline{b}_1 = \frac{P(\theta - k_2)}{k_1} = \frac{10(10 - 6.5)}{8.7} = 4$, and $\underline{b}_2 = \frac{P(\theta - k_1)}{k_2} = \frac{10(10 - 8.7)}{6.5} = 2$. By using that information, we will characterize the equilibrium in a discriminatory price auction.

Question 1. Can be a pure strategies Nash equilibrium in the bid interval $b \in [0, min\{\underline{b}_1, \underline{b}_2\}]$?

It is not possible to sustain a pure strategies Nash equilibrium in that interval, since both suppliers can increase their profits by submitting the maximum bid allowed by the auctioneer. Therefore, if $b < min\{\underline{b}_1, \underline{b}_2\}$, both suppliers have incentives to deviate by submitting the maximum bid allowed by the auctioneer.

Question 2. Can be a pure strategies Nash equilibrium in the bid interval $b \in [min\{\underline{b}_1, \underline{b}_2\}, max\{\underline{b}_1, \underline{b}_2\}]$?

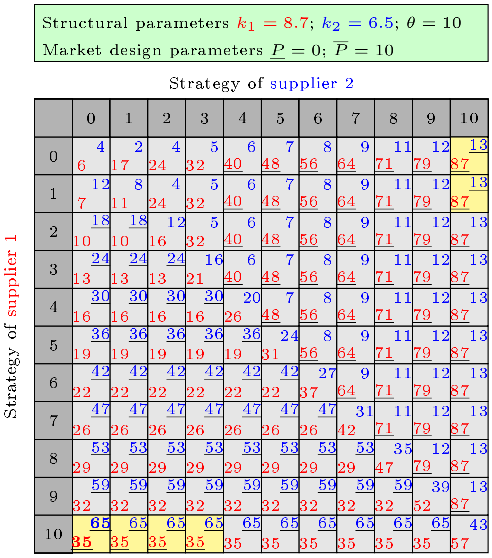
It is not possible to sustain a pure strategies Nash equilibrium in that interval, since the supplier with higher production capacity (supplier 1) can increase her profits by submitting the maximum bid allowed by the auctioneer. Therefore, if $min\{\underline{b}_1, \underline{b}_2\} \leq b \leq max\{\underline{b}_1, \underline{b}_2\}$, the supplier for which $\underline{b}$ is larger has incentives to deviate by submitting the maximum bid allowed by the auctioneer.

Question 3. Can be a pure strategies Nash equilibrium in the bid interval $b \in [max\{\underline{b}_1, \underline{b}_2\}, P]$?

No, since both suppliers have incentives to start a price war undercutting each other until they reach $max\{\underline{b}_1, \underline{b}_2\}$, where the supplier with higher $\underline{b}$ has incentives to deviate by submitting the maximum bid allowed by the auctioneer. At that bid, the price war starts again generating a circle in suppliers' strategies.

Question 4. In questions 1, 2 and 3, we have shown that when the auction is discriminatory, a pure strategies equilibrium does not exist. Can you define the bid interval in which the suppliers will randomize?

The suppliers never submit a bid lower than $b \leq min\{\underline{b}_1, \underline{b}_2\}$, since both suppliers can submit the maximum bid allowed by the auctioneer with probability one to increase their profits.



Structural parameters $k_1 = 8.7$; $k_2 = 6.5$; $\theta = 10$

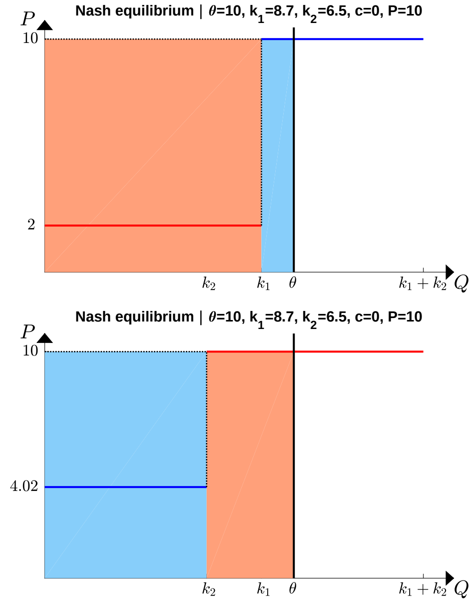
Market design parameters $\underline{P} = 0$; $\overline{P} = 10$

Strategy of supplier 2

Strategy of supplier 1

	0	1	2	3	4	5	6	7	8	9	10
0	4 6	2 17	4 24	5 32	6 40	7 48	8 56	9 64	11 71	12 79	13 87
1	12 7	8 11	4 24	5 32	6 40	7 48	8 56	9 64	11 71	12 79	13 87
2	18 10	18 10	12 16	5 32	6 40	7 48	8 56	9 64	11 71	12 79	13 87
3	24 13	24 13	24 13	16 21	6 40	7 48	8 56	9 64	11 71	12 79	13 87
4	30 16	30 16	30 16	30 16	20 26	7 48	8 56	9 64	11 71	12 79	13 87
5	36 19	36 19	36 19	36 19	36 19	24 31	8 56	9 64	11 71	12 79	13 87
6	42 22	42 22	42 22	42 22	42 22	42 22	27 37	9 64	11 71	12 79	13 87
7	47 26	47 26	47 26	47 26	47 26	47 26	47 26	31 42	11 71	12 79	13 87
8	53 29	53 29	53 29	53 29	53 29	53 29	53 29	53 29	35 47	12 79	13 87
9	59 32	59 32	59 32	59 32	59 32	59 32	59 32	59 32	59 32	39 52	13 87
10	**65** **35**	65 35	65 35	65 35	65 35	65 35	65 35	65 35	65 35	65 35	43 57

Figure 2.2: Nash equilibrium



The suppliers never submit a bid in the interval $min \{\underline{b}_1, \underline{b}_2\} \leq b \leq max \{\underline{b}_1, \underline{b}_2\}$, since the supplier with higher production capacity can submit the maximum bid allowed by the auctioneer with probability one to increase her profits. The supplier with lower production capacity knows that, and it never submits a bid lower than $\underline{b}_1$, since it can submit a bid an ϵ lower than $\underline{b}_1$ with probability one to increase her profits.

Therefore, both suppliers will randomize in the interval $b \in [max \{\underline{b}_1, \underline{b}_2\}, P]$.

Question 5. Find the probability that the suppliers assign to each bid in the support defined in question 4.

The probability distribution function has been calculated in the theory section. The cumulative distribution functions for each supplier are in figure 2.4.

2.2.2 Exercise 2. Uniform Price Auction

In an electricity market, there are two suppliers where supplier 1's production capacity is $k_1 = 8.7$, and supplier 2's production capacity is $k_2 = 6.5$, the demand is $\theta = 10$ and the maximum price allowed by the auctioneer is $\overline{P} = 10$.

Question 1: Find the two sets of pure Nash equilibrium if the auctioneer organize a uniform price auction.

In that case there are multiplicity of pure Nash strategies equilibrium. The first set of Nash equilibria is defined by $b_1 = P; b_2 \in [0, \underline{b}_1] = \left[0, \frac{10(10-6.5)}{8.7}\right] = [0, 4.02]$. The second set of Nash equilibria is defined by $b_1 = \in [0, \underline{b}_2] = \left[0, \frac{10(10-8.7)}{6.5}\right] = [0, 2]$.

Question 2: Assume that the players can submit only 11 bids equally split between 1 and 10, and that they have to offer their entire production capacity at that bid.

If the auction is uniform, build a matrix with those 11 bids. For each pair of bids, work out suppliers' payoffs. Find the Nash equilibrium in the uniform price auction using that payoff matrix.

Those two sets of Nash equilibria are highlighted in yellow in the matrix below.[2]

Question 3: By using a merit order curve, draw the combination of bids that define the two sets of pure Nash equilibria.

Those two sets of equilibria can also be represented using merit order curves (figure 2.2). The top graph in figure 2.2 represents the equilibrium in which supplier 2 submits the maximum price allowed by the auctioneer and supplier 1 submits a bid that makes undercutting unprofitable. The bottom graph in figure 2.2 represents the equilibrium in which supplier 1 submits the maximum price allowed by the auctioneer and supplier 2 submits a bid that makes undercutting unprofitable. As can be observed in the payoff matrix and in figure 2.2, supplier 1 prefers the equilibrium in which it submits the lower bid, and supplier 2 prefers the equilibrium in which it submits the lower bid.

Therefore, the relevant question is which supplier will submit the larger bid allowed by the auctioneer and which supplier will free ride? To address that question some equilibrium selection techniques can be applied: The risk dominance method proposed by Harsanyi and Selten (1988), the robustness to strategic uncertainty method proposed by Andersson, Argenton, and Weibull (2014), and the quantal response method proposed by McKelvey and Palfrey (1998). We will study those methods in the chapter that studies the equilibrium selection in uniform price auctions (hawk-dove games).

2.2.3 Exercise 3. Discriminatory Price Auction

In an electricity market, there are two suppliers where supplier 1's production capacity is $k_1 = 8.7$, and supplier 2's production capacity is $k_2 = 6.5$, the demand is $\theta = 10$ and the maximum price allowed by the auctioneer is $\overline{P} = 10$.

Question 1: Assume that the players can submit only 11 bids equally split between 1 and 10, and that they have to offer their entire production capacity at that bid. If the auction is discriminatory, build a matrix with those 11 bids. For each pair of bids, work out suppliers' payoffs. By using that matrix, can do you find a pure strategies Nash equilibrium?

As can be observed in the payoff matrix above, when the auction is discriminatory it doesn't exist a pure strategies equilibrium. When the suppliers compete in prices, they end up in a price war in which they undercut each other until arrive to a price low enough that makes undercutting unprofitable. At that price, one of the suppliers has incentive to deviate by submitting the maximum bid allowed by the auctioneer. Then, the suppliers start a price war again. Therefore, the suppliers play their strategies following circles.

Question 2: By using a merit order curve, draw any combination of bids and justify

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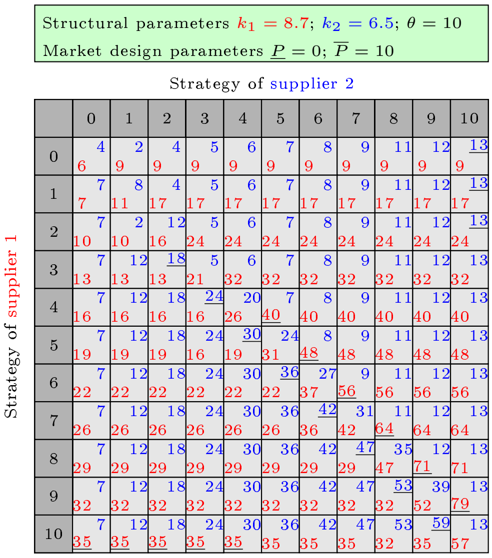
[2] It is important to notice that the set of strategies in that matrix are index and they do not correspond to the real strategies. We use indexes instead of real strategies to avoid the use of decimals and also because this matrix will be the one implemented in the experiment that we will play later, and it is important to keep the payoff matrix as "neutral" as possible. Therefore, the index 1 corresponds to the strategy 1, the index 1 corresponds to the strategy 1.9, the index 2 to the strategy 2.8, and so on until index 10 that corresponds to the strategy 10.



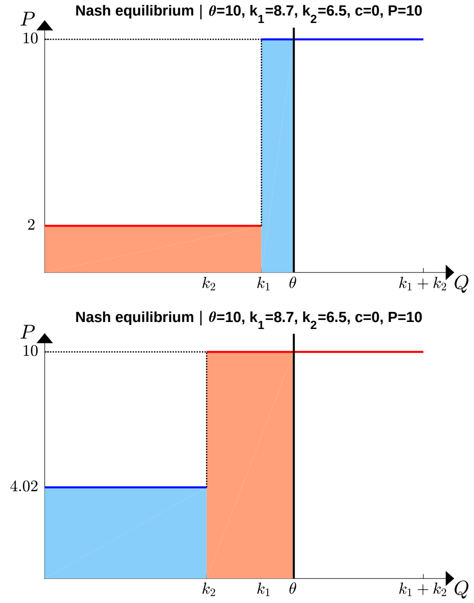
Structural parameters $k_1 = 8.7$; $k_2 = 6.5$; $\theta = 10$
Market design parameters $\underline{P} = 0$; $\overline{P} = 10$

Strategy of supplier 2

Strategy of supplier 1

	0	1	2	3	4	5	6	7	8	9	10
0	4 6	2 9	4 9	5 9	6 9	7 9	8 9	9 9	11 9	12 9	13 9
1	7 7	8 11	4 17	5 17	6 17	7 17	8 17	9 17	11 17	12 17	13 17
2	7 10	2 10	12 16	5 24	6 24	7 24	8 24	9 24	11 24	12 24	13 24
3	7 13	12 13	18 13	5 21	6 32	7 32	8 32	9 32	11 32	12 32	13 32
4	7 16	12 16	18 16	24 16	20 26	7 40	8 40	9 40	11 40	12 40	13 40
5	7 19	12 19	18 19	24 16	30 19	24 31	8 48	9 48	11 48	12 48	13 48
6	7 22	12 22	18 22	24 22	30 22	36 22	27 37	9 56	11 56	12 56	13 56
7	7 26	12 26	18 26	24 26	30 26	36 26	42 36	31 42	11 64	12 64	13 64
8	7 29	12 29	18 29	24 29	30 29	36 29	42 29	47 29	35 47	12 71	13 71
9	7 32	12 32	18 32	24 32	30 32	36 32	42 32	47 32	53 32	39 52	13 79
10	7 35	12 35	18 35	24 35	30 35	36 35	42 35	47 35	53 32	59 35	13 57

Figure 2.3: (Non-existence) Nash equilibrium



the existence (or not existence) of a pure Nash equilibria.

The non-existence of a Nash equilibrium can be also observed in figure 2.3, where any of the suppliers that submits the lower bid can increase its profit by undercutting the supplier that submits the maximum bid allowed by the auctioneer.[3]

Question 3: By using equation 2.6 draw the cumulative distribution functions for suppliers 1 and 2 that define the mixed strategies equilibrium.

As we have proved above, in a discriminatory price auction a pure strategies doesn't exist. However, the auction satisfies the properties established by Dasgupta and Maskin (1986) which guarantee that a mixed strategies equilibrium exists. Figure 2.4 represents the mixed strategies equilibrium for both suppliers. As can be observed, the supplier with higher production capacity (and higher residual demand) is the one that submits higher bids with higher probability.

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(Paper that has been quoted, but it is not necessary to read).

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(This paper is very useful to understand the discriminatory price auction).

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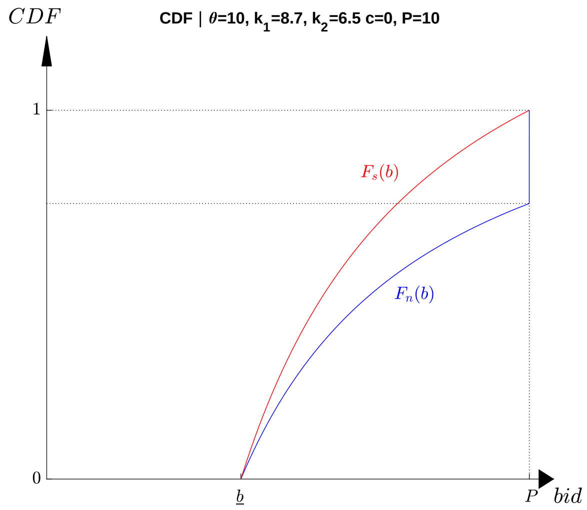
[3] For a complete proof of the non-existence of a pure Nash equilibrium check lemma 1 in the theory section.

Figure 2.4: Mixed strategies equilibrium



(∗) Fabra, N., von der Fehr N. H. and Harbord D., 2006, "Designing Electricity Auctions," *Rand Journal of Economics*, 37, 23-46.
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Harsanyi, J., and Selten R., 1988, "A General Theory of Equilibrium Selection in Games," *The MIT Press, Cambridge, MA*.
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(This paper is very useful to understand the discriminatory price auction).

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(Paper that has been quoted, but it is not necessary to read).

Chapter 3

Transmission

The purpose of this chapter is to present some of the key aspects in the integration process in Europe. It also introduces two relevant elements in the design of electricity auctions: the design of transmission tariffs and the delineation of bidding zones.

In the first three sections, we study some of the main documents that study the design of electricity markets in Europe. In the fourth section, we present a theoretical model to characterize the equilibrium in electricity markets in the presence of transmission constraints and transmission tariffs. We conclude the chapter with an exercise.

3.1 Integration of electricity markets in Europe. The role of the system operator.

This section is based on the next document:

European Commission, 2015, "Options for Future European Electricity System Operation."
($*$) Only the first 11th pages.

The European power sector is undergoing important changes. Especially the increasing penetration of renewable energy sources (RES), as part of the transition to a de-carbonised power system, results in a need to continuously assess and decide upon (the adoption of) alternative technologies, policies and practices. We study the main points presented in European Commission (2015) to improve the design of electricity markets.

The main **goals** that should lead any proposed changes to **system operations** and **planning**:

1. Security of supply (secure for everybody)
2. Market facilitation (affordable and competitive pricing)
3. Integration of RES (environmentally sustainable).

The **system planning** and **operations** that should be taken into account in the new regulation scenario are the next ones:

1. **Long-term network planning** (years),
2. **System operation before real-time** (months, day-ahead, intra-day),
3. **Real-time system operation** ($<$ 15 minutes)

According with the European Commission, the long-term planning and system operator before real time operations should be centralized. The **benefits from centralization** are mainly related to:

- Network planning,
- and system operations functions such as capacity calculation, congestion management, adequacy assessment and balancing.

To facilitate the **governance** of the system operator in the integration of electricity markets in Europe, the European commission proposed the next parties which will assume different roles:

- The **European Commission** to formulate general energy policy and directives
- **European regulatory body** (current ACER) with the power to independently check the formulation and execution of methodologies, processes and procedures in line with the general policy
- **Regional Operation Centres** (ROC) to execute prescribed tasks according to the formulated methodologies, processes and procedures; responsible for execution
- European entity (current **ENTSO-E**) for development and implementation of methods and tools for LT planning and SO. In consultation with ACER (who sets up guidelines by request of the EC) this body develops the framework (e.g. grid codes) for execution of the tasks by ROCs and ensures overall alignment between them, and with national TSOs.

The European entity for the development and implementation of methods and tools (current ENTSO-E) is responsible for development of the way of working of the foreseen ROCs in line with guidelines and/ or regulation. This is then monitored and enforced by the regulatory body (current ACER).

The European Commission propose the next **geographical partition** for TSO coordination (figure 3.1):

- CWE+CEE,
- Nordic + Baltics,
- UK + Ireland,
- Iberia and
- Italy + SEE.

3.2 Transmission tariffs

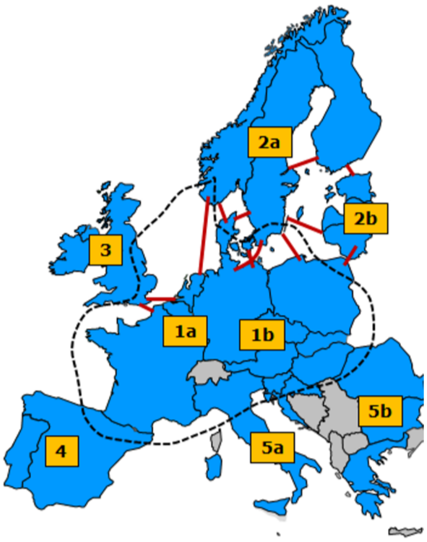
This section is based on the next documents:

ENTSO-E, 2016, "ENTSO-E ITC Overview of Transmission Tariffs in Europe: Synthesis 2014." (∗) Only the first 21st pages.

Nord Pool, 2007, "TSO Congestion Rent. How to Calculate the Congestion Rents."

Nord Pool, 2010, "Point Tariff System."

Figure 3.1: Geographical partition for TSO coordination



In the presence of transmission constraints the equilibrium prices differ across markets generating **congestion rents**.[1] In a **perfect competition scenario**, the congestion rents are enough to finance the investments in transmission capacity. However, in the presence of **uncertainty** or **lumpy investments** it is necessary to introduce tariffs to finance the investments in transmission capacity.

In the majority of the European countries, the tariff structure is based on a **point of connection tariff system** (ENTSO-E, 2016; Nord Pool, 2010). The **users of the grid** are charged for injection or outtake of electricity at a connection point in the transmission grid. The point of connection tariff consists of two parts, a **power charge** and an **energy charge**.

The **power charge** covers costs for expansion, operation and maintenance of the transmission grid. It is based on annual capacity subscription for injection and outtake of electricity at each connection point. The cost the subscriber has to pay is the product of the annual capacity subscription and the power charge in the connection point.

The **energy charge** is based on the transmission losses in the transmission grid caused by injection and outtake of electricity in different connection points. It is dependent on how the generation or load are distributed in the grid.

The power charge and the energy charge have a **geographical** (latitude, north/south) and a **time** component (day/night, winter/spring, peak/off-peak) that provide a long-term locational signal on where it is optimal to add generation and load capacity from a grid perspective.[2]

3.3 Bidding zones

This section is based on the next document:

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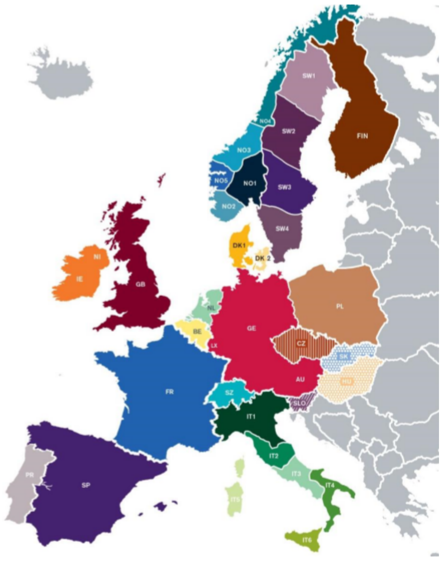
[1] The congestion rents are derived from the possibility to buy electricity in the cheap market and to sell it in the expensive market. Nord Pool (2007) explains the algorithm to work out the congestion rents. It also explains that in the Nord Pool, the TSO uses those rents to finance the investments in transmission lines (Finland and Sweden), or to reduce the price charged to consumers (Norway and Denmark).

[2] For a complete review of the tariff system in Europe and comparisons among countries see ENTSO-E (2016).

Figure 3.2: Bidding zones in Europe



Ofgem, 2014, "Bidding Zones Literature Review."

A bidding zone is the largest geographical area within which market participants are able to **exchange energy without capacity allocation**.

Bidding zones in Europe are currently defined according to differing **criteria** (figure 3.2).

- The majority are defined by national borders (eg, France or the Netherlands);
- however, some are larger than national borders (eg, Austria, Germany and Luxembourg or the Single Electricity Market for the island of Ireland)
- and some are smaller zones within individual countries (eg, Italy, Norway or Sweden).

How bidding zones can be delineated? Delineating bidding zones according to the location of network constraints may be undertaken in a number of ways.

- Nodal pricing: The equilibrium price across markets differs when the transmission line is congested.
- Zonal pricing: The equilibrium price across markets is the same even when the transmission line is congested.

Why does bidding zone configuration matter?

- An optimal delineation of bidding zones should promote robust price signals for **efficient short-term utilisation**
- and **long-term development** of the power system,
- whilst at the same time **limiting system costs**, including balancing costs and re-dispatch actions undertaken by TSOs.

The delineation of bidding zones has important impacts on **market efficiency**, **liquidity**, issues with **market power**, **investment** signals for new generation, **distributional** impacts and the cross border **flows**. We analyse one by one those impacts.

1. **Impact on efficient use of the network.** The configuration of bidding zones has important implications for system operation, providing **short run signals** to users of the network that impact on the utilisation of available capacity and ultimately the overall efficiency of the system. These short run signals also have a **long-term impact**, influencing the long-term investment decisions of market players.

    - Delineating bidding zones according to network constraints would allow these constraints to be managed by capacity allocation rather than re-dispatch (ie, ex- post modifications of generation schedules undertaken by the SO), **lowering constraint management costs for the SO**.
    - In export-constrained regions, the average wholesale price of electricity is likely to fall. Conversely, the average wholesale price of electricity is likely to rise in those areas at the other side of the constraint, where demand is higher. There is therefore a **distributional impact** to be considered through any reconfiguration of bidding zones.

2. **Impact on market liquidity and hedging.** The conventionally perceived impact on market liquidity arising from the configuration of bidding zones is that of **falling levels of liquidity as the number of zones increases**. This is a direct result of the smaller size of the markets, with fewer market players and as such a lower level of churn.

    Lower liquidity consequences:

    - Lower liquidity could mean a less clear indication of the **future value of power** from the market, which adds a layer of risk which could lead to inefficient investment, or efficient investment not taking place.
    - A fall in liquidity could also mean an increase in the **cost of risk**, due to lack of trading partners; this could well have a knock on effect on investment.

3. **Impact on investment.** The configuration of bidding zones is crucial to provide long-run signals that may affect investment decisions. The more the bidding zones configuration reflects the **physical network constraints**, the greater the efficiency of the price signals for cross-zonal network development and the price signals for generation and load investments.

    There are many **practical considerations** that should take into account to guarantee the correct investment decisions:

    - **Lumpiness** and **economies of scale** of transmission investments.
    - **Uncertainties** about future generation investments and demand growth.
    - Difficulty of decentralising charges for **reliability** and quality of service.
    - Transmission charges.

4. **Impact on market power.** The precise impact of the number of bidding zones on market power is unclear in the literature.

    - Fewer, larger bidding zones imply a large number of market players in any market and as such **greater competition** and liquidity.
    - Larger bidding zones **may create potential market power** in re-dispatch markets, if it is assumed that larger bidding zones implies greater need for managing congestion through re-dispatch.

5. **Impact on cross-border flows.**

    - If bidding zones are not delineated according to network constraints, there is an inherent **risk to the efficiency of power flows across borders**.
    - Prices in zones that are delineated according to network congestions are more reflective of local conditions whereas larger zones that suffer from internal network congestion do not tend to accurately reflect local conditions in their uniform wholesale prices, potentially resulting in sub-optimal interconnector flows.
    - **Optimal interconnector usage** between countries would be more likely if both sides of the interconnector use zonal pricing, as long as the zones are configured according to network constraints.

3.4 Theory. Electricity Auctions in the Presence of Transmission Constraints and Transmission Costs

This section is based on the paper:

Blázquez, M., 2018, "Electricity Auctions in the Presence of Transmission Constraints and Transmission Costs," *Energy Economics,* 74, 605-627.

3.4.1 Set up of the model

There exist two electricity markets, market North and market South, that are connected by a transmission line with capacity T. When suppliers transmit electricity through the grid from one market to the other, they face a symmetric and linear transmission tariff t.

There exist two duopolists with capacities k_n and k_s, where subscript n means that the supplier is located in market North and subscript s means that the supplier is located in market South. The suppliers' marginal costs of production are c_n and c_s for production levels less than the capacity, while production above the capacity is impossible (i.e., infinitely costly). Suppliers are symmetric in capacity $k_n = k_s = k > 0$ and symmetric in production costs $c_n = c_s = c = 0$. The level of demand in any period, θ_n in market North and θ_s in market South, is independent across markets and independent of market price, i.e., perfectly inelastic. Moreover, $\theta_i \in [\underline{\theta}_i, \overline{\theta}_i] \subseteq [0, k+T]$, $i = n, s$.

The capacity of the transmission line can be lower than the installed capacity in each market $T \leq k$, i.e., the transmission line could be congested for some realization of demands (θ_s, θ_n). When $T > k$, the transmission line is not congested and the equilibrium is as in Fabra et al. (2006).

3.4.2 Timing of the game.

Having observed the realization of demands $\theta \equiv (\theta_s, \theta_n)$, each supplier simultaneously and independently submits a bid specifying the minimum price at which it is willing to supply up to its capacity, $b_i \leq P$, $i = n, s$, where P denotes the "market reserve price", possibly determined by regulation. P can be interpreted as the price at which all consumers are indifferent between consuming and not consuming, or a price cap imposed by the regulatory authorities (von der Fehr and Harbord, 1993).

Let $b \equiv (b_s, b_n)$ denote a bid profile. On basis of this profile, the auctioneer calls suppliers into operation. If suppliers submit different bids, the capacity of the lower-bidding supplier is dispatched first.

The output allocated to supplier $i, i = n, s$, denoted by $q_i(b; \theta, T)$. Given the parameters of the model, suppliers' output functions are:

$$q_n(b; \theta, T) = \begin{cases} \theta_n + \theta_s & \text{if } b_n \leq b_s \\ \theta_n - T & \text{if } b_n > b_s \end{cases} \tag{3.1}$$

$$q_s(b; \theta, T) = \begin{cases} \theta_s + T & \text{if } b_s < b_n \\ 0 & \text{if } b_s \geq b_n \end{cases} \tag{3.2}$$

Finally, the payments are worked out by the auctioneer. When the auctioneer runs a discriminatory price auction, the price received by a supplier for any positive quantity dispatched by the auctioneer is equal to its own offer price, whenever a bid is wholly or partly accepted. Hence, for a given realization of demands $\theta \equiv (\theta_s, \theta_n)$ and a bid profile $b \equiv (b_s, b_n)$, supplier i's profits, $i = n, s$, can be expressed as

$$\pi_n^d(b; \theta, T) = \begin{cases} b_n(\theta_n + \theta_s) & \text{if } b_n \leq b_s \\ b_n(\theta_n - T) & \text{if } b_n > b_s \end{cases} \tag{3.3}$$

$$\pi_s^d(b; \theta, T) = \begin{cases} b_s(\theta_s + T) & \text{if } b_s < b_n \\ b_s(0) & \text{if } b_s \geq b_n \end{cases} \tag{3.4}$$

3.4.3 Equilibrium

As we have discussed in chapter 2, when the auction is discriminatory, the equilibrium is in mixed strategies. Then, we present the support of the mixed strategies equilibrium, the cumulative distribution function, the expected bid, and the expected profit.[3] First, we present the case where the transmission tariffs are zero ($t > 0$), then the case, where the transmission tariffs are positive ($t > 0$).

Case ($t = 0$). The characterization of the equilibrium is given by the next set of equations:

Support of the mixed strategies equilibrium. The lower bound of the support for the supplier located in the high-demand market is defined by: $\underline{b}_n = \dfrac{P(\theta_n - T)}{k}$ and the lower bound of the support for the supplier located in the low-demand market is defined by $\underline{b}_s = \dfrac{P(\theta_n + \theta_s - k)}{\theta_s + T}$. Therefore, the support of the mixed strategies equilibrium for both suppliers is defined by

$$b \in S = \left[\max\left\{\underline{b}_i, \underline{b}_j\right\}, P\right] = \left[\frac{P(\theta_n - T)}{k}, P\right] \tag{3.5}$$

The **cumulative distribution functions** are defined by:

---

[3] In the annex in this chapter is described the procedure to work out the mixed strategies equilibrium analytically.

$$F_s(b) = \begin{cases} 0 & \text{if } b < \underline{b} \\ \dfrac{\theta_n + \theta_s}{\theta_s + T}\dfrac{b - \underline{b}}{b} = C_n(\theta, k, T)\dfrac{b - \underline{b}}{b} & \text{if } b \in (\underline{b}, P) \\ 1 & \text{if } b = P \end{cases}$$

$$F_n(b) = \begin{cases} 0 & \text{if } b < \underline{b} \\ \dfrac{\theta_s + T}{\theta_s + T}\dfrac{b - \underline{b}}{b} = C_s(\theta, k, T)\dfrac{b - \underline{b}}{b} & \text{if } b \in (\underline{b}, P) \\ 1 & \text{if } b = P \end{cases} \tag{3.6}$$

Given that $\underline{b}_n > \underline{b}_s$, it is easy to show that $F_s(P)$ is continuous in the upper bound of the support, and that $F_n(P)$ is discontinuous in the upper bound of the support:

$$\begin{aligned} F_s(P) &= \frac{\theta_n + \theta_s}{\theta_s + T}\frac{P - \dfrac{P(\theta_n - T)}{\theta_n + \theta_s}}{P} = C_n(\theta, k, T)\frac{P - \dfrac{P(\theta_n - T)}{\theta_n + \theta_s}}{P} = 1 \\ F_n(P) &= \frac{\theta_s + T}{\theta_s + T}\frac{P - \dfrac{P(\theta_n - T)}{\theta_n + \theta_s}}{P} = C_s(\theta_n, k, T)\frac{P - \dfrac{P(\theta_n - T)}{\theta_n + \theta_s}}{P} < 1 \end{aligned}$$

The **expected bid** is determined by:

$$\begin{aligned} E_s(b) &= C_s(\theta, k, T)\underline{b}\left[ln(b)\right]_{\underline{b}}^{P} \\ E_n(b) &= C_n(\theta, k, T)\underline{b}\left[ln(b)\right]_{\underline{b}}^{P} + (1 - F_n(P))\,P \end{aligned} \tag{3.7}$$

Given that $F_n(b)$ is discontinuous in the upper bound of the support, to work out supplier n's expected bid is necessary to multiply the maximum bid allowed by the auctioneer by the probability that supplier n assigns to that bid $(1 - F_n(P))\,P$.

The **expected profit** is defined by:

$$\begin{aligned} \overline{\pi}_n &= \underline{b}(\theta_s + \theta_n) \\ \overline{\pi}_s &= \underline{b}(\theta_s + T) \end{aligned} \tag{3.8}$$

Case $(t > 0)$. The characterization of the equilibrium is given by the next set of equations:

Support of the mixed strategies equilibrium. The lower bound of the support for the supplier located in the high-demand market is defined by $\underline{b}_n = \dfrac{P(\theta_n - T) + t\theta_s}{\theta_n + \theta_s}$, and the lower bound of the support for the supplier located in the low-demand market is defined by $\underline{b}_s = \dfrac{tT}{\theta_s + T}$.

Therefore, the support of the mixed strategies equilibrium for both suppliers is defined by

$$b \in S = \left[\max\left\{\underline{b}_i, \underline{b}_j\right\}, P\right] = \left[\frac{P(\theta_n - T) + t\theta_s}{\theta_n + \theta_s}, P\right] \tag{3.9}$$

The **cumulative distribution functions** are defined by:

$$F_s(b) = \begin{cases} 0 & \text{if } b < \underline{b} \\ \dfrac{(b-\underline{b})(\theta_n+\theta_s)}{b\left[(\theta_s+\theta_n)-(\theta_n-T)\right]-t\ \theta_s} & \text{if } b \in (\underline{b}, P) \\ 1 & \text{if } b = P \end{cases}$$

$$F_n(b) = \begin{cases} 0 & \text{if } b < \underline{b} \\ \dfrac{(b-\underline{b})(\theta_s+T)}{b(\theta_s+T)-tT} & \text{if } b \in (\underline{b}, P) \\ 1 & \text{if } b = P \end{cases} \tag{3.10}$$

The **expected bid** is determined by:

$$\begin{aligned} E_s(b) &= \int_{\underline{b}}^{P} b f_s(b_s)\partial b = \int_{\underline{b}}^{P} b\frac{(\theta_n+\theta_s)(\underline{b}(\theta_s+T)-t\theta_s)}{(b(\theta_s+T)-t\theta_s)^2} + (1-F_s(P))P \\ &= \frac{(\theta_n+\theta_s)(\underline{b}(\theta_s+T)-t\theta_s)}{(\theta_s+T)^2} \\ &\quad \left[ln\left(\frac{P(\theta_s+T)-t\theta_s}{\underline{b}(\theta_s+T)-t\theta_s}\right) - \frac{t\theta_s}{P(\theta_s+T)-t\theta_s} + \frac{t\theta_s}{\underline{b}(\theta_s+T)-t\theta_s}\right] \\ E_n(b) &= \int_{\underline{b}}^{P} b f_n(b_s)\partial b = \int_{\underline{b}}^{P} b\frac{(\theta_s+T)(\underline{b}(\theta_s+T)-tT)}{(b(\theta_s+T)-tT)^2} + (1-F_n(P))P \\ &= \frac{(\underline{b}(\theta_s+T)-tT)}{(\theta_s+T)} \\ &\quad \left[ln\left(\frac{P(\theta_s+T)-tT}{\underline{b}(\theta_s+T)-tT}\right) - \frac{tT}{P(\theta_s+T)-tT} + \frac{tT}{\underline{b}(\theta_s+T)-tT}\right] \\ &\quad +(1-F_n(P))P \end{aligned} \tag{3.11}$$

Given that $F_n(b)$ is discontinuous in the upper bound of the support, to work out supplier n's expected bid is necessary to multiply the maximum bid allowed by the auctioneer by the probability that supplier n assigns to that bid $(1-F_n(P))\,P$.

The **expected profit** is defined by:

$$\begin{aligned} \overline{\pi}_n &= \underline{b}(\theta_s+\theta_n) - t\theta_s \\ \overline{\pi}_s &= \underline{b}(\theta_s+T) - tT \end{aligned} \tag{3.12}$$

3.4.4 Annex. Mixed strategies equilibrium

To work out the mixed strategies equilibrium it is necessary to work out the **support** in which the suppliers will randomize, and the **cumulative distribution function**.

First, we work out the **support**. Supplier n can always submit the maximum bid allowed by the auctioneer and satisfy the residual demand. Therefore, it will never submit a bid lower than $\underline{b}_n$, where $\underline{b}_n$ solves $\underline{b}_n(\theta+\theta_s) = P(\theta_n - T)$. The residual demand for supplier s is zero. Therefore, the lower bid of the support for supplier s is zero.

Supplier s knows that supplier n never submits bids lower than $\underline{b}_n$. Therefore, supplier s never randomize assign a positive probability to bids lower than $\underline{b}_n$ since it can increase its expected bid by submitting higher bids, but still undercutting supplier n.

Given that the demand is inelastic, the upper bound of the support is the maximum price allowed by the auctioneer P, since the suppliers can always increase their expected profit by raising their bids.

Therefore, the support of the mix strategies equilibrium is defined by $b \in [\underline{b}_n, P] = \left[\dfrac{P(\theta_n - T)}{\theta_n + \theta_s}\right]$.

Second, we work out the **cumulative distribution function** for each supplier.

The cumulative distribution function for supplier n.

First, the profit function for a given bid b is defined by

$$\pi_n(b) \quad = \quad b\left[F_s(b)(\theta_n - T) + (1 - F_s(b))(\theta_s + \theta_n)\right] \tag{3.13}$$

where, $F_s(b)$ is the probability that supplier s's bid is lower than supplier n's bid. Therefore, with probability $F_s(b)$ supplier n is dispatched last and it satisfies the residual demand. Therefore, in that case, supplier n's profits are $\pi_n(b) = F_s(b)b(\theta_n - T)$. With probability $(1 - F_s(b))$ supplier s is dispatched first, and it satisfies the total demand. Therefore, in that case, supplier n's profits are $\pi_n(b) = (1 - F_s(b))b(\theta_n - T)$.

By doing some algebra in equation 3.13, we obtain:

$$\begin{aligned} \pi_n(b) &= -bF_s(b)\left[(\theta_s + \theta_n) - (\theta_n - T)\right] + b(\theta_n + \theta_s) \Rightarrow \\ F_s(b) &= \frac{b(\theta_s + \theta_n) - \pi_n(b)}{b((\theta_s + \theta_n) + (\theta_n - T)} \end{aligned} \tag{3.14}$$

Second, in the lower bound the support, the value of the cumulative distribution function is zero. Otherwise, one supplier can undercut the other and it increases its profit. Therefore, in the lower bound of the support equation 3.14 becomes $\pi_n(\underline{b}) = \underline{b}(\theta_s + \theta_n)$.

Third, the profit for any bid in the support is the same. Otherwise, the suppliers increases its profit by reassigning probabilities. Therefore, $\pi_n(b) = \pi_n(\underline{b}) \forall b \in [\underline{b}, P]$. Therefore, equation 3.14 can be rewritten as:

$$F_s(b) \quad = \quad \frac{b(\theta_s + \theta_n) - \underline{b}(\theta_s + \theta_n)}{b((\theta_s + \theta_n) + (\theta_n - T)} = \frac{(\theta_s + \theta_n)}{(\theta_s + \theta_n) - (\theta_n - T)} \frac{(b - \underline{b})}{b} \tag{3.15}$$

Where, equation 3.15 and equation 3.6 are the same. Following the same approach it easy to derive the cumulative distribution function for supplier n.

3.5 Exercises

The set of parameters that we will use in the next two exercises are: $k = 60$, $T = 40$, $\theta_n = 55$, $\theta_s = 5$, $c = 0$, $P = 7$. In the first exercise (case $(t = 0)$), the parameter $t = 0$. In the second exercise (case $(t > 0)$), the parameter $t = 1.5$.

3.5.1 Case $(t = 0)$

Using the set of parameters defined above, and the set of equations that characterize mixed strategies equilibrium when $t = 0$, answer the next questions:

1. Work out the **support** of the mixed strategies equilibrium defined by equation 3.5.
2. Plot the **cumulative distribution functions** defined by equation 3.6.

   Which supplier will submit the higher bids? Which is the economic intuition behind those results?

   Hint: For the values of b defined by equation 3.5, plot $F_s(b)$ and $F_n(b)$.
3. Work out the **expected equilibrium price** defined by equation 3.7.

   Which supplier will submit the higher bid? In which direction will flow the electricity?
4. Knowing the demand and the expected equilibrium price, which will be the **(expected) consumer surplus**?
5. Work out the equilibrium **profit** for both suppliers.
6. Analyse the impact of an **increase in transmission capacity** on the lower bound of the support, the expected equilibrium price and the expected equilibrium profits.

   Hint: Plot equations 3.5, 3.7 and 3.8 for $T \in [\theta_s, \theta_n] = [5, 55]$. You can also plot the equations for $T \in [0, k]$ i.e., from isolated markets to fully integrated market. For illustrative purposes, I plot the equations for $T \in [\theta_s, \theta_n] = [5, 55]$.

3.5.2 Case $(t > 0)$

Using the set of parameters defined above, and the set of equations that characterize the mixed strategies equilibrium when $t > 0$, answer the next questions:

1. Work out the **support** of the mixed strategies equilibrium defined by equation 3.9.
2. Plot the **cumulative distribution functions** defined by equation 3.10.

   Which supplier will submit the higher bids? Which is the economic intuition behind those results?

   Hint: For the values of b defined by equation 3.9, plot $F_s(b)$ and $F_n(b)$.

   Which are the differences between the cumulative distribution functions when $t = 0$ and when $t = 1.5$? Which are the economic intuitions behind the results?
3. Work out the **expected equilibrium price** defined by equation 3.11.

   Which supplier will submit the higher bid? In which direction will flow the electricity?

   Compare the results when $t = 0$ and when $t = 1.5$.
4. Knowing the demand and the expected equilibrium price, which will be the **(expected) consumer surplus**?

   In which case the consumer surplus is larger?
5. Work out the equilibrium **profit** for both suppliers.

3.5.3 Solution

We present the solutions to the questions introduced in the two previous exercises simultaneously to compare both cases.

Support of the cumulative distribution function and **cumulative distribution function**.

When the transmission tariffs are zero ($t = 0$), in the lower bound of the support, the slope of cumulative distribution function (CDF) of the supplier located in the low-demand market is stepper than the slope of the cumulative distribution function of the supplier located in the high-demand market (left-hand panel, figure 3.3). The slope of the CDF is the probability distribution function. Therefore, the supplier located in the low-demand market submits lower bids with higher probability. Moreover, the slope of the CDF of the supplier located in the low-demand market is continuous in the upper bound of the support. In contrast, the CDF of the supplier located in the high-demand market is discontinuous. Therefore, the supplier located in the high-demand market submits the higher bid allowed by the auctioneer with a positive probability. Hence, the supplier located in the high demand market submits higher bids with higher probability.

The **intuition** behind this result is as follows: The supplier located in the high-demand market faces a high residual demand and it has incentives to submit higher bids.

When the transmission tariffs are positive ($t = 1.5$), in the lower bound of the support, the slope of cumulative distribution function (CDF) of the supplier located in the high-demand market is stepper than the slope of the cumulative distribution function of the supplier located in the low-demand market (right-hand panel, figure 3.3). Therefore, the supplier located in the high-demand market submits lower bids with higher probability. This is in contrast with the nil transmission tariffs case where the slope of the cumulative distribution functions follow the opposite pattern. In the upper bound of the support, as when $t = 0$, the CDF of the supplier located in the high-demand market is discontinuous.

The **intuition** behind this result is as follows: Since the supplier located in the high-demand market has to transmit a lower part of its production capacity to the other market, it faces lower transmission costs and thus, it has incentives to submit lower bids to extract the efficiency rents. Simultaneously, the supplier located in the high-demand market faces a high residual demand and it has incentives to submit high bids. Therefore, the supplier located in the high demand market assigns high probability to the extremes of the support and very little probability to the intermediate bids.

Expected equilibrium prices.

When the transmission tariffs are zero ($t = 0$), the supplier located in the high-demand market submits higher bids than the supplier located in the low-demand market $E_n(b) = 4.2 > E_s(b) = 3.2$ (column seven, table 3.1). Therefore, the electricity flows from the low-demand market to the high-demand market.

When the transmission tariffs are positive ($t = 1.5$), the supplier located in the high-demand market submits lower bids than the supplier located in the low-demand market $E_n(b) = 3.2 < E_s(b) = 3.3$ (column eight, table 3.1). Therefore, the electricity flows from the high-demand market to the low-demand market and the transmission losses are minimized (less electricity flows through the grid). Therefore, the introduction of a positive transmission tariff could **increase transmission efficiency**.

Figure 3.3: Cumulative Distribution Functions

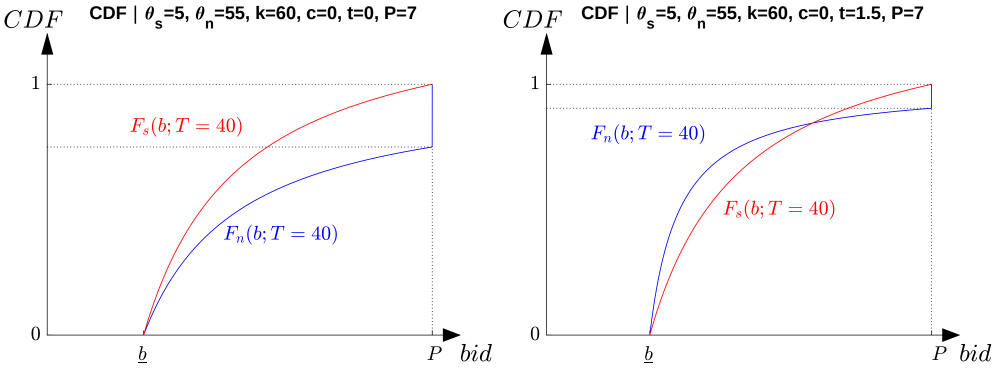


Table 3.1: Impact of transmission constraints and transmission tariffs on the equilibrium outcome ($\theta_s = 5,\ \theta_n = 55,\ k = 60,\ c = 0,\ P = 7$)

Model	T	$\underline{b}$	$\overline{\pi}_n$	$\overline{\pi}_s$	$\overline{\pi} = \overline{\pi}_n + \overline{\pi}_s$	$E_n(b)$	$E_s(b)$	$\theta_n E_n(b) + \theta_s E_s(b)$
$t = 0$	40	1.75	105	184	289	4.2	3.2	247
$t = 1.5$	40	1.87	105	24	129	3.1	3.3	187

Consumers surplus.

The introduction of a positive transmission tariff **increases consumers welfare**. As I have explained in the previous point, the introduction of a positive transmission tariff reduces equilibrium prices in the high-demand market and increases equilibrium prices in the low-demand market. Given that the majority of consumers are located in the high-demand market, the overall effect is an increase on consumers welfare (column nine, table 3.1).

Expected profits.

Figure 3.4: Increase transmission capacity

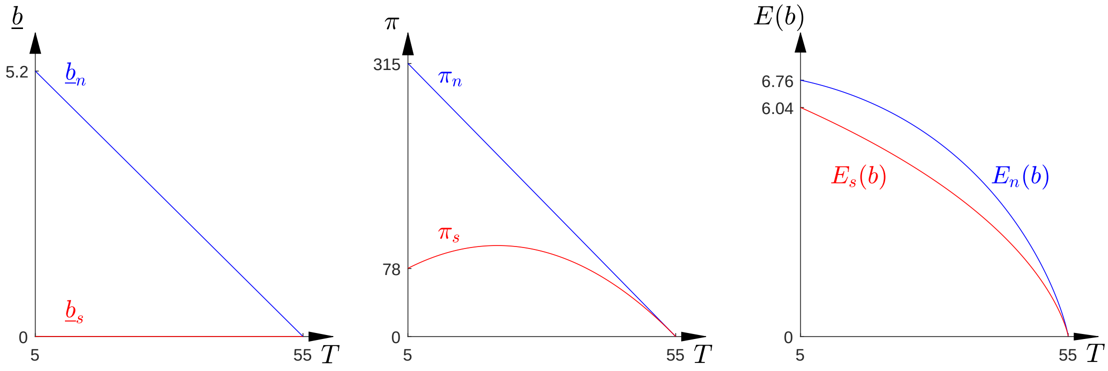


Table 3.2: *Zero* transmission tariffs. Increase in transmission capacity $\triangle T$ ($\theta_s = 5, \theta_n = 55, k = 60, c = 0, t = 0, P = 7$). Main variables.

T	$\underline{b}$	π_n	π_s	$E_n(b)$	$E_s(b)$
0	7	385.07	35	7	7
5	5.835	350.1	58.35	6.8963	6.3795
15	4.668	280.08	93.36	6.5587	5.6770
25	3.501	210.06	105.03	5.9261	4.8530
35	2.335	140.1	93.4	4.8981	3.8464
45	1.168	70.08	58.4	3.2589	2.5102
55	0	0	0	0	0

The introduction of a positive transmission tariff doesn't change the profits of the supplier located in the high-demand market, since the increase in transmission costs is compensated by the increase in demand derived by being dispatched first in the auction. In contrast, the introduction of a positive transmission tariff decreases the profits of the supplier located in the low-demand market since it faces an increase in transmission costs.

Increase in transmission capacity.

An increase in transmission capacity reduces the residual demand and, according to equation 3.5, the lower bound of the support decreases (left-hand panel, figure 3.4; column two, table 3.2). A decrease in the lower bound of the support implies that both suppliers randomize submitting lower bids and therefore, the expected bid decreases for both suppliers (right-hand panel, figure 3.4; columns five and six, table 3.2; equation 3.7). Finally, an increase in transmission capacity reduces the expected bid and the residual demand of the supplier located in the high-demand market as does its expected profit (central panel, figure 3.4; column three, table 3.2; equation 3.8). In contrast, an increase in transmission capacity reduces the expected bid and increases the total demand of the supplier located in the low-demand market. When the transmission capacity is low, the increase in demand dominates the decrease in the expected bid and its expected profit increases. However, when the transmission capacity is large enough, the decrease in bids dominates and its expected profit decreases (central panel, figure 3.4; column four, table 3.2; equation 3.8).

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Chapter 4

Auction Design in Zonal Pricing Electricity Markets

Electricity markets are organized as nodes, where a node is a market with consumers and suppliers. Those nodes are connected through transmission lines. In the presence of transmission constraints, electricity markets can be organized as nodal or as zonal pricing electricity markets. In a nodal pricing electricity market, the equilibrium price differs across nodes when the transmission line is congested. In contrast, in a zonal pricing electricity market, the equilibrium price is the same in all the nodes that belong to the same zone. In this chapter, we analyze the design of zonal pricing electricity markets when the **competition is perfect** and when the **competition is imperfect**.

First, following Holmberg and Lazarczyk (2015), we compare the equilibrium performance between nodal pricing electricity markets, zonal pricing electricity markets with counter-trading, and discriminatory pricing when the **competition is perfect** and the suppliers have information.

Second, following Blázquez (2019), we work out the equilibrium in a zonal pricing electricity market when the **competition is imperfect** and four different auction designs are implemented by the auctioneer.

We focus our study on zonal pricing electricity markets, since the zonal pricing electricity market is one of the most salient designs to organize electricity markets in the presence of transmission constraints. Moreover, the European Commission proposes that design for the integration of the European electricity markets. For a complete literature review of bidding zones, see Ofgem (2014) and ENTSO-E (2014).For a complete analysis of the delineation of regions for Transmission System Operators coordination and the governance of System Operators in the integration of European electricity markets, see ENTSO-E (2015), and European Commission (2015).

4.1 Theory

4.1.1 Zonal pricing with perfect competition: Holmberg and Lazarczyk (2015)

The set up, the timing and the equilibrium of the game are in Holmberg and Lazarczyk (2015). In exercise 1, I present the set up, and I characterize the equilibrium in three different cases: Nodal pricing, discriminatory pricing and zonal pricing.

4.1.2 Zonal pricing with imperfect competition: Blázquez (2019)

In this section, we characterize the equilibrium in a zonal market when three types of redispatch mechanisms are implemented by the auctioneer. First, we work out the equilibrium when the auction in the spot electricity market is uniform and the transmission constraint is taken into account ex-ante, i.e., it is not necessary to introduce a redispatch mechanism to alleviate the congestion in the transmission line. Second, we work out the equilibrium when the auction in the spot electricity market is discriminatory and the transmission constraint is taken into account ex-ante, i.e., it is not necessary to introduce a redispatch mechanism to alleviate the congestion in the transmission line. Third, we work out the equilibrium when the auction in the spot electricity market is uniform, and the auction in the redispatch market is discriminatory and the suppliers submit the same bid in the spot and in the redispatch market.[1]

Set up of the model

There exist two electricity nodes, node North and node South, that are connected by a transmission line with capacity T. Both nodes belong to the same zone, i.e., the equilibrium price in both nodes is the same even when the transmission line is congested (zonal pricing).

There exist two duopolists with capacities k_n and k_s, where subscript n means that the supplier is located in node North and subscript s means that the supplier is located in node South. The suppliers' marginal costs of production are c_n and c_s for production levels less than the capacity, while production above the capacity is impossible (i.e., infinitely costly). Suppliers are symmetric in production costs $c_n = c_s = c = 0$. The level of demand in any period, θ_n in node North and θ_s in node South, is independent of the node price, i.e., perfectly inelastic. I introduce two more assumptions on demand levels. First, $\theta_i \in [\underline{\theta}_i, \overline{\theta}_i] \subseteq [0, k_i + T]$, $i = n, s$, i.e., the installed production capacity in each node plus the electricity that flows from the other node is enough to satisfy the peak demand in each node. Second, $\overline{\theta}_i + \overline{\theta}_j < k_i + k_j$, i.e., the total installed production capacity is enough to satisfy the peak demand in both nodes.

The capacity of the transmission line can be lower than the installed capacity in each node $T \leq min\{k_s, k_n\}$, i.e., the transmission line could be congested for some realization of demands (θ_s, θ_n). When $T > min\{k_s, k_n\}$, the transmission line is not congested and the equilibrium is as in Fabra et al. (2006). We study the model when the transmission line is congested. A complete description of the model also when the transmission line is not congested is in Blázquez (2019).

Timing of the game

1. Both suppliers submit their bids independently and simultaneously.

Having observed the realization of demands $\theta \equiv (\theta_s, \theta_n)$, each supplier simultaneously and independently submits a bid in the spot electricity market specifying the minimum price at which it is willing to supply up to its capacity, $b_i^S \leq P$, $i = n, s$, where the super script S denotes the spot electricity market, and P denotes the "market reserve price", possibly determined by regulation. P can be interpreted as the price at which all consumers are indifferent between consuming and not consuming, or a price cap imposed by the regulatory authorities (von der Fehr and Harbord, 1993). Moreover, when the auction is discriminatory, the equilibrium in the discriminatory price auction is in mixed strategies. In that case, when the demand is inelastic, the introduction of a price cap guarantees the existence of the upper bound of the support in a

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[1] Blázquez (2019) also characterize the equilibrium when the suppliers can submit different bids in the spot and in the redispatch market. The characterization of the equilibrium is very similar. Therefore, to keep the analysis as simple as possible, we focus on the three market designs described above.

mixed-strategy equilibrium (Baye et al., 1992; Fabra et al., 2006).

2. The suppliers are called into operation.

Let $b^S \equiv (b_s^S, b_n^S)$ denote a bid profile in the spot electricity market. On basis of this profile, the auctioneer calls suppliers into operation and works out suppliers' outcomes and profits. If suppliers submit different bids, the capacity of the lower-bidding supplier is dispatched first. If the capacity of the lower-bidding supplier is not sufficient to satisfy total demand, the higher-bidding supplier's capacity is then dispatched to serve residual demand. If the two suppliers submit equal bids, then supplier i is ranked first with probability ρ_i, where $\rho_n + \rho_s = 1$, $\rho_i = 1$ if $\theta_i > \theta_j$, and $\rho_i = \frac{1}{2}$ if $\theta_i = \theta_j$, $i = n, s$, $i \neq j$.[2] Without loss of generality, we assume that node N is the importing node and node S is the exporting node, i.e, $b_n \geq b_s$.

2.a. When the auction is uniform and an ex-ante redispatch mechanism is introduced by the auctioneer.

When the auction is uniform and an ex-ante redispatch mechanism is introduced by the auctioneer, the output allocated to supplier n in the spot electricity market (supplier s's output function is symmetric), denoted by $q_n^{u1}(b^S;\theta)$, is given by:[3]

$$q_n^{u1}(b^S;\theta) = \begin{cases} \theta_n + T & \text{if } b_n^S \leq b_s^S \text{ and } \theta_s - T > \theta_s + \theta_n - k_n \\ \theta_n - T & \text{if } b_n^S > b_s^S \text{ and } \theta_n - T > \theta_s + \theta_n - k_s \end{cases} \tag{4.1}$$

When supplier n submits the lower bid in the spot electricity market and the transmission line is congested, supplier n cannot satisfy the demand in node South, even when it has enough production capacity. Therefore, the total demand that supplier n can satisfy is $(\theta_n + T)$. When supplier n submits the higher bid in the spot electricity market, and the transmission line is congested, supplier n's residual demand is defined by $(\theta_n - T)$.

2.b. When the auction is discriminatory and an ex-ante redispatch mechanism is introduced by the auctioneer.

When the auction is discriminatory, the output allocated to supplier n in the spot electricity market, denoted by $q_n^d(b^S;\theta)$, is as when the auction is uniform and an ex-ante redispatch mechanism is introduced by the auctioneer.

2.c. When the auction is uniform, an ex-post redispatch mechanism is introduced by the auctioneer and the suppliers submit the same bid in the spot and in the redispatch market.

When the auction is uniform and an ex-post redispatch mechanism is introduced by the auctioneer, the output allocated to supplier n in the spot electricity market, denoted by $q_n^{u2}(b^S;\theta)$, is given by:

$$q_n^{u2}(b^S;\theta) = \begin{cases} \min\{\theta_s + \theta_n, k_n\} & \text{if } b_n^S \leq b_s^S \text{ and } \theta_s - T > \theta_s + \theta_n - k_n \\ \theta_s + \theta_n - k_s & \text{if } b_n^S > b_s^S \text{ and } \theta_n - T > \theta_s + \theta_n - k_s \end{cases} \tag{4.2}$$

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[2] The implemented tie-break rule is such that if the bids of both suppliers are equal and demand in node i is larger than demand in node j, the auctioneer first dispatches the supplier located in node i. Moreover, when the auction is discriminatory, the equilibrium in this model is in mixed strategies. In that case, the tie-breaking rule ensures the existence of a mixed strategies equilibrium (Dasgupta and Maskin, 1986).

[3] We use the super script $u1$ to denote the uniform price auction when an ex-ante redispatch mechanism is introduced by the auctioneer, the super script $u2$ to denote the uniform price auction when an ex-post redispatch mechanism is introduced by the auctioneer, and the super script d to denote the discriminatory price auction

When an ex-post redispatch mechanism is introduced, the congestion is not taken into account when the spot electricity market is cleared. Therefore, when supplier n submits the lower bid in the spot electricity market, it satisfies the total demand $(\theta_s + \theta_n)$ up to its production capacity (k_n). When it submits the higher bid, it satisfies the residual demand $(\theta_s + \theta_n - k_s)$.

When the transmission line is congested and an ex-post redispatch mechanism is introduced by the auctioneer to alleviate the congestion in the line, the outcome allocated to supplier n in the redispatch market is denoted by (I use the super script R to denote the redispatch market):

$$q_n^R(b^S;\theta) = \begin{cases} min\{\theta_s + \theta_n, k_n\} - (\theta_n + T) & \text{if } b_n^S \leq b_s^S \text{ and } \theta_s - T > \theta_s + \theta_n - k_n \\ (\theta_n - T) - (\theta_s + \theta_n - k_s) & \text{if } b_n^S > b_s^S \text{ and } \theta_n - T > \theta_s + \theta_n - k_s \end{cases} \tag{4.3}$$

When supplier n submits the lower bid in the spot electricity market, it is dispatched first, but due to the transmission constraint it cannot satisfy the total demand or sell its entire production capacity $(min\{\theta_s + \theta_n, k_n\})$, but only $(\theta_n + T)$. Therefore, in the redispatch market it has to buy back the difference between what it wants to sell and what it can sell $(min\{\theta_s + \theta_n, k_n\} - (\theta_n + T))$. When supplier n submits the higher bid in the spot electricity market, it is dispatched last. Due to the transmission constraint, it faces a high residual demand and it can sell more electricity $(\theta_n - T)$ that what it sells in the spot electricity market $(\theta_s + \theta_n - k_s)$. Therefore, in the redispatch market it can sell all the electricity that it could not sell in the spot electricity market $((\theta_n - T) - (\theta_s + \theta_n - k_s))$.

3. The payments are worked out by the auctioneer.

3.a. When the auction is uniform and an ex-ante redispatch mechanism is introduced by the auctioneer.

When the auction is uniform and an ex-ante redispatch mechanism is introduced by the auctioneer, the price received by a supplier in the spot electricity market for any positive quantity dispatched by the auctioneer is equal to the higher bid accepted in the auction. Hence, for a given realization of $\theta \equiv (\theta_s, \theta_n)$ and a bid profile $b \equiv (b_s, b_n)$, supplier n's profits can be expressed as $\pi_n^{u1}(b^S;\theta)$:

$$\begin{cases} b_s^S(\theta_n + T) & \text{if } b_n^S \leq b_s^S \text{ and } \theta_s - T > \theta_s + \theta_n - k_n \\ b_n^S(\theta_n - T) & \text{if } b_n^S > b_s^S \text{ and } \theta_n - T > \theta_s + \theta_n - k_s \end{cases} \tag{4.4}$$

When supplier n submits the lower bid in the spot electricity market and the transmission line is congested, supplier s sets the price and supplier n's profits are defined by $(b_s^S(\theta_n + T))$. When supplier n submits the higher bid in the spot electricity market and the transmission line is congested, it sets the price and its profits are defined as $(b_n^S(\theta_n - T))$.

3.b. When the auction is discriminatory and an ex-ante redispatch mechanism is introduced by the auctioneer.

When the auctioneer runs a discriminatory price auction, the price received by a supplier in the spot electricity market for any positive quantity dispatched by the auctioneer is equal to its own offer price, whenever a bid is wholly or partly accepted. Hence, for a given realization of demands $\theta \equiv (\theta_s, \theta_n)$ and a bid profile $b \equiv (b_s, b_n)$, supplier n's profits can be expressed as $\pi_n^d(b^S;\theta)$:

$$\begin{cases} b_n^S(\theta_n + T) & \text{if } b_n^S \leq b_s^S \text{ and } \theta_s - T > \theta_s + \theta_n - k_n \\ b_n^S(\theta_n - T) & \text{if } b_n^S > b_s^S \text{ and } \theta_n - T > \theta_s + \theta_n - k_s \end{cases} \tag{4.5}$$

3.c. When the auction is uniform, an ex-post redispatch mechanism is introduced by the auctioneer and the suppliers submit the same bid in the spot and in the redispatch market.

When the auction is uniform and an ex-post redispatch mechanism is introduced by the auctioneer and when the bid that supplier n submits in the spot electricity market is also used in the redispatch market, supplier n's profits can be expressed as $\pi_n^{u2}(b^S;\theta)$:

$$\begin{cases} b_s^S \min\{\theta_s+\theta_n, k_n\} - ... & \\ \quad b_n^S\left(\min\{\theta_s+\theta_n, k_n\} - (\theta_n+T)\right) & \text{if } b_n^S \leq b_s^S \text{ and } \theta_s - T \leq \theta_s+\theta_n-k_n \\ b_n^S(\theta_s+\theta_n-k_s) + ... & \\ \quad b_n^S((\theta_n-T)-(\theta_s+\theta_n-k_s)) & \text{if } b_n^S > b_s^S \text{ and } \theta_n - T > \theta_s+\theta_n-k_s \end{cases} \tag{4.6}$$

When supplier n submits the lower bid in the spot electricity market and the transmission line is congested, supplier s sets the price and supplier n's profits in that market are defined as $(b_s^S \min\{\theta_s+\theta_n, k_n\})$. However, due to the transmission constraint, supplier s cannot satisfy the demand in both nodes, and it has to use the redispatch market to buy back the capacity that cannot be sold in the spot electricity market $(\min\{\theta_s+\theta_n, k_n\} - (\theta_n+T))$. Given that redispatch market is designed as a discriminatory price auction, supplier n's expenses in the redispatch market are determined by $b_n^S\left(\min\{\theta_s+\theta_n, k_n\} - (\theta_n+T)\right)$. By summing and subtracting the term $b_s^S(\theta_n+T)$ in the first equation in 4.6, we can rewrite it as $b_s^S(\theta_n+T) + (b_s^S - b_n^S)\left(\min\{\theta_s+\theta_n, k_n\} - (\theta_n+T)\right)$. This last expression has an useful economic interpretation, the first term represents supplier n's profits in the spot electricity when the transmission constraint is taken into account, the second term represents supplier n's compensation for not being able to satisfy the demand in both nodes.

When supplier n submits the higher bid in the spot electricity market and the transmission line is congested, supplier n's profits in that market are defined as $(b_n^S(\theta_s+\theta_n-k_s))$. However, due to the transmission constraint, supplier n can sell more electricity that what it sells in the spot electricity market. Therefore, in the redispatch market it can sell all the electricity that it could not sell in the spot electricity market $((\theta_n-T)-(\theta_s+\theta_n-k_s))$. Given that the redispatch market is organized as a discriminatory price auction, supplier n' profits in that market are defined as $b_n^S((\theta_n-T)-(\theta_s+\theta_n-k_s))$.

After the algebra transformations described above, equation 4.6 can be rewritten as $\pi_n^{u2}(b^S;\theta)$:

$$\begin{cases} b_s^S(\theta_n+T) + ... & \\ \quad (b_s^S - b_n^S)\left(\min\{\theta_s+\theta_n, k_n\} - (\theta_n+T)\right) & \text{if } b_n^S \leq b_s^S \text{ and } \theta_s - T \leq \theta_s+\theta_n-k_n \\ b_n^S(\theta_s+\theta_n-k_s) + ... & \\ \quad b_n^S((\theta_n-T)-(\theta_s+\theta_n-k_s)) & \text{if } b_n^S > b_s^S \text{ and } \theta_n - T > \theta_s+\theta_n-k_s \end{cases} \tag{4.7}$$

As can be observed by comparing equations 4.4, 4.5 and 4.7, the introduction of different auction designs in the zonal market change suppliers' profits functions. These equations present a lot of similarities, but also important differences that will affect the characterization of the equilibrium. In the rest of the paper, we work out the equilibrium when different auction designs are implemented by the auctioneer.

Equilibrium

In this section we characterize the equilibrium for each of the three auction designs presented in the model section. As in the model section, we assume that node N is the importing node and node S is the exporting node.

In lemma 1, we study the type of equilibrium in the spot electricity market when a uniform and discriminatory price auction are implemented by the auctioneer.

Lemma 1. When the transmission line is congested, the equilibrium price in the spot electricity market is in pure strategies when the auction is uniform and an ex-ante or an ex-post redispatch mechanism are implemented, but a pure strategies equilibrium does not exist when the auction is discriminatory.

Proof. When the transmission line is congested, the supplier located in the importing node faces a positive residual demand. In that case, when the auction is uniform and an ex-ante or an ex-post redispatch mechanism are introduced by the auctioneer, the supplier located in the importing node submits the maximum bid allowed by the auctioneer, and the supplier located in the low-demand node submits a bid that makes undercutting unprofitable.

In contrast, when the auction is discriminatory, a pure strategies equilibrium does not exist, since the suppliers has incentives to undercut each other to be dispatched first in the auction. ☐

Based on the ancillary result presented in lemma 1, we present the main result of this section.

Proposition 1. When the transmission line is congested, the characterization of the equilibrium falls in one of the next three categories depending on the auction design:

i. When the auction is uniform and an ex-ante redispatch mechanism is introduced by the auctioneer, there are multiplicity of Nash equilibria in the spot electricity market.

ii. When the auction is uniform, an ex-post redispatch mechanism is introduced by the auctioneer and the suppliers submit the same bid in the spot and in the redispatch market, there is a unique Nash equilibrium in the spot and in the redispatch market.

iii. When the auction is discriminatory and an ex-ante redispatch mechanism is introduced by the auctioneer, the equilibrium is in mixed strategies.

Proof. We prove the results for the four auction designs.

i. When the auction is uniform and an ex-ante redispatch mechanism is introduced by the auctioneer. By using lemma 1, the proof is as follows:

The pure strategies equilibrium is defined by

$$b_s^S \in \left[0, \frac{P(\theta_n - T)}{min\left\{\theta_s + \theta_n, k_n\right\}}\right]; \;\; b_n^S = P. \tag{4.8}$$

The equilibrium price in the spot electricity market is P.

The profits are defined by:

$$\overline{\pi}_s = P(\theta_s + T); \;\; \overline{\pi}_n = P(\theta_n - T). \tag{4.9}$$

The electricity flows from the low-demand node to the high-demand node, and the transmission line is congested.

Consumers' surplus is defined by:

$$CS = (P - P)(\theta_s + \theta_n) = 0 \tag{4.10}$$

ii. When the auction is uniform, an ex-post redispatch mechanism is introduced by the auctioneer, and the suppliers submits the same bid in the spot and in the redispatch market. By using lemmas 1 and 2, the proof is as follows:

Solving by backward induction, we characterize the equilibrium in the redispatch market. According with equation 4.7, supplier s's profits are given by $b_n^S(\theta_n+T)+(b_n^S-b_s^S)(\min\{\theta_s+\theta_n, k_s\}-(\theta_s+T))$, where $(b_n^S-b_s^S)(\min\{\theta_s+\theta_n, k_s\}-(\theta_s+T))$ represents the compensation for the electricity that supplier s wants to sell in the spot electricity market, but that it cannot sell because of the transmission constraint. If supplier s could participate in the redisptach market it would submit a bib equal to zero to maximize that compensation.

According with equation 4.7, supplier n's profits are given by $b_n^S(\theta_s+\theta_n-k_s)+b_n^S((\theta_n-T)-(\theta_s+\theta_n-k_s))$, where $(b_n^S((\theta_n-T)-(\theta_s+\theta_n-k_s)))$ represents supplier n's profits in the redispatch market. If supplier n could participate in the redisptach market it would submit the maximum bid allowed by the auctioneer to maximize those profits.

Given that the bid submitted by the suppliers in the redispatch market has to be the same as the one in the spot electricity market, it is necessary to check that the bid that the suppliers want to submit in the redispatch market is also the one that they want to submit in the spot electricity market. Otherwise, it does not exist a pair of strategies that clear both markets simultaneously.

When the transmission line is congested, the unique possible equilibrium is the spot electricity market is the one in which supplier n submits the maximum bid, and supplier s submits a bid that makes undercutting unprofitable. Therefore, the unique pair of strategies that makes compatible an equilibrium in the spot and in the redispatch market simultaneously is defined by:

$$b_s^S = 0; \;\; b_n^S = P, \tag{4.11}$$

The equilibrium price in the spot electricity market is P.

By plugging those values in equation 4.7, the profits are defined by:

$$\begin{aligned} \overline{\pi}_s &= P(\theta_s+T)+(P-0)\left(\min\{\theta_s+\theta_n, k_n\}-(\theta_s+T)\right); \\ \overline{\pi}_n &= P(\theta_s+\theta_n-k_s)+P((\theta_n-T)-(\theta_s+\theta_n-k_s)). \end{aligned} \tag{4.12}$$

The electricity flows from the low-demand node to the high-demand node, and the transmission line is congested.

Consumers' surplus is defined by:

$$CS = (P - P)(\theta_s + \theta_n) = 0 \tag{4.13}$$

iii. When the auction is discriminatory and an ex-ante redispatch mechanism is introduced by the auctioneer. The equilibrium is as in Blázquez (2018). However, I present the main equations that characterize the equilibrium to facilitate the comparison with the other three auction designs.

First, the *lower bound of the support* is defined by:

$$\underline{b}_s^S = \underline{b}_n^S = \frac{P(\theta_n - T)}{k_n} \tag{4.14}$$

Second, I work out the *cumulative distribution functions.*

$$F_s(b^S) = \begin{cases} 0 & \text{if } b^S < \underline{b}^S \\ \dfrac{k_n}{k_n - (\theta_n - T)} \dfrac{b^S - \underline{b}^S}{b^S} & \text{if } b^S \in \left(\underline{b}^S, P\right) \\ 1 & \text{if } b^S = P \end{cases}$$

$$F_n(b^S) = \begin{cases} 0 & \text{if } b^S < \underline{b}^S \\ \dfrac{\theta_s + T}{(\theta_s + T) - (\theta_s + \theta_n - k_n)} \dfrac{b^S - \underline{b}^S}{b^S} & \text{if } b^S \in (\underline{b}, P) \\ 1 & \text{if } b^S = P \end{cases} \tag{4.15}$$

Given that $\underline{b}_n^S > \underline{b}_s^S$, it is easy to show that $F_s(P)$ is continuous in the upper bound of the support, and that $F_n(P)$ is discontinuous in the upper bound of the support:

$$\begin{aligned} F_s(P) &= \frac{k_n}{k_n - (\theta_n - T)} \frac{P - \dfrac{P(\theta_n - T)}{k_n}}{P} = 1 \\ F_n(P) &= \frac{\theta_s + T}{(\theta_s + T) - (\theta_s + \theta_n - k_n)} \frac{P - \dfrac{P(\theta_n - T)}{k_n}}{P} < 1 \end{aligned}$$

Third, the *probability distribution function* is equal to:

$$\begin{aligned} f_s(b^S) &= \frac{\partial F_s(b^S)}{\partial b^S} = \frac{k_n}{k_n - (\theta_n - T)} \frac{\underline{b}^S}{(b^S)^2} \\ f_n(b^S) &= \frac{\partial F_n(b^S)}{\partial b^S} = \frac{\theta_s + T}{(\theta_s + T) - (\theta_s + \theta_n - k_n)} \frac{\underline{b}^S}{(b^S)^2} \end{aligned} \tag{4.16}$$

Fourth, the *expected bid* is determined by:

$$\begin{aligned} E_s(b^S) &= \int_{\underline{b}^S}^{P} b^S f_s(b_s^S) \partial b^S = \int_{\underline{b}^S}^{P} \frac{k_n}{k_n - (\theta_n - T)} \frac{\underline{b}^S}{b^S} \partial b^S = \frac{k_n}{k_n - (\theta_n - T)} \underline{b}^S \left[ln(b^S)\right]_{\underline{b}^S}^{P} \\ E_n(b^S) &= \int_{\underline{b}^S}^{P} b^S f_n(b_n^S) \partial b^S = \int_{\underline{b}^S}^{P} \frac{\theta_s + T}{(\theta_s + T) - (\theta_s + \theta_n - k_n)} \frac{\underline{b}^S}{(b^S)^2} \partial b^S = \\ &= \frac{\theta_s + T}{(\theta_s + T) - (\theta_s + \theta_n - k_n)} \underline{b}^S \left[ln(b^S)\right]_{\underline{b}^S}^{P} + (1 - F_n(P))\, P \end{aligned} \tag{4.17}$$

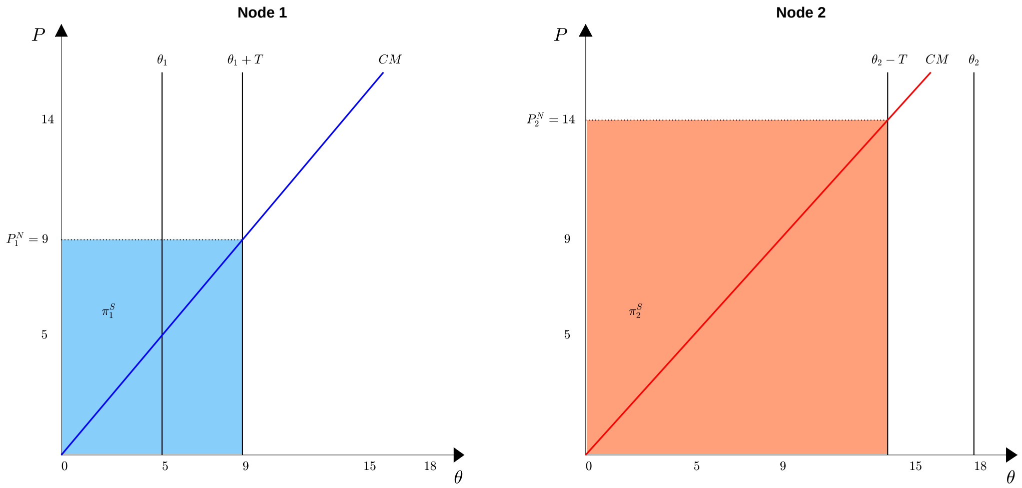
Given that $F_n(b^S)$ is discontinuous in the upper bound of the support, to work out supplier n's expected bid is necessary to multiply the maximum bid allowed by the auctioneer by the probability that supplier n assigns to that bid $(1 - F_n(P))\, P$, where $F_n(P) = F_n(b^S)$, when $b^S \longrightarrow P$.

When the auction is discriminatory, the expected equilibrium price in the spot market is defined by:

$$E(b^S) = \frac{E(b_s^S)\theta_s}{(\theta_s + \theta_n)} + \frac{E(b_n^S)\theta_n}{(\theta_s + \theta_n)} \tag{4.18}$$

Fifth, the expected profit is defined by:

Figure 4.1: Nodal Pricing



$$\begin{aligned} \overline{\pi}_n &= \underline{b}^S(\theta_s + \theta_n) \\ \overline{\pi}_s &= \underline{b}^S(\theta_s + T) \end{aligned} \tag{4.19}$$

The electricity flows in expectation from the low-demand node to the high-demand node, and the transmission line is congested.

Consumers' surplus is defined by:

$$CS = (P - E(b^S))(\theta_s + \theta_n) \geq 0 \tag{4.20}$$

□

4.2 Exercises

4.2.1 Exercise 1. Zonal pricing with perfect competition: Holmberg and Lazarczyk (2015)

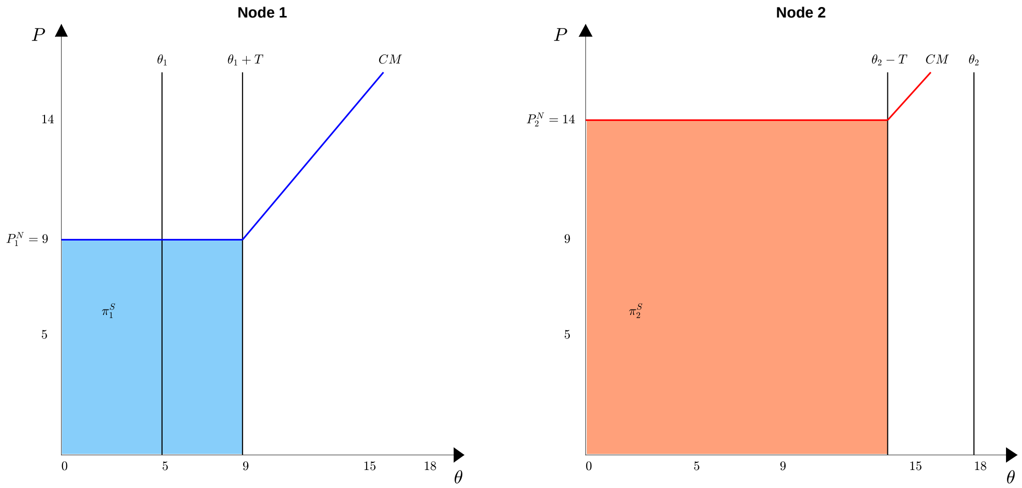
Set up: There are many suppliers located in two different nodes, node 1 and node 2. Both nodes are connected through a transmission line with capacity $T = 4$. The total production capacity installed in each node is equal to 15 ($k_1 = k_2 = k = 15$). The marginal cost of the suppliers located in node 1 and 2 is linear and increasing starting from $MC = 0$ when $k = 0$, and ending with $MC = 15$ when $k = 15$. The demand in each node is inelastic ($\theta_1 = 5$), and ($\theta_2 = 18$).

Question 1 (Nodal Pricing). By using a similar approach that in the example that appears in page 156 in the paper. Work out the equilibrium bids, the equilibrium price and the equilibrium profits.

The market is organized as a uniform price auction. Therefore, the suppliers that are dispatched in the auction sell their production capacity at the last offer bid accepted in the auction.

The suppliers located in node 1 satisfy the demand in their node (θ_1) and sell T units of their demand to node 2. Therefore, the equilibrium price in node 1 is set by the last supplier

Figure 4.2: Discriminatory Pricing



dispatched in that node, and the equilibrium price in node 1 is $\theta_1 + T = 9$.

The equilibrium bids coincide with the marginal costs. By submitting a bid equal to their marginal costs the suppliers with a marginal cost lower than the equilibrium price will be dispatched and will sell their production at that price. The profits of the suppliers dispatched in node 1 are represented by the blue area in figure 4.1. The suppliers with a marginal cost higher than the equilibrium price will not be dispatched and they do not have incentives to deviate.

The suppliers located in node 2 satisfy the demand in that node minus the electricity that flows from node 1. Therefore, the equilibrium price in node 2 is set by the last supplier dispatched in that node, and the equilibrium price in node 2 is $\theta_2 - T = 14$.

The equilibrium bids in node 2 coincide with the marginal cost curve, since nor the suppliers that are dispatched, neither the suppliers that are not dispatched have incentives to deviate. The profits of the suppliers located in node 2 are equal to the orange area in figure 4.1.

Question 2 (Discriminatory Pricing). By using a similar approach that in the example that appears in page 156 in the paper. Work out the equilibrium bids, the equilibrium price and the equilibrium profits.

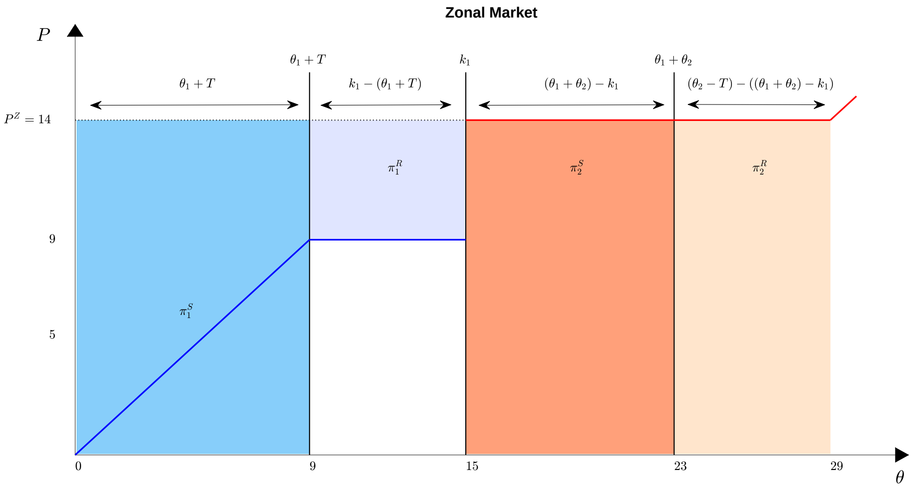
When the auction is discriminatory, the equilibrium price in each node is set by the last supplier dispatched in each node. Therefore, the equilibrium price in each node is as when the auction is discriminatory.

The equilibrium bids are different, since now the suppliers that are dispatched sell their production capacity at their own bid. Therefore, the inframarginal suppliers in each node maximize their bid by submitting a bid equal to the equilibrium price in each node. The overmarginal suppliers submits a bid equal to their marginal cost, since if they are disptached they incur in losses (figure 4.2).

Suppliers' profits are as in the nodal pricing model.

Question 3 (Zonal Pricing) If the market is organized as a zonal pricing electricity market. Work out the equilibrium bids and suppliers' profits in the spot and in the

Figure 4.3: Zonal Pricing



redispatch market in the exporting and in the importing node.

The equilibrium price in the zonal market is 14 since the last supplier that will be dispatched in that market is $\theta_2 - T$.

In the exporting node: The inframarginal suppliers, the ones which a production capacity below $\theta_1 + T$ submits a bid equal to their marginal cost, since they well sell their production capacity at the price 14 in the spot electricity market (dark blue area, figure 4.3). The overmarginal suppliers in the exporting node submits a bid equal to $\theta_1 + T$ so that they will be never dispatched in the spot electricity market, but they will be compensated in the redispatch market, since they cannot sell their production capacity in the spot electricity market. Their profits in the redispatch market are: $\pi_1^R = (14 - 9)(k_1 - (\theta_1 + T)))$ (light-blue area, figure 4.3).

In the importing node: The inframarginal suppliers, the ones with production capacity below $\theta_1 + \theta_2$ submits a bid equal to 14 since that will be the maximum bid accepted in the auction and they will send their entire production at that price. Therefore, their profits in the spot electricity market are $\pi_2^S = 14((\theta_1 + \theta_2) - k_1)$ (dark red area, figure 4.3). The overmarginal suppliers also submit a bid equal to 14, since they know that due to the transmission constraint, they will be called into operation in the redispatch market, and they want to maximize their profit by submitting a bid equal to 14. Therefore, their profits in the redispatch market are $\pi_2^R = 14((\theta_2 - T) - ((\theta_1 + \theta_2) - k_1))$ (light-red area, figure 4.3).

It is important to notice that the sum of the profit areas in the nodal and the discriminatory pricing markets are the same. In contrast, the sum of the profits areas in the zonal pricing market is larger due to the compensation received by the suppliers located in the exporting node (light-blue are, figure 4.3).

4.2.2 Exercise 2. Zonal pricing with imperfect competition: Blázquez (2019)

Set up: There is a nodal market with two nodes, node S and node N connected by a transmission line with capacity $T = 40$. There are two suppliers with production capacities $k_s = k_n = 60$ and production costs $c_s = c_n = 0$, where supplier s is located in node S, and supplier n is located in node N. The demand in node S is inelastic an equal to $\theta_s = 4$, and the demand in node N is also inelastic an equal to $\theta_n = 65$. The reservation price is equal to $P = 7$.

Figure 4.4: Model comparison. Suppliers' profits functions ($\theta_s = 5, \theta_n = 65$, $k_s = k_n = k = 60$, $T = 40$, $c_s = c_n = 0$, $P = 7$)

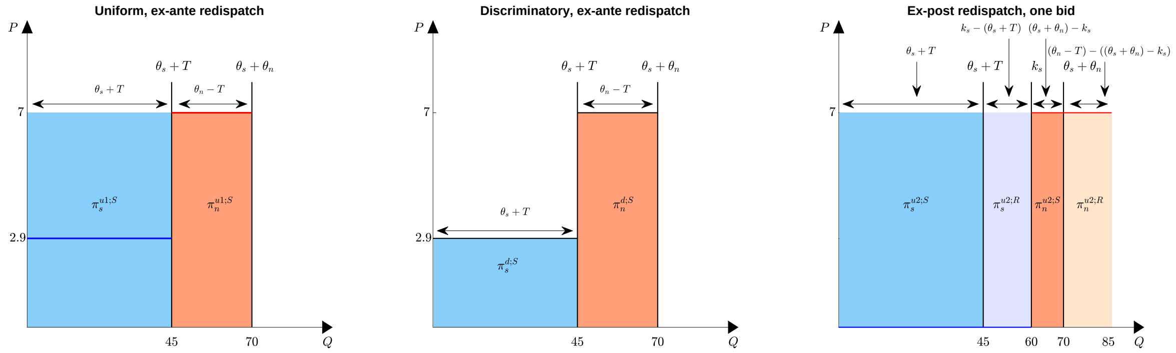


Table 4.1: Model comparison ($\theta_s = 5, \theta_n = 65$, $k_s = k_n = k = 60$, $T = 40$, $c_s = c_n = 0$, $P = 7$)

Design	b_s^S	b_n^S	P^S	b_s^R	b_n^R	$\underline{b}$	$E(b_s^S)$	$E(b_n^S)$	$E(b^S)$	π_n	π_s	CS
I	$[0, 2.9]$	7	7	–	–	–	–	–	–	175	315	0
II	–	–	–	–	–	2.9	4.4	5.03	4.98	175	131.2	140.9
III	0	7	7	–	–	–	–	–	–	175	420	0

I: Ex-ante redispatch, uniform. II: Ex-ante redispatch, discriminatory. III: Ex-post redisptach, one bid.

Question 1. Work out the equilibrium when the auction is uniform and an ex-ante redispatch mechanism is introduced by the auctioneer.

When the auction in the spot electricity market is uniform and an ex-ante redispatch mechanism is introduced by the auctioneer, the supplier located in the high-demand node submits the maximum bid allowed by the auctioneer setting the price in the zonal market (columns 3 and 4, table 4.1). The supplier located in the low-demand node submits a bid that makes undercutting unprofitable (column 2, table 4.1). In that case, the supplier located in the high demand node satisfies the residual demand, the supplier in the low-demand node satisfies the demand in its own node and the demand in the other node up to the transmission capacity, and suppliers' profits are defined by equation 4.9 (columns 11 and 12, table 4.1; left-hand panel, figure 4.4). Finally, given that the equilibrium price is equal to the reserve price, consumers' surplus is zero (equation 4.10; column 13, table 4.1).

Question 2. Work out the equilibrium when the auction is discriminatory and an ex-ante redispatch mechanism is introduced by the auctioneer.

When the auction in the spot electricity market is discriminatory and an ex-ante redispatch mechanism is introduced by the auctioneer, the equilibrium is in mixed strategies and the supplier located in the high-demand node submits higher bids in expectation (equation 4.17; columns 9 and 10, table 4.1). Suppliers' profits are defined by equation 4.19 (column 13, table 4.1; central panel, figure 4.4). Consumers' surplus is positive, since the expected equilibrium price is lower than the reserve price (equations 4.18 and 4.20; columns 10 and 13, table 4.1).

Question 3. Work out the equilibrium when the auction is uniform, an ex-post re-

dispatch mechanism is introduced by the auctioneer, and the suppliers submits the same bid in the spot and in the redispatch market.

When the auction is uniform, an ex-post redispatch mechanism is introduced by the auctioneer and the suppliers submit the same bid in the spot and in the redispatch market, the supplier located in the high-demand node submits the maximum bid allowed by the auctioneer setting the price in the zonal market (columns 3 and 4, table 4.1). Supplier n satisfies the demand in the spot electricity market and its profits in that market are defined as $\pi_n^{u2;S} = 7(65+5-60) = 70$ (dark-red area, right-hand panel, figure 4.4). Due to the transmission constraint, supplier s cannot sell its entire production capacity, and in the redispatch market, supplier n is called into operation again to sell the production capacity that it could not sell in the spot electricity market, and its profit in that market are defined as $\pi_n^{u2;R} = 7((65-40)-(65+5-60)) = 105$ (light-red area, right-hand panel, figure 4.4). Supplier n's profits in the spot and in the redispatch market are defined in equation 4.12. The sum of supplier n's profits in both markets is $\pi_n^{u2;S} + \pi_n^{u2;R} = 175$ (column 11, table 4.1).

The supplier located in the low-demand node submits a bid equal to zero in the spot electricity market to make undercutting unprofitable (column 2, table 4.1). By using equation 4.7 to work out the equilibrium profits, we obtain an useful economic interpretation of supplier s's profits. Supplier s' profits can be calculated as the profits that it obtains by selling its production capacity up to transmission capacity in the spot electricity market $\pi_s^{u2;S} = 7(5+40) = 315$, plus the compensation that it receives for not being able to sell its entire production capacity in the spot electricity market $\pi_s^{u2;R} = (7-0)(60-(5+40)) = 105$ (dark-blue area and light-blue area, right-hand panel, figure 4.4). By summing the profits in both markets, we obtain supplier s's profits $\pi_s^{u2;S} + \pi_s^{u2;R} = 315 + 105 = 420$ (equation 4.12; column 12, table 4.1).

By using equation 4.6 instead of equation 4.7, supplier s it is dispatched first in the spot electricity market, selling its entire production capacity at the price set by supplier n, and its profits are defined as $\pi_s^{u2;S} + \pi_s^{u2;R} = 7(60) = 420$ (sum of the dark-blue and light-blue areas, right-hand panel, figure 4.4).[4] Due to the transmission constraint, supplier s cannot sell its entire production capacity in the high-demand node, and it has to buy back the production capacity that it cannot sell in that node. Given that supplier s has to submit the same bid in both markets, and that the auction in the redispatch market is discriminatory, supplier s's expenses in that market are defined as $e_s^{u2;R} = 0(60-(5+40)) = 0$. By subtracting the expenses in the redispatch market from the profits in the spot electricity market, I obtain supplier s's profits $\pi_s^{u2;S} + \pi_s^{u2;R} - e_s^{u2;R} = 420 - 0 = 420$ (equation 4.12; column 12, table 4.1).

As when a redispatch mechanism is introduced ex-ante by the auctioneer, consumers' surplus is zero since the equilibrium price is equal to consumers' reserve price (equation 4.13, annex 2; column 13, table 4.1).

Question 4. By comparing the three market designs, which will be the impact of those designs on long-term investment decisions and consumers' welfare?

The change on supplier s's profits induced by the changes on the design could induce distortions on investment decisions. In particular, when the auction in the spot electricity market is discriminatory and an ex-ante redispatch mechanism is introduced by the auctioneer, the suppliers want to invest in the high-demand node, since the equilibrium profits in that node are larger $\pi_n^{d;S} \geq \pi_s^{d;S}$. When the auction in the spot electricity market is uniform and an ex-ante

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[4] It is important to notice that supplier n's profits in the spot electricity market should be a single area that covers the dark-blue and the light-blue areas. We explain supplier n's profits by summing $\pi_s^{u2;S} + \pi_s^{u2;R}$ to avoid introducing more graphs.

redispatch mechanism is introduced by the auctioneer, the suppliers want to invest in the low-demand node since $\pi_s^{u1;S} \geq \pi_n^{u1;S}$. The introduction of an ex-post redispatch market makes even more attractive to invest in the low-demand node ($\pi_s^{u2;S} + \pi_s^{u2;R} > \pi_s^{u1;S} > \pi_n^{u1;S}$). Therefore, the introduction of different redispatch designs change suppliers' profits and that could have important investment implications in the long-term.

The introduction of different redispatch designs also affect consumers' surplus. In particular, when the auction is uniform and for any type of redispatch mechanism, consumers' surplus is zero, since the equilibrium price is equal to consumers' reserve price. In contrast, when the auction in the spot electricity market is discriminatory and an ex-ante redispatch mechanism is introduced by the auctioneer, the equilibrium price is lower than consumers' reserve price and consumers' surplus is positive.

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Chapter 5

Experiments in Electricity Markets

In chapter 2, we characterize the equilibrium in an uniform and in a discriminatory price auction. When the suppliers face a positive residual demand, the characterization of the equilibrium depends crucially on the type of auction. In particular, when the auction is discriminatory, an unique mixed strategies equilibrium exists. In contrast, when the auction is uniform multiplicity of pure strategies equilibrium exist.

In this chapter, we study different equilibrium selection techniques to study which equilibrium will be selected in an uniform price auction. In particular, we apply the risk dominance method (Harsanyi and Selten, 1988), the robustness to strategic uncertainty method (Andersson, Argenton and Weibull, 2014) and the quantal response method (McKelvey and Palfrey, 1998) to predict which equilibrium is selected by the players. We also play a game that simulate a uniform price auction, and analyze the strategies selected by the players in that game to evaluate if those results are in line with the theoretical predictions.

The chapter is organized as follows: In section 5.1, we present the game, we also play that game in the lectures to get adquantied with the game. In section 5.3, we present the main equilibrium selection techniques to predict which equilibrium will be played in the game. In section 5.4 we present the statistical analysis of the game played by the students in section 5.1.

5.1 The game

The set up and the characterization of the equilibrium is similar to the one in chapter 2. However, we present the set up and the characterization of the equilibrium, since this chapter present some particularities that it is necessary to explain.

Set up: There are two players with production capacity k_i and k_j, where $k_i > k_j$. The level of demand, θ is independent of market price, i.e., perfectly inelastic. Moreover, $\theta \in [k_i, k_i + k_j]$, i.e., the demand is large enough to guarantee that both players face a positive residual demand. We introduce this assumption because when the demand is very low, both players have enough production capacity to satisfy the demand, and in that case the Nash equilibrium is unique, and it has no sense to study any type of equilibrium selection technique.

Timing: Having observed the realization of demand θ, each player simultaneously and independently submits a bid specifying the minimum price at which it is willing to supply up to its capacity, $b_i \in [b_{min}, P]$, $i = 1, 2$, where b_{min} and P are determined by the auctioneer. The players can only submit bids higher or equal than b_{min} and lower or equal than $b_{max} = P$.[1]

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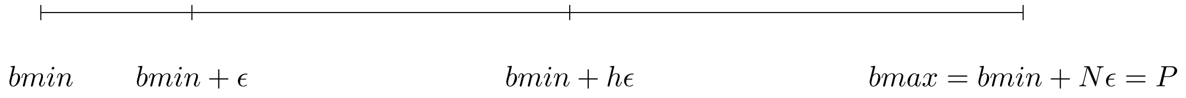
[1] The minimum bid in the auction (b_{min}) and the maximum bid (P) are determined by the auctioneer. The minimum bid guarantees a minimum profit for the players. The maximum bid represents the reservation price

The number of bids in that interval (N) is determined exogenously and it can be as large as we wanted. The distance between one bid and the next one is defined by $\epsilon = \frac{P - b_{min}}{N}$. The set of strategies is represented in figure 5.1.

Figure 5.1: Strategies set



Let $b \equiv (b_1, b_2)$ denote a bid profile. On basis of this profile, the auctioneer calls players into operation. The output allocated to player $i, i = 1, 2$, denoted by $q_i(b; \theta, k)$, is given by[2]

$$q_i(b; \theta, k) = \begin{cases} k_i & \text{if } b_i < b_j \\ \dfrac{k_i \theta}{k_i + k_j} & \text{if } b_i = b_j \\ \theta - k_j & \text{if } b_i > b_j \end{cases} \tag{5.1}$$

When player i submits the lower bid, it sells her entire production capacity ($q_i = k_i$). When both players submit the same bid, the demand is split among them in proportion to their production capacity $\left(q_i = \frac{k_i \theta}{k_i + k_j}\right)$. When player i submits the higher bid in the auction, it satisfies the residual demand ($q_i = \theta - k_j$).

Finally, the payments are worked out by the auctioneer. When the auctioneer runs a uniform price auction, the price received by a player for any positive quantity dispatched by the auctioneer is equal to the higher offer price accepted in the auction. Hence, for a given realization of demand θ and a bid profile $b \equiv (b_1, b_2)$, player i's payoffs, $i = 1, 2$, are expressed as[3]

$$\pi_i(b; \theta, k) = \begin{cases} b_j k_i & \text{if } b_i < b_j \\ b_i \dfrac{k_i}{k_i + k_j} \theta & \text{if } b_i = b_j \\ b_i(\theta - k_j) & \text{if } b_i > b_j \end{cases} \tag{5.2}$$

When player i submits the lower bid, it sets the price in the auction and sells her entire production capacity $\pi_i = b_i k_i$. These are the payoffs over the diagonal in figure 5.3. When players i and j submit the same bid, the payoff is split among them in proportion to their production capacity $\pi_i = b_i \frac{\theta k_i}{k_i + k_j}$. These are the payoffs on the diagonal in figure 5.3. When player i submits the higher bid, it sets the price and satisfies the residual demand $\pi_i = b_i(\theta - k_j)$. These are the payoffs below the diagonal in figure 5.3.

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for the consumers of the good.

[2] It is important to emphasize that $q_i(b; \theta, k)$ is only valid under the assumptions of the model which stablish that $\theta \in [k_j, k_i + k_j]$. When $\theta < k_j$, $q_i(b; \theta, k)$ is slightly different, since in that case both players have enough production capacity to satisfy the entire demand and the equilibrium is unique. For a complete analysis of the uniform price auction when the demand is low see Fabra, von der Fehr and Harbord (2006).

[3] As with $q_i(b; \theta, k)$, $\pi_i(b; \theta, k)$ is slightly different when the assumptions of the model are relaxed.

Figure 5.2: Generalized Hawk-Dove and Battle of the Sexes payoff matrices

Hawk-Dove game ($a > b > c > d$)

b_j b_i	H	D
H	d, d	a, c
D	c, a	b, b

Battle of the sexes game ($a, b > c, d, e, f$)

b_j b_i	H	D
H	a, b	c, d
D	e, f	b, a

The tie breaking rule implemented in this game is crucial since it determines if the game is a Hawk-Dove or a Battle of the Sexes game.[4] According with the footnote 1 in Cabrales, García-Fontes and Motta (2000), the Battle of the Sexes game defined in Luce and Raiffa (1957) and the Hawk-Dove game defined in Binmore (1992) are equivalent. However, the payoff matrix in Benndorf, Martínez-Martínez and Normann (2016) (left-hand panel, figure 5.2) and the one in Belleflamme and Peitz (2015) (right-hand panel, figure 5.2) show that those games are different. Moreover, Tirole and Fudenberg (1991) study the Hawk-Dove and the Battle of the Sexes games, but the matrix that they present to characterize the Hawk-Dove game does not coincide with the one in Benndorf, Martínez-Martínez and Normann (2016). In this paper, we assume that a game has the structure of a Hawk-Dove game when it follows the structure presented in Benndorf, Martínez-Martínez and Normann (2016).

Equilibrium: The uniform price auction described above has multiplicity of pure strategies equilibria in which one player submits the maximum bid allowed by the auctioneer (dove strategy), and the other submits a bid that makes undercutting unprofitable (hawk strategy). When the players are asymmetric in production capacity, the number of equilibria in which the player with higher production capacity submits the maximum bid are larger than the number of equilibria in which the player with lower production capacity submits the maximum bid. Those sets of equilibria are represented in the dark grey cells in figure 5.3. To provide a better understanding of the uniform price auction and the set of equilibria in that game, in section 5.2, we provide an illustrative example taken from Bigoni, Blázquez and le Coq (2018).

The players have opposite preferences on both set of equilibria. Both players prefer the set of equilibria in which the other player submits the higher bid allowed by the auctioneer (dove strategy), since in that case the player that is dispatched first sells her entire production capacity at the maximum price allowed by the auctioneer. It could be possible that both players coordinate by submitting the maximum bid allowed by the auctioneer. In that case, the price perceived by the players is the maximum price allowed by the auctioneer and the players split the profit in proportion to their production capacity. However, it is very difficult to coordinate in that pair of strategies, since both players have incentives to deviate to sell their entire production capacity at the maximum price allowed by the auctioneer.

In section 5.3, we apply the tracing procedure method proposed by Harsanyi and Selten (1988), the robustness to strategic uncertainty method proposed by Andersson, Argenton, and Weibull (2014), and the quantal response method proposed by McKelvey and Palfrey (1998) to analyze which of those equilibria is played in the game.

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[4] When the auction is discriminatory, the tie-breaking rule is also very important, but for different reasons. In that case, the tie-breaking rule is important to guarantee the existence of the equilibrium (Blázquez, 2018; Dasgupta and Maskin, 1986; Fabra, von der Fehr and Harbord, 2006; Osborne and Pitchik, 1986).

Figure 5.3: Payoff matrix in a uniform price auction

b_j b_i	b_{min}	$\cdots$	$b_{min} + h\epsilon$	$\cdots$	$b_{min} + N\epsilon = P$
b_{min}	$\pi_i(b_{min}, b_{min}) =$ $b_{min}\frac{\theta k_i}{k_i + k_j}$ $\pi_j(b_{min}, b_{min}) =$ $b_{min}\frac{\theta k_j}{k_i + k_j}$	$\cdots$	$\pi_i(b_{min}, b_{min} + h\epsilon) =$ $(b_{min} + h\epsilon)k_i$ $\pi_j(b_{min}, b_{min} + h\epsilon) =$ $(b_{min} + h\epsilon)(\theta - k_i)$	$\cdots$	$\pi_i(b_{min}, b_{min} + N\epsilon) =$ $(b_{min} + N\epsilon)k_i$ $\pi_j(b_{min}, b_{min} + N\epsilon) =$ $b_{min}(\theta - k_i)$
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$b_{min} + h\epsilon$	$\pi_i(b_{min} + h\epsilon, b_{min}) =$ $(b_{min} + h\epsilon)(\theta - k_j)$ $\pi_j(b_{min} + h\epsilon, b_{min}) =$ $(b_{min} + h\epsilon)k_j$	$\cdots$	$\pi_i(b_{min} + h\epsilon, b_{min} + h\epsilon) =$ $(b_{min} + h\epsilon)\frac{\theta k_i}{k_i + k_j}$ $\pi_j(b_{min} + h\epsilon, b_{min} + h\epsilon) =$ $(b_{min} + h\epsilon)\frac{\theta k_j}{k_i + k_j}$	$\cdots$	$\pi_i(b_{min} + h\epsilon, b_{min} + N\epsilon) =$ $(b_{min} + N\epsilon)k_i$ $\pi_j(b_{min} + h\epsilon, b_{min} + N\epsilon) =$ $(b_{min} + h\epsilon)(\theta - k_i)$
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$b_{min} + N\epsilon = P$	$\pi_i(b_{min} + N\epsilon, b_{min}) =$ $(b_{min} + N\epsilon)(\theta - k_j)$ $\pi_j(b_{min} + N\epsilon, b_{min}) =$ $(b_{min} + N\epsilon)k_j$	$\cdots$	$\pi_i(b_{min} + N\epsilon, b_{min} + h\epsilon) =$ $(b_{min} + N\epsilon)(\theta - k_j)$ $\pi_j(b_{min} + N\epsilon, b_{min} + h\epsilon) =$ $(b_{min} + N\epsilon)k_j$	$\cdots$	$\pi_i(b_{min} + N\epsilon, b_{min} + N\epsilon) =$ $(b_{min} + N\epsilon)\frac{\theta k_i}{k_i + k_j}$ $\pi_j(b_{min} + N\epsilon, b_{min} + N\epsilon) =$ $(b_{min} + N\epsilon)\frac{\theta k_j}{k_i + k_j}$

5.2 An example

In this section, we present an example of an uniform price auction that follows the set up and the timing described in the previous section.

To facilitate the understanding of the game and the experiment results presented in section 5.4, we play several rounds of the game summarized in the payoff matrix 5.4. The students will play the game in pairs using the payoff matrix in figure 5.4. One of the students will be the red player, and the other will be the blue player. Each player has to select one of the strategies in the matrix independently and simultaneously. Then, the two players show their strategies and write their profits.

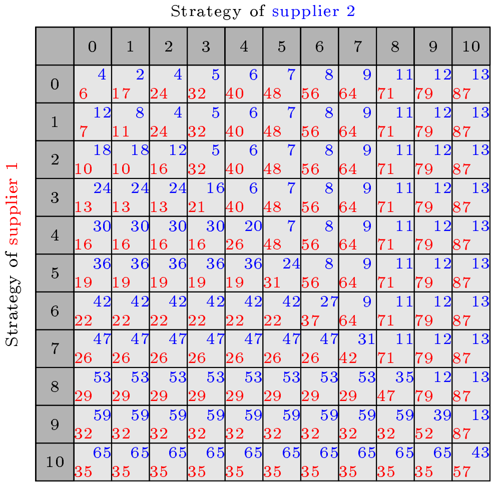
The students repeat this game during ten rounds. After those ten rounds, new pairs are created and the students play the game another ten rounds.

In section 5.3, we apply different equilibrium selection techniques to understand which equilibrium will be played in this game. In section 5.4, we analyze statistically the results of this game that have been studied in detail in Bigoni et al. (2019).

5.3 Equilibrium selection techniques

In this section, we apply the risk dominance method (Harsanyi and Selten, 1988), the robustness to strategic uncertainty method (Andersson, Argenton and Weibull, 2014) and the quantal response method (McKelvey and Palfrey, 1998) to predict which equilibrium will be played in the uniform price auction.

Figure 5.4: Example. Payoff matrix



Strategy of supplier 2

Strategy of supplier 1

	0	1	2	3	4	5	6	7	8	9	10
0	4 6	2 17	4 24	5 32	6 40	7 48	8 56	9 64	11 71	12 79	13 87
1	12 7	8 11	4 24	5 32	6 40	7 48	8 56	9 64	11 71	12 79	13 87
2	18 10	18 10	12 16	5 32	6 40	7 48	8 56	9 64	11 71	12 79	13 87
3	24 13	24 13	24 13	16 21	6 40	7 48	8 56	9 64	11 71	12 79	13 87
4	30 16	30 16	30 16	30 16	20 26	7 48	8 56	9 64	11 71	12 79	13 87
5	36 19	36 19	36 19	36 19	36 19	24 31	8 56	9 64	11 71	12 79	13 87
6	42 22	42 22	42 22	42 22	42 22	42 22	27 37	9 64	11 71	12 79	13 87
7	47 26	47 26	47 26	47 26	47 26	47 26	47 26	31 42	11 71	12 79	13 87
8	53 29	53 29	53 29	53 29	53 29	53 29	53 29	53 29	35 47	12 79	13 87
9	59 32	59 32	59 32	59 32	59 32	59 32	59 32	59 32	59 32	39 52	13 87
10	65 35	65 35	65 35	65 35	65 35	65 35	65 35	65 35	65 35	65 35	43 57

5.3.1 Risk dominance method

In this section, we use the risk dominance method proposed by Harsanyi and Selten (1988) to study which equilibrium is selected by the players.

The risk dominance method assumes that players' payoffs are a linear combination of the original payoff matrix and the expected payoff matrix based on players' believes.

$$\pi_i = t\pi_i(b_i, b_j) + (1-t)\pi_i(b_i, p_j), \tag{5.3}$$

where p_j is the probability that player j assigns to each strategy based on player i's believes. Therefore, when $t = 0$, players' payoffs are determined only by players' expected profit based on their prior believes. When $t = 1$, players' payoffs are determined only by the original payoff matrix.

In general, at $t = 0$ the players choose a pair of strategies that is not an equilibrium of the original game. When t increases the players change their strategies. At some $t \in [0, 1]$, the players chose a pair of strategies that is an equilibrium of the original game. That pair of strategies (b_i^*, b_j^*) will be the equilibrium selected by the tracing procedure. Therefore, the key point in the tracing procedure is to find the player that first deviates to a Nash equilibrium in the original game, and to find the parameter t for which that player deviates to the equilibrium.

If we assume that the players' believes follow a uniform probability distribution, the players maximize their expected payoffs by submitting the lower bid in the auction (left-panel, figure 5.5).[5] Therefore, when $t = 0$, the equilibrium selected by the risk dominance method is the one in which both players submit the lower bid in the auction ($b_i = b_j = b_{min}$), (right-panel, figure 5.5).

As can be observed in figure 5.5, the pair of strategies ($b_i = b_j = b_{min}$) it is not an equilibrium of the original game. According with equation 5.3, when the parameter t increases, the players give more importance to the payoff of the original game (figures 5.3 and 5.4). Therefore, when t increases one of the players deviates from b_{min} by increasing its bid. In particular the player

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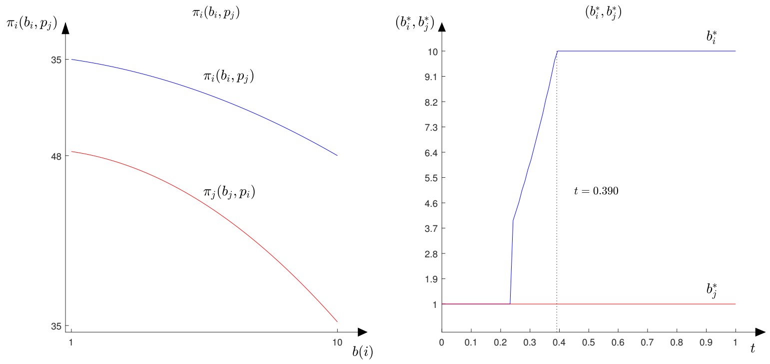
[5] This statement is only true if the demand is high enough to guarantee that both players face a positive residual demand, but the demand is not to high. For a complete characterization of the equilibrium for any type of demand, see Blázquez and Koptyug (2019).

Figure 5.5: Expected payoffs and equilibrium ($k_i = 8.7$, $k_j = 6.5$, $b_{min} = 1$, $b_{max} = 10$, $N = 110$)



with higher production capacities deviates first, since it faces a high residual demand and it is less "risky" for it to submit a high bid. The player with higher production capacity continues increasing its bid until both players select the equilibrium in which the player with higher production capacity submits the maximum bid allowed by the auctioneer, and the player with lower production capacity submits the minimum bid allowed by the auctioneer ($b_i = b_{min}, b_j = P$) (left-hand panel, figure 5.5).

5.3.2 Robustness to strategic uncertainty method

In this section we apply the robustness to strategic uncertainty method presented by Andersson, Argenton and Weibull (2014) to determine the equilibria played in the game.

In the robustness to strategic uncertainty method the players face some uncertainty on the strategies played by other players. Player i's uncertainty on player j's strategy is modelled as follows:

$$b_{ij}^t = b_j^t + t\epsilon_{i,j} \;\forall j \neq i, \tag{5.4}$$

where the random variables $\epsilon_{i,j} \sim \phi_{ij}$ are statistically independent.

Equation 5.4 can be interpreted as follows: player i thinks that player j will play strategy b_j plus some random perturbation. When the uncertainty parameter (t-parameter) goes to zero, the players do not face any type of uncertainty.

For $t > 0$, the random variable $b_{i,j}$ has probability density defined by:

$$f_{i,j}^t = \frac{1}{t}\phi_{i,j}\left(\frac{x - b_j^t}{t}\right)$$

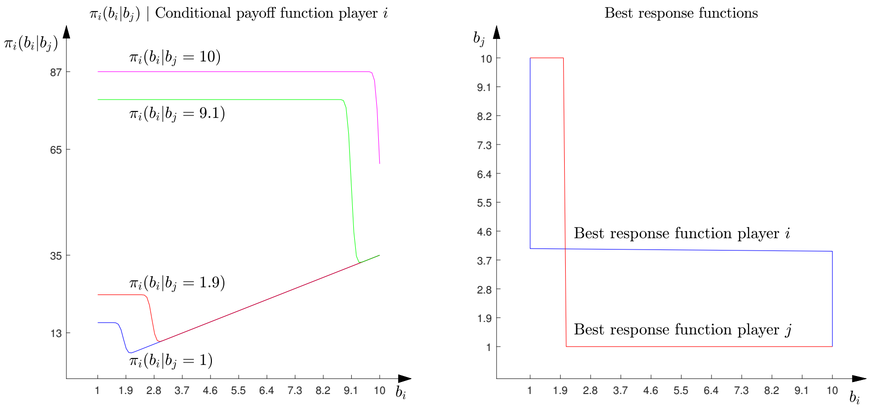
And the profit function is defined by:

$$\pi_i^t(b) = E\left[\pi(b_i, b_{-i}^t)\right] = \int \left[\pi_i(b_i, x_{-i}) f_{i,j}^t(b_j)\right] \partial x_{-i} \tag{5.5}$$

A pair of strategies (b_i^*, b_j^*) is a t-equilibrium of the game if b_i^* and b_j^* maximize 5.5. Therefore, to find the t-equilibrium of the game is enough to work out the best response functions and to find the intersection between them.

If we apply equation 5.5 to the payoff function in the uniform price auction model defined by equation 5.2 in the model section we obtain:

Figure 5.6: Players' conditional payoff functions ($k_i = 8.7$, $k_j = 6.5$, $\theta = 10$, $b_{min} = 1$, $b_{max} = 10$, $N = 110$)



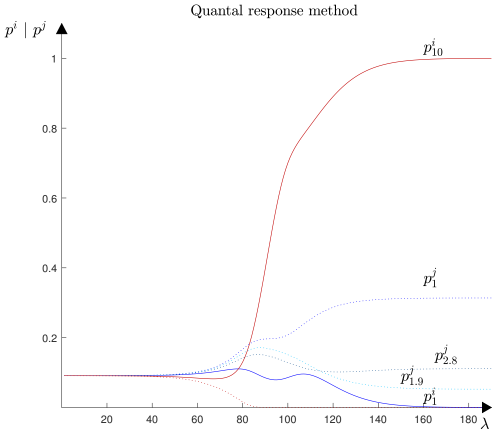
$$
\begin{aligned}
\pi_i^t(b_i, b_j^t) &= E\left[\pi(b_i, b_j^t)\right] = \int \left[\pi_i(b_i, x_{-i}) f_{i,j}^t(b_j)\right] \partial x_{-i} \\
&= b_j k_i \left[1 - \Phi_{i,j}\left(\frac{b_i - b_j}{t}\right)\right] + b_i(\theta - k_j)\left[\Phi_{i,j}\left(\frac{b_i - b_j}{t}\right)\right],
\end{aligned} \tag{5.6}
$$

where $\Phi_{i,j}$ is the cumulative distribution function of $\phi_{i,j}$. The first term in equation 5.6 $\left(b_j k_i \left[1 - \Phi_{i,j}\left(\frac{b_i - b_j}{t}\right)\right]\right)$ represents player i's expected payoff when it submits the lower bid in the auction. With probability $\left[1 - \Phi_{i,j}\left(\frac{b_i - b_j}{t}\right)\right]$ player i submits the lower bid in the auction. In that case, player j sets the price (b_j), and player i's sells its entire production capacity (k_i). The second term in equation 5.6 $\left(b_i(\theta - k_j)\left[\Phi_{i,j}\left(\frac{b_i - b_j}{t}\right)\right]\right)$ represents supplier i's expected payoff when it submits the higher bid in the auction. With probability $\left[\Phi_{i,j}\left(\frac{b_i - b_j}{t}\right)\right]$ player i submits the higher bid in the auction. In that case, player i sets the price (b_i) and satisfies the residual demand ($\theta - k_j$).

To find the equilibrium selected by the robustness to strategic method, it is enough to work out players' best response functions. By using equation 5.6, it is easy to work out one player's expected payoff given the strategy of the other player. In particular, we set b_j and vary b_i between b_{min} and b_{max}. Knowing that the random variable $b_{i,j}$ has probability density defined by $f_{i,j}^t = \frac{1}{t}\phi_{i,j}\left(\frac{x - b_j^t}{t}\right)$, and if we assume that $f_{i,j}^t \sim N(0,1)$, we work out player i' expected payoff $\pi_i(b_i \mid b_j)$, and we choose b_i that maximizes that payoff. Repeating that process for every $b_j \in [b_{min}, b_{max}]$, we work out player i's best response function. By using the same approach we work out player j's best response function. The intersection between both players' best response functions determines the equilibrium selected by the robustness to strategic uncertainty method.

To understand the best response functions, it is useful to work out the expected payoff for one player when we set the strategy played by the other player. In the left-hand panel of figure 5.6 we plot four of the expected payoff functions for player i. When player j sets a low bid, player i maximizes its expected payoff by submitting the maximum bid allowed by the auctioneer. In contrast, when player j sets a high bid, player i maximizes its expected payoff by submitting

Figure 5.7: Quantal Response Equilibria ($k_i = 8.7$, $k_j = 6.5$, $\theta = 10$, $b_{max} = 10$, $N = 11$)



low bids. In both cases, as can be observed in figure 5.6, players' expected payoff functions are concave and therefore, players maximize their expected payoff by submitting low or high bid, but never by submitting intermediate bids.

The analysis of players' expected payoff functions is useful to understand the equilibrium selected by the robustness to strategic uncertainty method. In figure 5.6 we plot players' best response functions. The intersection of the best response functions selects two of the Nash equilibria in the original game. In each of these two equilibria, one player submits the higher bid allowed by the auctioneer and the other player submits the lower bid allowed by the auctioneer.

5.3.3 Quantal response method

As in the previous sections, we present the quantal response method and we study the equilibrium selected by this method when we apply it to the uniform price auction defined in the model section.

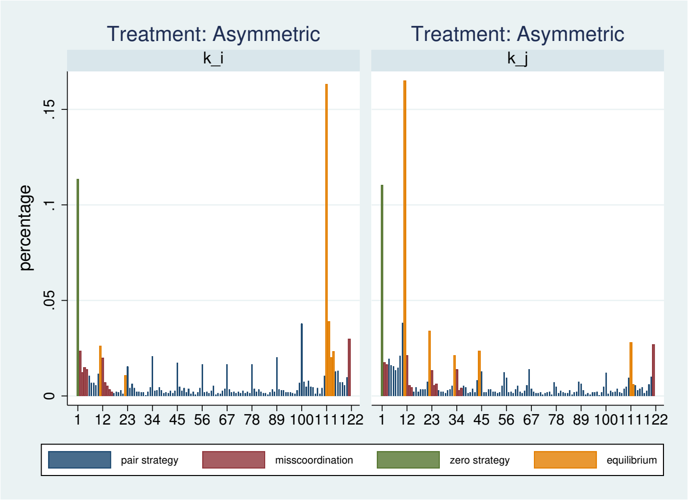
The quantal response method assumes that the players choose among the strategies in the game based on their relative expected payoff. The key idea in the quantal response method is that when the players calculate their expected payoff, they make calculation errors according to some random process. The players assign more probability to the strategies that give them a higher expected payoff. The Nash equilibrium in the quantal response method is the set of probabilities for which none of the players wants to deviate.

In the seminal paper to study the quantal response method, McKelvey and Palfrey (1998) use the logistic quantal response function. That specific function is a particular parametric class of quantal response functions that has a long tradition in the study of individual choice behaviour. The logit equilibrium is the correspondence $\pi^* : \Re \longrightarrow 2^{\triangle}$ given by:

$$\pi^*(\lambda) = \left\{ \pi \in \triangle : \pi_{ij} = \frac{e^{\lambda \overline{u}_{ij}(\pi)}}{\sum_{k=1}^{J_i} e^{\lambda \overline{u}_{ik}(\pi)}} \forall i, j \right\}, \tag{5.7}$$

where the term in the numerator $\left(e^{\lambda \overline{u}_{ij}(\pi)}\right)$ is one of the players' expected payoff when it selects strategy i and the other player selects strategy j. The term in the denominator $\left(\sum_{k=1}^{J_i} e^{\lambda \overline{u}_{ik}(\pi)}\right)$ is the sum of one of the players' expected payoff when it selects strategy i and the other player selects all the strategies in its strategies set. Therefore, by using equation 5.7 each player assigns

Figure 5.8: $k_i = 8.7$; $k_j = 6.5$; $\theta = 10$; $b_{min} = 1$; $b_{max} = 10$, $N = 11$



more probability to the strategies that give it higher expected payoff.

The Nash equilibrium in the quantal response method is defined as follows: Given $\{\lambda_1, \lambda_2, ...\}$ a sequence such that $\lim_{t\longrightarrow\infty} \lambda_t = \infty$, and $\{p_1, p_2, ...\}$ a corresponding sequence with $p_t \in \pi^*(\lambda_t)$ for all t, such that $\lim_{t\longrightarrow\infty} p_t = p^*$, then p^* is a Nash equilibrium.

When we apply the quantal response method to the uniform price auction presented in the model section we observe that the player with larger production capacity (player i) plays the maximum bid with a probability close to one. In contrast, the player with lower production capacity (player j) assigns higher probabilities to the lower bids (figure 5.7). Therefore, the quantal response method selects the equilibrium in which the player with higher production capacity submits the higher bid and the player with lower production capacity submits the lower bid.

The equilibrium selected by the quantal response method is in line with the equilibrium selected by the tracing and the robustness to strategic uncertainty methods. Moreover, the pattern that appears in the equilibrium selected by the quantal response method is very similar to the pattern that appears in the other two methods, since in the three methods the players tend to select extreme strategies. In particular, the player with higher production capacity submits the maximum bid allowed by the auctioneer and the player with lower production capacity submits the lower bid allowed by the auctioneer.

5.4 Experimental results

In this section we study if the theoretical results presented in section 5.3 are in line with the results of the game played in section 5.2.[6]

According with the theoretical analysis conducted in section 5.3, there are three different theoretical predictions that can be tested: First, according with the risk dominance method, the equilibrium selected by the players is $(b_i = b_{max}, b_j = b_{min})$; second, according with the robustness to strategic uncertainty method, the equilibrium selected by the players is either $(b_i = b_{min}, b_j = b_{max})$, or $(b_i = b_{max}, b_j = b_{min})$; finally, according with the quantal response method the equilibrium selected by the players is $(b_i = b_{max}, b_j = p_j)$, where p_j is the probability

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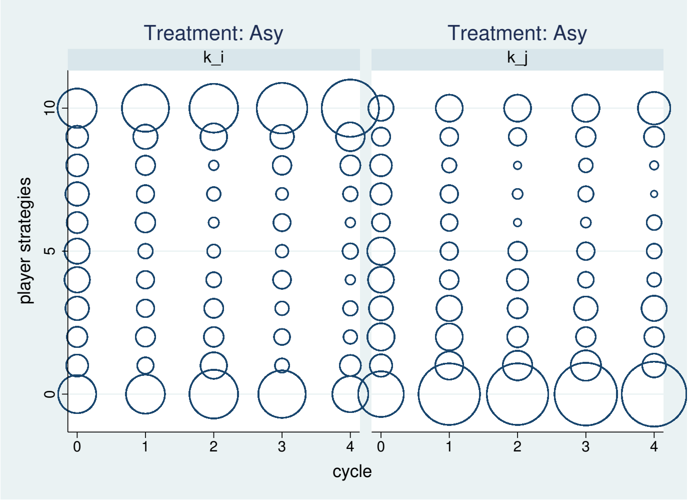
[6] For a detailed statistical analysis of the experiment conducted in section 5.2 see Bigoni et al. (2019).

Figure 5.9: $k_i = 8.7$; $k_j = 6.5$; $\theta = 10$; $b_{min} = 1$; $b_{max} = 10$, $N = 11$



that player j assigns to each strategy in the quantal response equilibrium.

To test those theoretical predictions we follow two different approaches: First, we use an histogram to visualize which pairs of strategies are played in the game; second, we use a scatter plot to study which strategies are played in the game.

In figure 5.8, we have plot the histogram for each player and for each pair of strategies. We have enumerate each pair of strategies as follows: Pair 1 corresponds with the strategies $(b_i = b_{min} = 1, b_j = b_{min} = 1)$; pair 2 corresponds with the strategies $(b_i = b_{min} = 1, b_j = 1.81)$; pair 3 corresponds with the strategies $(b_i = b_{min} = 1, b_j = 2.63)$; pair 11 corresponds with the strategies $(b_i = b_{min} = 1, b_j = b_{max} = 10)$; pair 110 corresponds with the strategies $(b_i = b_{max} = 10, b_j = b_{min} = 1)$; pair 111 corresponds with the strategies $(b_i = b_{max} = 10, b_j = 1.81)$; pair 112 corresponds with the strategies $(b_i = b_{max} = 10, b_j = 2.63)$; pair 121 corresponds with the strategies $(b_i = b_{max} = 10, b_j = b_{max} = 10)$.

Based on the histogram information that appears in figure 5.8 we can tested the theoretical predictions. In particular, according with the risk dominance method, the pair of strategies that is more played by the players is the pair 111 $(b_i = b_{max}, b_j = b_{min})$ seen from player i perspective, or the pair 11 $(b_i = b_{max}, b_j = b_{min})$ seen from player j perspective, and that is the result that we obtain in figure 5.8. According with the robustness to strategic method, the players select either the pair of strategies $(b_i = b_{min}, b_j = b_{max})$ (pair 11 seen from player i perspective, or pair 111 seen from player j perspective), or $(b_i = b_{max}, b_j = b_{min})$ (pair 111 seen from player i perspective, or pair 11 seen from player j perspective), and that is the result that we obtain in figure 5.8. However, it is important to notice that the pair of strategies $(b_i = b_{max}, b_j = b_{min})$ is more prominent. Finally, according with the quantal response method, the equilibrium selected by the players is $(b_i = b_{max}, b_j = p_j)$, where p_j is the probability that player j assigns to each strategy in the quantal response equilibrium. That pair of strategies corresponds with the pair of strategies 111 to 122 seen from player i perspective, or the pair of strategies 1 to 11 seen from player j perspective, and that is the result that we obtain in figure 5.8.

We test the theoretical predictions by using the scatter plot presented in figure 5.9. In the horizontal line of that graph are the cycles[7] in the vertical axes appears the frequency with which each player plays each of the strategies in the game. As can be observed in figure 5.9, player i plays

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[7] In the experiment, the students play the game in five cycles of 15 rounds each.

the maximum bib more frequently than player j, that is in line with the theoretical predictions of risk dominance method. The players also select the pair of strategies $(b_i = b_{min}, b_j = b_{max})$. However, that pair of strategies is played with less frequency. Finally, the theoretical predictions of the quantal response method are only partially validated by the scatter plot analysis.

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Chapter 6

Regulation of Natural Monopolies

In this chapter we define the characteristics of a natural monopoly, and we introduce and discuss different price regulation regimes when the regulator has full information.

6.1 Introduction

Textbook discussions of price and entry regulation typically are motivated by the asserted existence of an industry with **"natural monopoly"** characteristics. These characteristics make it economical for a single firm to supply services in the relevant market rather than two or more competing. Markets with natural monopoly characteristics are thought to lead to a variety of economic performance problems: excessive prices, production inefficiencies, costly duplication of facilities, poor service quality, and to have potentially undesirable distributional impacts.

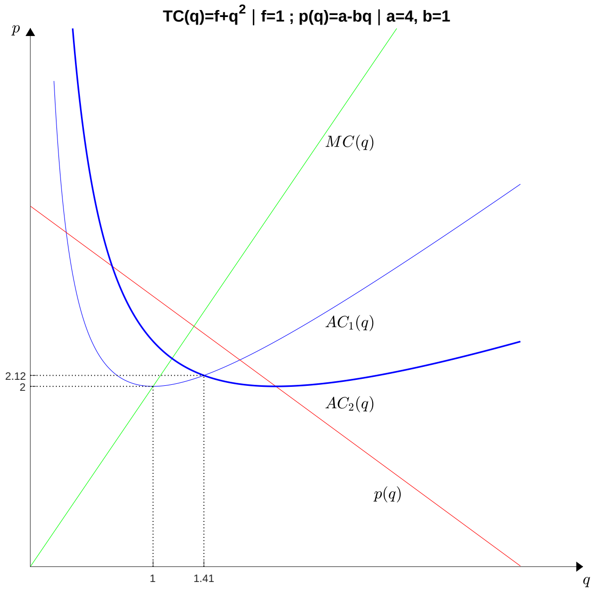
Economic analysis of natural monopoly has focused on several questions which, while related, are somewhat different.

1. One question is a **normative question**: What is the most efficient number of sellers (firms) to supply a particular good or service given firm cost characteristics and market demand characteristics? This question leads to technological or cost-based definitions of natural monopoly.
2. A second and related question is a **positive question**: What are the firm production or cost characteristics and market demand characteristics that lead some industries "naturally" to evolve to a point where there is a single supplier (a monopoly) or a very small number of suppliers (an oligopoly)? This question leads to behavioral and market equilibrium definitions of natural monopoly which are in turn related to the technological attributes that characterize the cost-based definitions of natural monopoly.
3. A third question is also a **normative question**: If an industry has "a tendency to monopoly" what are the potential economic performance problems that may result and how do we measure their social costs? This question leads to an evaluation of the losses in economic efficiency and other social costs resulting from an "unregulated" industry with one or a small number of sellers.
4. This question in turn leads to a fourth set of questions: When is government regulation justified in an industry with natural monopoly characteristics and how can regulatory mechanisms best be designed to mitigate the performance problems of concern?

6.2 Technological definition of natural monopoly

A natural monopoly is characterized by the presence of increasing returns to scale. A production function exhibits increasing returns to scale when the average cost curve is decreasing. In that

Figure 6.1: Technological definition of natural monopoly



case the average cost decrease with the production and therefore, it is less costly than a unique supplier serves the entire demand.

To illustrate the previous point, assume what we have a cost function defined by $TC(q) = f + q^2$, where f is the fix cost. The average cost is the total cost divided by the quantity. Therefore, $AC(q) = \frac{f}{q} + q$. The average cost is represented by the blue line $AC_1(q)$ in figure 6.1. The marginal cost of production is defined by $MC(q) = \frac{\partial TC(q)}{\partial q} = 2q$. As can be observed, if the production region where the average cost is decreasing, the marginal cost is lower than the average cost and thus, if the supplier sets a price equal to the marginal cost (perfect competition scenario), it doesn't cover the total cost.

It is important to notice, that from an economic perspective, it can be less costly that only one supplier satisfies the total demand even when the production function exhibits decreasing returns to scale (increasing average cost). This happens when the average cost of producing with only one supplier is lower that the average cost of producing with two suppliers. This property of the cost functions is called sub-additivity, and mathematically implies that $TC(q) \leq TC\left(\frac{q}{2}, \frac{q}{2}\right)$. To work out the quantity that equalize both production costs, it is enough to compute q such that $f + q^2 = f + \left(\frac{q}{2}\right)^2 + f + \left(\frac{q}{2}\right)^2 \Rightarrow \frac{2q^2}{4} = f \Rightarrow q = \sqrt{2f}$.

As can be observed in figure 6.1, when the production capacity is lower than 1.4, even when the production function exhibits decreasing returns to scale, it is less costly that one supplier satisfies the total demand. In that case the average cost of producing with only one supplier $AC_1(q)$ is lower that the average cost of producing with two suppliers $AC_2(q)$.

6.3 Price regulation by a fully informed regulator

Fully efficient pricing (price equal to marginal cost) is typically not feasible for a private firm that must meet a break-even constraint in the presence of economies of scale. Accordingly, the traditional literature on price regulation of natural/legal monopolies focused on **normative issues** related to the development of **second-best pricing rules** for the regulated firm given a break-even constraint (or given a cost of government subsidies that ultimately rely on a tax system that also creates inefficiencies). A secondary focus of the literature has been on **pricing of services like electricity** which are non-storable, have widely varying temporal demand, have high capital intensities and capital must be invested to provide enough capacity to meet the peak demand, the so-called peak-load or variable-load pricing (PLP) problem.

The traditional literature on **second-best pricing** for natural monopolies assumes that the regulator is fully informed about the regulated firm's costs and knows as much about the attributes of the demand for the services that the firm supplies as does the regulated firm. The **regulator's goal** is to identify and implement normative pricing rules that **maximize total surplus given a budget constraint faced by the regulated firm**. Neither the regulated firm nor the regulator acts strategically.

6.3.1 Optimal linear price Ramsey-Boiteux pricing

In order for the firm with increasing returns to break-even it appears that the prices the firm charges for the services it provides will have to exceed marginal cost. One way to proceed in the single product context is simply to set a single price for each unit of the product equal to its average cost (p_{AC}). Then the expenditures made by each consumer i will be equal to $E_i = p_{AC}q_i$. In this case p_{AC} is a **uniform linear price** schedule since the firm charges the same price for each unit consumed and each consumer's expenditures on the product varies proportionately with the output she consumes.

The deadweight loss associated with uniform linear prices is represented in the left-hand side in figure 6.2. As can be observed in that figure, since the elasticity of the demand during the day is lower than during the night, the deadweight loss for the consumers during the day is larger than the deadweight loss for the consumers during the night.

The first question to address is whether, **within the class of linear prices**, we can do better than charging a uniform price per unit supplied that embodies an equal mark-up over marginal cost to all consumers for all products sold by the regulated firm? Alternatively, can we do better by engaging in **third degree price discrimination**?[1] in the case of a single product firm, by charging different unit prices to different types of consumers (e.g. residential and industrial and assuming that resale is restricted) charging a constant unit price for each product but where each unit price embodies a different markup over its incremental cost?

Set up: Following Laffont and Tirole ((2000), p. 64), the regulated firm produces n products whose quantities supplied are represented by the vector $q = (q_1, ..., q_n)$. Assume that the demand functions for the price vector $p = (p_1, ..., p_n)$ are $q_k = D_k(p_1, ..., p_n)$. The firm's total revenue function is then $R(q) = \sum_{i=1}^{n} p_k q_k$. Let the firm's total cost function be $C(q) = C(q_1, ..., q_n)$ and denote the marginal cost for each product k as $c_k(q_1, ..., q_n)$. Let $S(q) = \sum_{i=1}^{n} \int_0^{q_i(p_i)} p_i(q)\partial q_i$ denote the gross surplus for output vector q with $\dfrac{\partial S}{\partial q_i} = p_i$.

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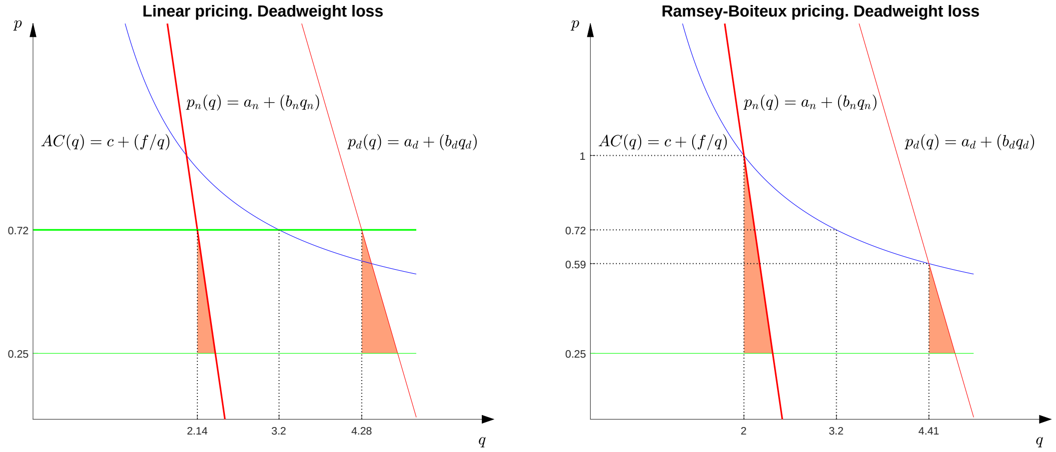
[1] In a third degree price discrimination, the suppliers can offer different price to different types of consumers. The types of price discrimination are explained in the annex in this section

Figure 6.2: Linear and Ramsey-Boiteux pricing $c = 0.25, f = 1.5, a_n = a_d = 5, b_n = 2, b_d = 1$



The Ramsey-Boiteux pricing problem (Ramsey 1927; Boiteux, 1971) is then to find the vector of contant unit (linear) prices for the n products that maximizes net social surplus subject to the regulated firm's break-even or balanced budget constraint:

$$\begin{array}{ll} \max_q & S(q) - C(q) \\ \text{subject to} & R(q) - C(q) \geq 0 \end{array}$$

This problem may be solved using a Lagrange multiplier. FOC on q are:

$$\begin{aligned}
p_n - c_n(q) &= -\lambda\left(\frac{\partial R}{\partial q_n} - c_n(q)\right) \\
&= -\lambda\left(p_n + \frac{\partial p_n}{\partial q_n} q_n \frac{p_n}{p_n} - c_n(q)\right) \\
&= -\lambda\left(p_n\left(1 + \frac{\partial p_n}{p_n}\frac{q_n}{\partial q_n}\right) - c_n(q)\right) \\
&= -\lambda\left(p_n\left(1 - \frac{1}{|\epsilon_n|}\right) - c_n(q)\right) \\
&= -\lambda(p_n - c_n(q)) + \frac{\lambda p}{|\epsilon_n|} \Rightarrow \\
(p_n - c_n(q))(1+\lambda) &= \frac{\lambda p_n}{|\epsilon_n|} \Rightarrow \\
\frac{p_n - c_n(q)}{p_n} &= \frac{\lambda}{1+\lambda}\frac{1}{|\epsilon_n|} = \frac{k}{|\epsilon_n|} \qquad (6.1)
\end{aligned}$$

Equation 6.1 is often referred to as the inverse elasticity rule (Baumol and Bradford (1970)). Prices are set so that the difference between a product's price and its marginal cost varies inversely with the elasticity of demand for the product. **The margin is higher for products that have less elastic demands** than for products that have more elastic demand.

Note that Ramsey-Boiteux prices involve **third-degree price discrimination** that results in a set of prices that lie between marginal cost pricing and the prices that would be set by a pure monopoly engaging in third-degree price discrimination. For example, rather than being different products, assume that q_1 and q_2 are the same product consumed by two groups of consumers who have different demand elasticities (e.g. residential and industrial consumers) and that resale can be blocked eliminating the opportunity to arbitrage away differences in prices charged to the two groups of consumers. Then the price will be higher for the group with the less elastic demand despite the fact that the product and the associated marginal cost of producing it are the same.

The Ramsey-Boiteux prices are represented in the right-hand side in figure 6.2. As can be observed, the consumers with the less elastic demand curve pay a higher price. However, for this particular example, the deadweight loss with linear pricing (0.164) is lower than the deadweight loss with Ramsey-Boiteux pricing (0.198).

6.3.2 Non-linear prices: Simple two-part tariffs

Ramsey-Boiteux prices are still only **second-best prices** because the per unit usage prices are not equal to marginal cost. The distortion is smaller than for uniform ($p = AC$ in the single product case) pricing since we are taking advantage of differences in the elasticities of demand for different types of consumers or different products to satisfy the budget constraint yielding a **smaller dead-weight loss** from departures from marginal cost pricing. That is, there is still a wedge between the price for a product and its marginal cost leading to an associated dead-weight loss.

The question is whether we **can do better** by further relaxing the restriction on the kinds of prices that the regulated firm can charge? Specifically, can we do better if we were to allow the regulated firm to charge a **"two-part" price** that includes a **non-distortionary uniform fixed "access charge"** (A) and then a **separate per unit usage price** (p). A price schedule or tariff of this form would yield a consumer expenditure or outlay schedule of the form: $T_i = A + pq_i$.

Such a price schedule is "non-linear" because the average expenditure per unit consumed T_i/q_i is no longer constant, but falls as q_i increases. We can indeed do (much) better from an efficiency perspective with two-part prices than we can with second-best (Ramsey-Boiteux) linear prices (Brown and Sibley 1986, pp. 167-183).

Set up: Assume that there are N identical consumers in the market each with demand $q_i = d(p)$ and gross surplus of S_i evaluated at $p = 0$. The regulated firm's total cost function is given by $C = f + cq$. That is, there is a fixed cost f and a marginal cost c. Consider a tariff structure that requires each consumer to pay an access charge $A = f/N$ and then a unit charge $p = c$. Consumer i's expenditure schedule is then: $T_i = A + pq_i = f/N + cq_i$.

Economic analysis: This two-part tariff structure is first-best. On the margin, each consumer pays a usage price equal to marginal cost and the difference between the revenues generated from the usage charges and the firm's total costs are covered with a fixed fee that acts as a lump sum tax. As long as $A < (S_i - pq_i)$ then consumers will pay the access fee and consume at the efficient level. If $A > (S_i - pq_i)$ then it is not economical to supply the service at all because the gross surplus is less than the total cost of supplying the service (recall S_i is the same for all consumers and $p_i = c$).

6.3.3 Peak-Load Pricing

Many public utility services **cannot be stored** and the demand for these services may vary widely from hour to hour, day to day and season to season. Because these services cannot be stored, the physical capacity of the network must be expanded sufficiently to meet peak demand. Services like electricity distribution and generation, gas distribution, and telephone networks are very capital intensive and the carrying costs (depreciation, interest on debt, return on equity investment) of the capital invested in this capacity is a relatively large fraction of total cost.

The intuition behind the basic peak load pricing results is quite straightforward. If capacity must be built to meet peak demand then when demand is below the peak there will be surplus capacity available.

- The long run marginal cost of increasing supply to meet an increment in **peak demand** includes both the **additional capital** and **operating costs** of building and operating an increment of peak capacity.
- The long run marginal cost of increasing supply to meet an increment in **off peak demand** reflects only the additional **operating costs** or short run marginal cost of running more of the surplus capacity to meet the higher demand as long as off-peak demand does not increase to a level greater than the peak capacity on the system.

Accordingly, the peak price should be relatively high, reflecting both marginal operating and capital costs, and the off-peak prices low to reflect only the off-peak marginal costs of operating the surplus capacity more intensively.

Set up: Let $q_d = q_d(p_d)$, the demand during the day-time hours, and $q_n = q_n(p_n)$, the demand for electricity during night-time hours for any $p_d = p_n$ day-time demand is higher than night-time demand $(q_d(p_d)) > q_n(p_n))$. The gross surplus during each period (area under the demand curve) is given by $S(q_i)$ and $\dfrac{\partial S_i}{\partial q_i} = p_i$.

Assume that the production of electricity is characterized by a simple fixed-proportions technology composed of a unit rental cost c_k for each unit of generating capacity (k) and a marginal operating cost c_e for each unit of electricity produced. We will assume that there are **no economies of scale**, recognizing that any budget balance constraints can be handled with second-best linear or non-linear prices. Demand in any period must be less than or equal to the amount of capacity installed so that $q_d < k$ and $q_n < k$.

Equilibrium analysis: The optimal prices are then given by solving the following program which maximizes net surplus subject to the constraints that output during each period must be less than or equal to the quantity of capacity that has been installed:

$$L = S(q_d) + S(q_n) - c_k k - c_e(q_d + q_n) + \lambda_d(k - q_d) + \lambda_n(k - q_n) \tag{6.2}$$

where λ_d and λ_n are the shadow prices on capacity. The first order conditions are then given by:

$$\frac{\partial L}{\partial q_d} = p_d - c_e - \lambda_d = 0 \tag{6.3}$$

$$\frac{\partial L}{\partial q_n} = p_n - c_e - \lambda_n = 0 \tag{6.4}$$

$$\frac{\partial L}{\partial k} = \lambda_d + \lambda_n - c_k = 0 \tag{6.5}$$

with complementary slackness conditions:

$$\lambda_d(k - q_d) = 0 \tag{6.6}$$

$$\lambda_n(k - q_n) = 0 \tag{6.7}$$

There are then **two interesting cases:**

Case 1: Classic peak load pricing results.

If $q_n < k$, then by equation 6.7, $\lambda_n = 0$.

If $q_d = k$, then by equation 6.6, $\lambda_d \geq 0$.

By equation 6.5, $\lambda_d = c_k$. Finally, to obtain the prices that the consumers have to pay, we use equations 6.3 and 6.4 to obtain that $p_d = c_e + c_k$ and that $p_n = c_e$. Therefore, the consumers during the peak (day), pay the marginal operation costs (c_e) and the marginal capacity costs (c_k). The consumers during the base (night) only pay the marginal operation costs (c_e).

Case 2: Shifting peak case.

If $q_n = k$, then by equation 6.7, $\lambda_n \geq 0$.

If $q_d = k$, then by equation 6.6, $\lambda_d \geq 0$.

By equation 6.5, $\lambda_d + \lambda_n = c_k$. Finally, to obtain the prices that the consumers have to pay, we use equations 6.3 and 6.4 to obtain that $p_d = c_e + \lambda_d$ and that $p_n = c_e + \lambda_n$. Therefore, the consumers during the day and during the night are consuming at peak demand. Therefore, both consumers contribute to pay the marginal operation costs (c_e) and the marginal capacity costs (c_k). If the demand elasticity is the same during the day and during the night, the share of the marginal capacity costs paid by the consumers during the day is larger than the share paid by the consumers during the night.

Example. When the demand is linear, equation 6.2 becomes:

$$\begin{aligned} L &= S(q_d) + S(q_n) - c_k k - c_e(q_d + q_n) + \lambda_d(k - q_d) + \lambda_n(k - q_n) \\ &= b_d q_d^2 \frac{1}{2} + b_n q_n^2 \frac{1}{2} - c_k k - c_e(q_d + q_n) + \lambda_d(k - q_d) + \lambda_n(k - q_n) \qquad (6.8) \end{aligned}$$

Where λ_d and λ_n are the shadow prices on capacity. The first order conditions are then given by:

$$\begin{aligned} \frac{\partial L}{\partial q_d} &= b_d q_d - c_e - \lambda_d = 0 \qquad (6.9) \\ \frac{\partial L}{\partial q_n} &= b_n q_n - c_e - \lambda_n = 0 \qquad (6.10) \\ \frac{\partial L}{\partial k} &= \lambda_d + \lambda_n - c_k = 0 \qquad (6.11) \end{aligned}$$

with complementary slackness conditions:

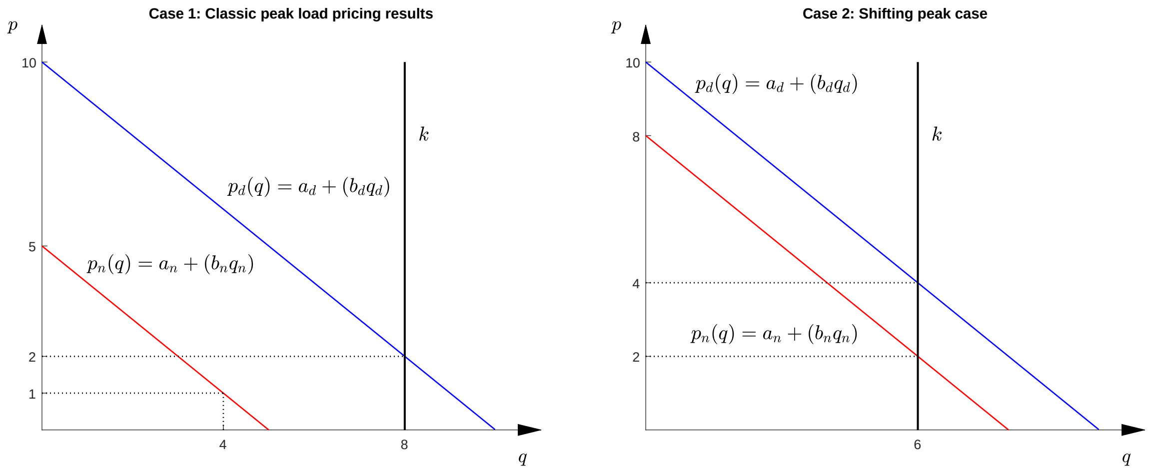
$$\lambda_d(k - q_d) = 0 \qquad (6.12)$$

$$\lambda_n(k - q_n) = 0 \qquad (6.13)$$

There are then **two interesting cases:**

Case 1: Classic peak load pricing results.

Figure 6.3: Peak-Load pricing


Case 1: Classic peak load pricing results

Case 2: Shifting peak case

If $q_n < k$, then by equation 6.13, $\lambda_n = 0$.

If $q_d = k$, then by equation 6.12, $\lambda_d \geq 0$.

By equation 6.11, $\lambda_d = c_k$. Finally, to obtain the quantities and the prices we use equations 6.9 and 6.10. By equation 6.9:

$$q_d b_d = c_e + c_k \Rightarrow q_d = \frac{c_e + c_k}{b_d} \tag{6.14}$$

$$p_d = a_d - b_d \frac{c_e + c_k}{b_d} = a_d - (c_e + c_k) \tag{6.15}$$

By equation 6.10:

$$q_n b_n = c_e \Rightarrow q_n = \frac{c_e}{b_n} \tag{6.16}$$

$$p_n = a_n - b_n \frac{c_e}{b_n} = a_n - (c_e) \tag{6.17}$$

When $a_n = 5, a_d = 10, b_n = b_d = 1, c_3 = c_k = 4$, where subindex n refers to night, and subindex d refers to day, the values of equations 6.14, 6.15, 6.16, 6.17 are represented in the left-hand side in figure 6.3. The price paid by the consumers during the day is larger than the price paid by the consumers during the night. However, this is not necessarily true for different parameters. Moreover, equations 6.3 and 6.4 are different from equations 6.15 and 6.17. Therefore, it is very important to take carefully the results presented in the example because they could be wrong.

Case 2: Shifting peak case.

If $q_n = k$, then by equation 6.13, $\lambda_n \geq 0$.

If $q_d = k$, then by equation 6.12, $\lambda_d \geq 0$.

By equation 6.11, we know that $c_k = \lambda_n + \lambda_n$.

Summing equations 6.12 and 6.13 taken into account the previous results, we obtain:

$$(b_d + b_n)k - 2c_e - c_k = 0 \Rightarrow k = q_d = q_n \frac{2c_e + c_k}{b_d + b_n} \tag{6.18}$$

By plugging 6.18 equations 6.9 and 6.10, we obtain:

$$\lambda_d = b_d \frac{2c_e + c_k}{b_d + b_n} - c_e$$
$$\lambda_n = b_n \frac{2c_e + c_k}{b_d + b_n} - c_e$$

Finally, by using the demand functions, we obtain the equilibrium prices:

$$p_d = a_d - b_d \frac{2c_e + c_k}{b_d b_n} \tag{6.19}$$
$$p_n = a_n - b_n \frac{2c_e + c_k}{b_d b_n} \tag{6.20}$$

When $a_n = 8, a_d = 10, b_n = b_d = 1, c_3 = c_k = 4$, where subindex n refers to night, and subindex d refers to day, the values of equation 6.18, 6.19, 6.20 are represented in the right-hand side in figure 6.3.

6.4 Exercises

6.4.1 Natural Monopoly. Linear pricing and Ramsey-Boiteux pricing

Set up: We have a supplier with production cost $TC(q) = f + cq$, where f is the fixed cost and c is the marginal cost of production. We have two groups of consumers with different demand functions: $q_i = a_i - b_i q \forall i = d, n$, where subindex n refers to night, and subindex d refers to day. Assume the next set of parameters: $f = 1.5, c = 0.25, a_n = a_d = 5, b_n = 2, b_d = 1$.

Questions:

1. Work out the average cost and the marginal cost.
2. Is this cost function sub-additive?
3. Work out the perfect competition quantities for the day and night consumers. Work out the profits in that case? Are the profits positive?
4. Given the perfect competition solution, work out the linear price that makes that the supplier can cover the fix cost. Work out the deadweight loss in that case.
5. Work out the Ramasey-Boiteux pricing and work out the deadweight loss in that case. Compare it with the linear price case.
6. Program this exercise in GAMS and analyze how the changes in the amount of electricity consumed (a_n, a_d) and elasticity of the demand (b_n, b_d) modify the equilibrium prices and the deadweight loss.

6.4.2 Peak-Load pricing

Set up: We have a supplier with production cost $TC(q,k) = c_k k + c_e q$, where c_k is the marginal cost of installing production capacity and c_e is the marginal cost of producing energy. We have two groups of consumers with different demand functions: $q_i = a_i - b_i q \forall i = d, n$, where subindex n refers to night, and subindex d refers to day. Assume the next set of parameters: $f = 1.5, c = 0.25, a_n = 8, a_d = 10, b_n = b_d = 1, c_e = c_k = 2$. In this case, the supplier doesn't face a fixed investment cost and therefore, the perfect competition solution is feasible. The main questions in this problem are: First, to find the right investment in capital to satisfy the demand during the day and the demand during the night. Second, to find the prices that the consumers have to pay to cover the cost of capital and electricity.

Questions:

1. Work out the optimal solution for investments in production capacity, and the optimal production during the day and during the night.
2. Write a program in GAMS and work out the optimal investment in capacity, the optimal production during the day, during the night and the prices the consumers have to pay.
3. Using your program, change the parameters a_n, b_n, a_d, b_d and analyze how that affect the main variables of the model (optimal investment in capacity, optimal production during the day, optimal production during the night, and prices faced by the consumers).

Annex. Price discrimination

Taken from the wikipedia (link). We can differentiate three types of price discrimination.

First degree:

Exercising first degree (or perfect/Primary) price discrimination requires the monopoly **seller of a good or service to know the absolute maximum price** (or reservation price) that every consumer is willing to pay. By knowing the reservation price, the seller is able to sell the good or service to each consumer at the maximum price he is willing to pay, and thus transform the consumer surplus into revenues. So the profit is equal to the sum of consumer surplus and producer surplus. The marginal consumer is the one whose reservation price equals to the marginal cost of the product. The seller produces more of his product than he would to achieve monopoly profits with no price discrimination, which means that there is no deadweight loss.

Second degree:

In second degree price discrimination, price varies according to **quantity demanded**. Larger quantities are available at a lower unit price. This is particularly widespread in sales to industrial customers, where bulk buyers enjoy higher discounts.

Additionally to second degree price discrimination, **sellers are not able to differentiate between different types of consumers**. Thus, the suppliers will provide incentives for the **consumers to differentiate themselves** according to preference, which is done by quantity "discounts", or non-linear pricing. This allows the supplier to set different prices to the different groups and capture a larger portion of the total market surplus.

In reality, different pricing may apply to differences in **product quality** as well as quantity. For example, airlines often offer multiple classes of seats on flights, such as first class and economy class, with the first class passengers receiving wine, beer and spirits with their ticket and the economy passengers offered only juice, pop and water. This is a way to differentiate consumers based on preference, and therefore allows the airline to capture more consumer's surplus.

Third degree:

Third degree price discrimination, means **charging a different price to different consumer groups**. For example, rail and tube (subway) travellers can be subdivided into commuter and casual travellers, and cinema goers can be subdivided into adults and children, which some theatres also offering discounts to full-time students and seniors. Splitting the market into peak and off peak use of a service is very common and occurs with gas, electricity, and telephone supply, as well as gym membership and parking charges. Some parking lots charge less for "early bird" customers who arrive at the parking lot before a certain time.

(Some of these examples are not pure "price discrimination", in that the differential price is related to production costs: the marginal cost of providing electricity or car parking spaces is very low outside peak hours. Incentivizing consumers to switch to off-peak usage is done as much to minimize costs as to maximize revenue.)

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Chapter 7

Investments

The increasing weight of renewable capacity in the production mix increases the volatility of production capacity. Moreover, the presence of price caps can induce a scarcity in production capacity investments. The increase in production volatility, and the lack of production capacity investments could generate blackouts that have important economic consequences. In this chapter, we study the main models that analyze generation capacity investment decisions, and the influence that production volatility and price caps have on those decisions.

We study Kreps and Scheinkman (1984), where the suppliers invest in production capacity and then they compete in prices. We broaden that analysis by analyzing the papers that extend that analysis by introducing different rationing rules and demand uncertainty. We complement this study by analyzing other models that study investment decisions in transmission and in production capacities.

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Chapter 8

Forward Contracts

Forward contracts have been implemented historically to exacerbate competition in spot electricity markets. However, the effects of those contracts enhancing competition in spot electricity markets depend crucially on the type of competition between firms in those markets. In particular, the effect on competition varies if the strategies are complementary or substitutes. In this chapter, we study the main models that analyze the effects that the introduction of forward contracts has on spot electricity markets.

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Chapter 9

Measuring Market Power

Electricity is a commodity with special characteristics that facilitate the exercise of market power as we have studied in chapter 1. Moreover, electricity markets are characterized for the presence of few competitors and the presence of transmission constraints. Therefore, it is necessary to develop tools to study if the suppliers are exercising market power. In this chapter, we study different techniques to measure market power in electricity markets.

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Chapter 10

Collusion

Electricity is a commodity with special characteristics that facilitate the exercise of market power as we have studied in chapter 1. Moreover, electricity markets are characterized for the presence of few competitors and the presence of transmission constraints. In electricity markets, the suppliers interact with each other and that could facilitate collusion. In this chapter, we study the main papers that study collusion in electricity markets.

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Chapter 11

Exams

11.1 Exam November 2019

Question 1:

Electricity is a special commodity, and electricity markets operates with some particular rules that do not apply to other markets. Could you enumerate some of the particularities of electricity and electricity markets that facilitates the exercise of market power in those markets?

- Demand is difficult to **forecast**.
- Demand is **insensitive** to price fluctuations.
- Supply faces **binding constraints** at peak times.
- **Storage** is prohibitively costly.
- Demand and supply have to **match** all the time.
- Electricity markets are connected by **transmission lines that could be congested**.

Question 2:

In an electricity market there are two suppliers with production capacities $k_s = k_n =$ 50, and marginal cost of production $c_s = c_n = 0$, and the price cap is $P = 5$. Work out the Nash equilibrium in an uniform and in a discriminatory price auction when the demand is equal to $\theta = 40$.

When the demand is $\theta = 40$ both suppliers have enough production capacity to satisfy the total demand and they compete fiercely to be dispatched in the auction. Therefore, there exists an **unique** Nash equilibrium where both suppliers submit a bid equal to the their marginal costs. Hence, the pair of strategies that define the equilibrium are $b_s^* = b_n^* = 0$.

Question 3:

In an electricity market there are two suppliers with production capacities $k_s = k_n =$ 50, and marginal cost of production $c_s = c_n = 0$, and the price cap is $P = 5$. Work out the close form solution that define the Nash equilibrium in an uniform price auction when the demand is equal to $\theta = 60$.

When the demand is $\theta = 60$ both suppliers face a positive residual demand and there are **multiplicity of Nash equilibrium**. One of the sets of Nash equilibria is the one in which supplier n submits the higher bid, and supplier s submits a bid that makes undercutting unprofitable. That set of Nash equilibria is defined by:

$$\begin{aligned} b_s^* &\in \left[0, \frac{P(\theta - k_s)}{k_n}\right] = \left[0, \frac{5(60-50)}{60}\right] = [0,1]\,; \\ b_n^* &= P = 5 \end{aligned}$$

The other set of Nash equilibria is the one in which supplier s submits the higher bid, and supplier n submits a bid that makes undercutting unprofitable. That set of Nash equilibria is defined by:

$$\begin{aligned} b_s^* &= P = 5; \\ b_n^* &\in \left[0, \frac{P(\theta - k_s)}{k_n}\right] = \left[0, \frac{5(60-50)}{60}\right] = [0,1] \end{aligned}$$

Question 4:

In an electricity market there are two suppliers with production capacities $k_s = k_n = 50$, and marginal cost of production $c_s = c_n = 0$, and the price cap is $P = 5$. Work out the close form solution that define the Nash equilibrium in a discriminatory price auction when the demand is equal to $\theta = 60$.

When the demand is $\theta = 60$ both suppliers face a positive residual demand. In that case, **a pure Nash equilibrium does not exist**. The proof of this statement is as follows:

First, any pair of bids (b_s, b_n) in the interval $\left[0, \frac{P(\theta - k_s)}{k_n}\right]$, it is not a Nash equilibrium, since both suppliers prefer to deviate to the price cap and satisfy the residual demand.

Second, any pair of bids (b_s, b_n) in the interval $\left[\frac{P(\theta - k_s)}{k_n}, P\right]$, it is not a Nash equilibrium, since the suppliers undercut each other to be dispatched first in the auction.

Therefore, the Nash equilibrium is in mixed strategies. To work out the Nash equilibrium, it is necessary to work out the **support of the mixed strategies equilibrium**, and suppliers' cumulative distribution functions.

Both suppliers assign probability equal to zero to the bids in the interval $\left[0, \frac{P(\theta - k_s)}{k_n}\right]$, since they can assign probability one to the price cap increasing their profits. Therefore, the support of the mixed strategies equilibrium is defined by $\left[\frac{P(\theta - k_s)}{k_n}, P\right] = [1, 5]$.

To work out the **cumulative distribution function**, we proceed in three steps:

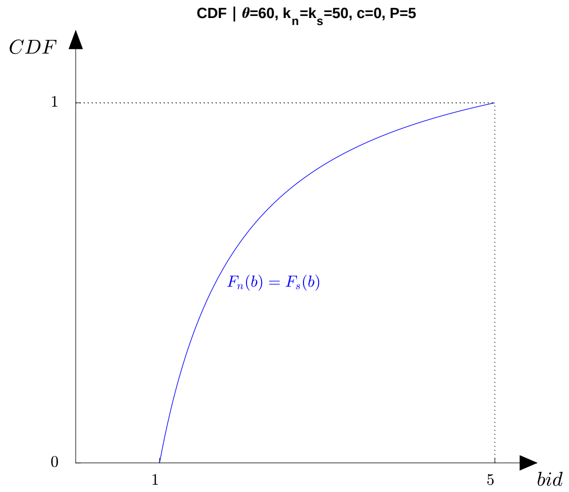
First, supplier n's profit function is defined by:[1]

$$\begin{aligned} \pi_n(b;\theta) &= b\left[F_s(b)(\theta - k_s)\right] + b\left[(1 - F_s(b))k_n\right] \\ &\quad -bF_s(b)\left[k_n - (\theta - k_s)\right] + bk_n \end{aligned} \tag{11.1}$$

---

[1] Suppliers' production capacities and suppliers' production costs are identical. Therefore, the Nash equilibrium is also symmetric and it is only necessary to work out the cumulative distribution function of one of the suppliers.

Figure 11.1: Mixed strategies equilibrium



The first term in equation 12.1 ($b\left[F_s(b)(\theta - k_s)\right]$) represents supplier n' profits when $b_s \leq b_n$. The second term in equation 12.1 ($b\left[(1 - F_s(b))k_n\right]$) represents supplier n' profits when $b_s > b_n$.

Therefore, supplier s's cumulative distribution function is defined by:

$$F_s(b) = \frac{bk_n - \pi_n(b;\theta)}{b\left[k_n - (\theta - k_s)\right]} \tag{11.2}$$

Second, at the lower bound of the support ($\underline{b}$), the cumulative distribution function is zero. Plugging that information in equation 12.2, we can work out supplier n's profits:

$$F_s(\underline{b};\theta) = 0 \Longrightarrow \pi_n(\underline{b};\theta) = \underline{b}k_n \tag{11.3}$$

Moreover, we know that $\pi_n(\underline{b};\theta) = \underline{b}k_n = \pi_n(b;\theta)\ \forall b \in [\underline{b}, P]$. The last inequality holds because all the bids in the support give in expectation the same payoff. Otherwise, the suppliers can reallocate probabilities to increase their expected profits.

Third, by using the information obtained in the second step, and plug in the value of supplier n's profits in equation 12.1, we obtain supplier s's cumulative distribution function:

$$F_s(b) = \frac{bk_n - \underline{b}k_n}{b\left[k_n - (\theta - k_s)\right]} = \frac{k_n}{k_n - (\theta - k_s)} \frac{b - \underline{b}}{b} \tag{11.4}$$

The graphical representation of the cumulative distribution functions for both suppliers are in figure 12.1.

Optional:

To complete the analysis of the equilibria, it is important to work out the expected bid submitted by each supplier. First, we work out the probability distribution function:

$$f_s(b) = \frac{\partial F_s(b)}{\partial b} = \frac{k_n}{k_n - (\theta - k_s)} \frac{\underline{b}}{b^2} \tag{11.5}$$

Second, we work out the expected bid for each supplier:

$$
\begin{aligned}
E_s(b) &= \int_{\underline{b}}^{P} b f_s(b) \partial b = \frac{k_n}{k_n - (\theta - k_s)} \int_{\underline{b}}^{P} \frac{b\, \underline{b}}{b^2} \\
&= \frac{k_n}{k_n - (\theta - k_s)} \left[ln(b) \right]_{\underline{b}}^{P} = 2.0118 \qquad (11.6)
\end{aligned}
$$

Question 5:

In an electricity market there are two nodes connected by a transmission line with capacity $T = 20$. There is one supplier with capacity $k_n = 50$ located in node North, and a supplier with capacity $k_s = 50$ located in node South. The marginal cost of production $c_s = c_n = 0$ and the price cap is $P = 5$. Work out the close form solution that define the Nash equilibrium in a discriminatory price auction when the demand in node South is $\theta_s = 10$, and the demand in node North is $\theta_n = 60$.

We have to prove that **a pure strategies equilibrium does not exis**t. We proceed in three steps:

First, we find the bid that makes indifferent each supplier between to submit the maximum bid allowed by the auctioneer and to undercut the other supplier. We start from supplier n. Supplier n's residual profits are determined by $P(\theta_n - T)$, and the bid that makes undercutting unprofitable is defined by:

$$
P(\theta_n - T) = \underline{b}_n k_n \Longrightarrow \underline{b}_n = \frac{P(\theta_n - T)}{k_n} = 2.5
$$

We work out the bid that makes undercutting unprofitable for supplier s. In that case, supplier s's residual profits are determined by $P(\theta_n + \theta_s - k_n)$, and the bid that makes undercutting unprofitable is defined by:

$$
P(\theta_s + \theta_n - k_n) = \underline{b}_s(\theta_s + T) \Longrightarrow \underline{b}_s = \frac{P(\theta_s + \theta_n - k_n)}{(\theta_s + T)} = 0.83
$$

Second, we prove that a pair of bids in the interval $[0, max\left\{\underline{b}_s, \underline{b}_n\right\}]$ cannot be a Nash equilibrium. A pure strategies equilibrium does not exist in that interval, since at least one of the suppliers prefers to submit the maximum bid allowed by the auctioneer and satisfy the residual demand.

Third, a pair of bids in the interval $[max\left\{\underline{b}_s, \underline{b}_n\right\}, P]$ cannot be a Nash equilibrium, since both suppliers undercut each other to be dispatched first in the auction.

Therefore, the Nash equilibrium is in mixed strategies. Hence, it is necessary to find **the support of the mixed strategies equilibrium** and the cumulative distribution function. The support of the mixed strategies equilibrium is defined by $b \in [max\left\{\underline{b}_s, \underline{b}_n\right\}, P]$, since as we show above, the suppliers never randomize in the interval $b \in [0, max\left\{\underline{b}_s, \underline{b}_n\right\}]$.

To work out the **cumulative distribution function**, we proceed in three different steps:

First, supplier n's profit function is defined by:

$$
\begin{aligned}
\pi_n(b;\theta) &= b\left[F_s(b)(\theta_n - T)\right] + b\left[(1 - F_s(b))k_n\right] \\
&\quad -bF_s(b)\left[k_n - (\theta_n - T)\right] + bk_n \qquad (11.7)
\end{aligned}
$$

The first term in equation 12.7 ($b\left[F_s(b)(\theta_n - T)\right]$) represents supplier n' profits when $b_s \leq b_n$. The second term in equation 12.7 ($b\left[(1-F_s(b))k_n\right]$) represents supplier n' profits when $b_s > b_n$.

Therefore, supplier s's cumulative distribution function is defined by:

$$F_s(b) = \frac{bk_n - \pi_n(b;\theta)}{b\left[k_n - (\theta\_T)\right]} \tag{11.8}$$

Second, at the lower bound of the support ($\underline{b}$), the cumulative distribution function is zero. Plugging that information in equation 12.7, we can work out supplier n's profits:

$$F_s(\underline{b};\theta) = 0 \Longrightarrow \pi_n(\underline{b};\theta) = \underline{b}k_n \tag{11.9}$$

Moreover, we know that $\pi_n(\underline{b};\theta) = \underline{b}k_n = \pi_n(b;\theta)\ \forall b \in [\underline{b}, P]$. The last inequality holds because all the bids in the support give in expectation the same payoff. Otherwise, the suppliers can reallocate probabilities to increase their expected profits.

Third, by using the information obtained in the second step and plug in the value of supplier n's profits in equation 12.8, we obtain supplier s's cumulative distribution function:

$$F_s(b) = \frac{bk_n - \underline{b}k_n}{b\left[k_n - (\theta_n - T)\right]} = \frac{k_n}{k_n - (\theta_n - T)}\frac{b - \underline{b}}{b} \tag{11.10}$$

We repeat the same steps to work out supplier n's cumulative distribution function:

First, supplier s's profit function is defined by:

$$\begin{aligned} \pi_s(b;\theta) &= b\left[F_s(b)(\theta_s + \theta_n - k_n)\right] + b\left[(1 - F_s(b))(\theta_s + T)\right] \\ &\quad -bF_s(b)\left[(\theta_s + T) - (\theta_s + \theta_n - k_n)\right] + b(\theta_s + T) \end{aligned} \tag{11.11}$$

Therefore, supplier n's cumulative distribution function is defined by:

$$F_n(b) = \frac{b(\theta_s + T) - \pi_s(b;\theta)}{b\left[(\theta_s + T) - (\theta_s + \theta_n - k_n)\right]} \tag{11.12}$$

Second, at the lower bound of the support ($\underline{b}$), the cumulative distribution function is zero. Plugging that information in equation 12.12, we can work out supplier S's profits:

$$F_n(\underline{b};\theta) = 0 \Longrightarrow \pi_s(\underline{b};\theta) = \underline{b}(\theta_s + T) \tag{11.13}$$

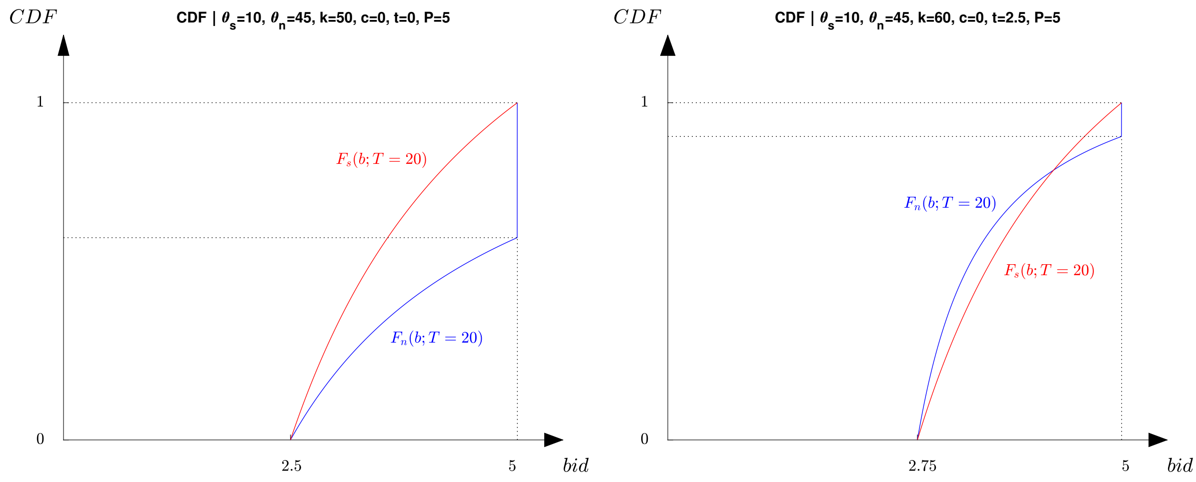
Moreover, we know that $\pi_s(\underline{b};\theta) = \underline{b}(\theta_s + T) = \pi_s(b;\theta)\ \forall b \in [\underline{b}, P]$. The last inequality holds because all the bids in the support give in expectation the same payoff. Otherwise, the suppliers can reallocate probabilities to increase their expected profits.

Third, by using the information obtained in the second step and plug in the value of supplier s's profits in equation 12.12, we obtain supplier n's cumulative distribution function:

$$F_n(b) = \frac{b(\theta_s + T) - \underline{b}(\theta_s + T)}{b\left[(\theta_s + T) - (\theta_s + \theta_n - k_n)\right]} = \frac{(\theta_s + T)}{(\theta_s + T) - (\theta_s + \theta_n - k_n)}\frac{b - \underline{b}}{b} \tag{11.14}$$

The graphical representation of the cumulative distribution functions of both suppliers are in the left-hand side of figure 12.2. As can be observed in that figure, the cumulative distribution function of the supplier located in the high-demand node stochastic dominate the cumulative distribution function of the supplier located in the low-demand node, i.e., the supplier located in the high-demand node submits higher bids with higher probabilities. This result is very intuitive, the supplier located in the high-demand node faces a high residual demand and it prefers

Figure 11.2: Mixed strategies equilibrium



to satisfy that demand by submitting higher bids.

Optional:

To complete the analysis of the equilibria, it is important to work out the expected bid submitted by each supplier. First, we work out the probability distribution function:

$$
\begin{aligned}
f_s(b) &= \frac{\partial F_s(b)}{\partial b} = \frac{k_n}{k_n - (\theta_n - T)} \frac{\underline{b}}{b^2} \\
f_n(b) &= \frac{\partial F_n(b)}{\partial b} = \frac{(\theta_s + T)}{(\theta_s + T) - (\theta_s + \theta_n - k_n)} \frac{\underline{b}}{b^2}
\end{aligned} \tag{11.15}
$$

Second, we work out the expected bid for each supplier:

$$
\begin{aligned}
E_s(b) &= \int_{\underline{b}}^{P} b f_s(b) \partial b = \frac{k_n}{k_n - (\theta_n - T)} \int_{\underline{b}}^{P} \frac{b\,\underline{b}}{b^2} \\
&= \frac{k_n}{k_n - (\theta_n - T)} \left[ln(b)\right]_{\underline{b}}^{P} = 3.51 \\
E_n(b) &= \int_{\underline{b}}^{P} b f_n(b) \partial b = \frac{(\theta_s + T)}{(\theta_s + T) - (\theta_s + \theta_n - k_n)} \int_{\underline{b}}^{P} \frac{b\,\underline{b}}{b^2} + P\left[1 - F_n(P)\right] \\
&= \frac{(\theta_s + T)}{(\theta_s + T) - (\theta_s + \theta_n - k_n)} \left[ln(b)\right]_{\underline{b}}^{P} + P\left[1 - F_n(P)\right] = 4.109
\end{aligned} \tag{11.16}
$$

It is important to notice that the supplier n's cumulative distribution function is discontinuous in the upper bound of the support. Therefore, to work out the supplier n's expected bid, it is necessary to add the term $P\left[1 - F_n(P)\right]$ to work out supplier n's expected bid. In contrast supplier s's cumulative distribution function is continuous in the upper bound of the support, and the probability assigned to the maximum bid allowed by the auctioneer is zero. Therefore, it is not necessary to add the term $P\left[1 - F_s(P)\right]$ to work out supplier s's expected bid.

Question 6:

Use the same parameters than in question 5, but now assume that the suppliers have to pay a tariff for the electricity that they sell into the other market, but not for the

electricity that they sell in their own market (transmission tariff) $t = 2.5$. **Work out the close form solution that define the Nash equilibrium in a discriminatory price auction.**

We have to prove that **a pure strategies equilibrium does not exist**. We proceed in three steps:

First, we find the bid that makes indifferent each supplier between to submit the maximum bid allowed by the auctioneer and to undercut the other supplier. We start from supplier n. Supplier n's residual profits are determined by $P(\theta_n - T)$, and the bid that makes undercutting unprofitable is defined by:

$$P(\theta_n - T) = \underline{b}_n k_n - t(k_n - \theta_n) \Longrightarrow \underline{b}_n = \frac{P(\theta_n - T) + t(k_n - \theta_n)}{k_n} = 2.75$$

We work out the bid that makes undercutting unprofitable for supplier s. In that case, supplier s's residual profits are determined by $P(\theta_n + \theta_s - k_n)$, and the bid that makes undercutting unprofitable is defined by:

$$P(\theta_s + \theta_n - k_n) = \underline{b}_s(\theta_s + T) - tT \Longrightarrow \underline{b}_s = \frac{P(\theta_s + \theta_n - k_n) + tT}{(\theta_s + T)} = 2.5$$

Second, we prove that a pair of bids in the interval $[0, max\left\{\underline{b}_s, \underline{b}_n\right\}]$ cannot be a Nash equilibrium. A pure strategies equilibrium does not exist in that interval, since at least one of the suppliers prefers to submit the maximum bid allowed by the auctioneer and satisfy the residual demand.

Third, a pair of bids in the interval $[max\left\{\underline{b}_s, \underline{b}_n\right\}, P]$ cannot be a Nash equilibrium, since both suppliers undercut each other to be dispatched first in the auction.

Therefore, the Nash equilibrium is in mixed strategies. Hence, it is necessary to find the support of the mixed strategies equilibrium and the cumulative distribution function. **The support of the mixed strategies equilibrium** is defined by $b \in [max\left\{\underline{b}_s, \underline{b}_n\right\}, P]$, since as we show above, the suppliers never randomize in the interval $b \in [0, max\left\{\underline{b}_s, \underline{b}_n\right\}]$.

To work out the **cumulative distribution function**, we proceed in three different steps:

First, supplier n's profit function is defined by:

$$\begin{aligned} \pi_n(b;\theta) \quad = \quad & F_s(b)\left[b(\theta_n - T)\right] + (1 - F_s(b))\left[bk_n - t(k_n - \theta_n)\right] \\ & - F_s(b)\left[b(k_n - (\theta_n - T)) - t(k_n - \theta_n)\right] + bk_n - t(k_n - \theta_n) \end{aligned} \tag{11.17}$$

The first term in equation 12.17 $F_s(b)\left[b(\theta_n - T)\right]$ represents supplier n' profits when $b_s \leq b_n$. The second term in equation 12.17 $(1 - F_s(b))\left[bk_n - t(k_n - \theta_n)\right]$ represents supplier n' profits when $b_s > b_n$. It is important to notice that when supplier n submits the lower bid in the auction it has to pay a tariff for the electricity that sells to the other node $(k_n - \theta_n)$.

Therefore, supplier s's cumulative distribution function is defined by:

$$F_s(b) = \frac{bk_n - t(k_n - \theta_n) - \pi_n(b;\theta)}{b\left[k_n - (\theta\_T)\right] - t(k_n - \theta_n)} \tag{11.18}$$

Second, at the lower bound of the support $(\underline{b})$, the cumulative distribution function is zero. Plugging that information in equation 12.17, we can work out supplier n's profits:

$$F_s(\underline{b};\theta) = 0 \Longrightarrow \pi_n(\underline{b};\theta) = \underline{b}k_n \tag{11.19}$$

Moreover, we know that $\pi_n(\underline{b};\theta) = \underline{b}k_n = \pi_n(b;\theta)\ \forall b \in [\underline{b}, P]$. The last inequality holds because all the bids in the support give in expectation the same payoff. Otherwise, the suppliers can reallocate probabilities to increase their expected profits.

Third, by using the information obtained in the second step and plug in the value of supplier n's profits in equation 12.8, we obtain supplier s's cumulative distribution function:

$$F_s(b) = \frac{bk_n - \underline{b}k_n}{b\left[k_n - (\theta_n - T)\right] - t(k_n - \theta_n)} \tag{11.20}$$

We repeat the same steps to work out supplier n's cumulative distribution function:

First, supplier s's profit function is defined by:

$$\begin{aligned} \pi_s(b;\theta) &= F_n(b)\left[b(\theta_s + \theta_n - k_n)\right] + (1 - F_n(b))\left[b(\theta_s + T) - tT\right] \\ &\quad -F_n(b)\left[b((\theta_s + T) - (\theta_s + \theta_n - k_n)) - tT\right] + b(\theta_s + T) - tT \end{aligned} \tag{11.21}$$

Therefore, supplier n's cumulative distribution function is defined by:

$$F_n(b) = \frac{b(\theta_s + T) - tT - \pi_s(b;\theta)}{b\left[(\theta_s + T) - (\theta_s + \theta_n - k_n) - tT\right]} \tag{11.22}$$

Second, at the lower bound of the support ($\underline{b}$), the cumulative distribution function is zero. Plugging that information in equation 12.22, we can work out supplier s's profits:

$$F_n(\underline{b};\theta) = 0 \Longrightarrow \pi_s(\underline{b};\theta) = \underline{b}(\theta_s + T) - tT \tag{11.23}$$

Moreover, we know that $\pi_s(\underline{b};\theta) = \underline{b}(\theta_s + T) = \pi_s(b;\theta)\ \forall b \in [\underline{b}, P]$. The last inequality holds because all the bids in the support give in expectation the same payoff. Otherwise, the suppliers can reallocate probabilities to increase their expected profits.

Third, by using the information obtained in the second step and plug in the value of supplier s's profits in equation 12.22, we obtain supplier n's cumulative distribution function:

$$F_n(b) = \frac{b(\theta_s + T) - \underline{b}(\theta_s + T)}{b\left[(\theta_s + T) - (\theta_s + \theta_n - k_n)\right] - Tt} \tag{11.24}$$

The graphical representation of the cumulative distribution functions of both suppliers are in the right-hand side of figure 12.2. The cumulative distribution functions present two important characteristics:

First, the cumulative distribution function of the supplier located in the high-demand node is stepper in the lower bound of the support. Therefore, supplier n submits lower bids with higher probability. The intuition is as follows: supplier n has to sell a lower portion of its electricity into the other node and its costs are lower. Therefore, it submits lower bids to extract the efficiency rents.

Second, the supplier located in the high-demand node submits the maximum bid with a positive probability. The intuition of this result is as follows: the supplier located in the high-demand node faces a high residual demand and it submits the price cap with a positive probability to satisfy that demand.

This contrast with the case in with the transmission tariff is zero. In that case, the supplier located in the high-demand node has no advantages in cost, and it submits in expectation higher bid than the supplier located in the low-demand node (left-hand side vs. right-hand side in figure 12.2).

Question 7:

By using the parameters in question 5, and only for the case in which the transmission line is congested, answer the next questions:
Question 7.1: Work out the Nash equilibrium in a zonal market when the auction is uniform and the transmission is taken into account ex-ante.

The transmission line is congested only when the supplier located in the high demand market submits the highest bid in the spot electricity market $b_n^S > b_s^S$. In that case, suppliers' profits are equal to:

$$\begin{aligned}\pi_s &= b_n^S(\theta_s + T) \\ \pi_n &= b_n^S(\theta_n - T)\end{aligned}$$

The Nash equilibrium in that case is defined by the pair of strategies:

$$\left(b_s \in \left[0, \frac{P(\theta_n - T)}{k_n}\right]; b_n = P\right)$$

The profits of the suppliers in the equilibrium are:

$$\begin{aligned}\pi_s^* &= P(\theta_s + T) \\ \pi_n^* &= P(\theta_n - T)\end{aligned} \tag{11.25}$$

Question 7.2: Work out the Nash equilibrium in a zonal market when the auction in the spot electricity market is uniform, and a discriminatory redispatch market is introduced ex-post to alleviate the congestion in the line. Assume that the suppliers submit different bids in the spot and in the redispatch market.

The suppliers submit the pair of bids (b_s^S, b_n^N) in the spot electricity market and the pair of bids (b_s^R, b_n^R) in the redispatch market, and suppliers' profits are defined by:

$$\begin{aligned}\pi_s &= b_n^S(k_s) - b_s^R(k_s - (\theta_s + T)) \\ \pi_n &= b_n^S(\theta_s + \theta_n - k_s) + b_n^R((\theta_n - T) - (\theta_s + \theta_n - k_s))\end{aligned} \tag{11.26}$$

Where the term $b_n^S(k_s)$ in equation 12.26 represents supplier s's profits in the spot electricity market, and $-b_s^R(k_s - (\theta_s + T))$ represents supplier s's expenses in the redispatch market, since it has to buy back the electricity that it cannot sell in the spot electricity market due to the transmission constraint. The term $b_n^S(\theta_s + \theta_n - k_s)$ in equation 12.26 represents supplier n's profits in the spot electricity market, and the term $+b_n^R((\theta_n - T) - (\theta_s + \theta_n - k_s))$ represents supplier n's profits in the redispatch market, since it is called into operation to satisfy the demand that supplier s cannot satisfy in the spot electricity market due to the transmission constraint.

To work out the subgame perfect Nash equilibrium, we have to proceed by backward induction. First, we have to work out the equilibrium in the redispatch market. By using equation 12.26, it is easy to find that in equilibrium $b_s^{R*} = 0$, since that bid minimizes supplier s's expenses in the redispatch market. By using equation 12.26, it is easy to find that in equilibrium $b_n^{R*} = P$, since that bid maximizes supplier n's profits in the redispatch market.

We plug in the pair of values of $b_s^{R*} = 0$, and $b_n^{R*} = P$ in equation 12.26, and we obtain:

$$\begin{aligned} \pi_s &= b_n^S(k_s) \\ \pi_n &= b_n^S(\theta_s + \theta_n - k_s) + P((\theta_n - T) - (\theta_s + \theta_n - k_s)) \end{aligned} \tag{11.27}$$

By using equation 12.27, it is easy to work out the equilibrium in the spot electricity market. The pair of bids that define that equilibrium is:

$$\left(b_s^{S*} \in \left[\frac{P(\theta_s + \theta_n - k_s)}{k_n} \right], b_n^{S*} = P \right)$$

By plug in the equilibrium bid in equation 12.26, we work out suppliers' profits in the equilibrium:

$$\begin{aligned} \pi_s &= P(k_s) \\ \pi_n &= P(\theta_n - T) \end{aligned} \tag{11.28}$$

Question 7.3: Based on the your answers to questions 7.1 and 7.2, what is the effect of a change on the design on suppliers' profits?

By comparing equation 12.25 and 12.28, it is easy to check that the equilibrium profits of the supplier located in the importing node are the same with the two market designs. However, the profits of the supplier located in the exporting node are much higher when an expost redispatch mechanism is introduced by the auctioneer. That can induce long-term investment distortions, since the suppliers could invest in the exporting node where the production capacity is less valuable. These distortions receive the name of increase-decrease game.