Can Intraday Market be Designed as a Congestion Management Tool?

Somayeh Rahimi Alangi *[1], Mette Bjørndal [1], and Endre Bjørndal [1]

[1] *Department of Business and Management Science, NHH Norwegian School of Economics, N-5045 Bergen, Norway*

Abstract

For dealing with the large-scale penetration of intermittent generation resources in the Europe, intraday market has been designed and now the integration of all European intraday markets is on the agenda. As both day-ahead and intraday markets are based on zonal pricing wherein physical characteristic of transmission network is not taken into account, large amount of unscheduled flows originating specifically from wind generators, makes it very difficult that network to be in balance close to the delivery time. In this paper, we suggest a new design for intraday market based on a coordinated multilateral trade approach. In our customized approach, participants submit their buy and sell orders to the shared order book continuously whenever they find it profitable and (batch) auctions are conducted by power exchange at frequent but discrete-time intervals. Each batch auction result is accepted by the TSO if no violation occurs in the network or is curtailed until no violation occurs. After each curtailment, TSO sends some signals to the power exchange to be considered for the next (batch) auction in order to help future orders meet network constraints. This approach shows all the benefits of the nodal pricing while is compatible to the European electricity market structure. In this way, intraday transmission capacity is allocated more efficiently and the value of the scarce capacity can be signified.

Keywords: Coordinated Multilateral Trades (CMT), Congestion management, Integrated Intraday market, Renewable integration

1 Introduction

The growth of intermittent generation capacity has increased the importance of efficient intraday markets, seeing that it becomes more challenging for market participants to be in balance between the day-ahead and real-time balancing markets. As investigated by many authors intraday market, if properly designed, can be an effective market mechanism not only for facilitating the large-scale integration of wind power generation but also for increasing wind power generators' competitiveness (Weber (2010) and Jafari et al. (2014)). Hence, there will be an increasing interest in trading in the intraday markets. It is very lucrative for both market participants and power systems that network to be in balance closer to the delivery time, to reduce the need for reserves

*Corresponding Author: somayeh.alangi@nhh.no

and associated costs. In this regard, the European commission has established a target model to integrate all intraday markets based on continuous trading. Therefore, a project called XBID (cross-border intraday) was launched to create a joint integrated intraday cross-zonal market. As mentioned in XBID documents the main aims of integration are promoting effective competion and pricing, increasing liquidity and enabling a more efficient utilization of generation resources across Europe.

Currently, two different exchange-based forms of intraday markets have been designed in Europe: auction-based (discrete auctions) and continuous trading intraday markets. In continuous trading, power exchange provides a 'limit order book'-based platform wherein market participants can submit bid (for buy) and ask (for sell) orders. Whenever they find it profitable in a period between intraday market opening until 15 minutes before delivery time. A trade occurs when the bid price is higher than or equal to the ask price. Hence, each trade has its own specific price and this property substantially differentiates continuous trading from discrete auction (with a unique market clearing price) (Nordpool (2016)).

Advantages and disadvantages of discrete auction versus continuous trading intraday markets have been debated in many papers. As continuous trading allows market participants to trade 24 hours/7 days a week, they find an immediate opportunity to trade their imbalances. Thus, as soon as new information receives (either their own situation, like updated wind power forecast, or signals from others that can be reflected in bid-ask spread), it can be used immediately which is especially important for intermittent generators (Henriot (2012)). Conversely, Hagemann (2013) points out that in discrete auction intraday market, participants have to wait until the next auction is cleared. Therefore, they are not allocated to do immediate self-balancing. Hence, continuous trading is superior to discrete auction from ease of trade point of view.

By simulating the behaviour of a zero-intelligent trader, Weber and Schröder (2011) assess the efficiency of continuous trading versus discrete auctions. They conclude that since continuous trading adhere the first-come-first-serve principle, it entails a lower allocative efficiency, means that depending on the order arrivals some trades with negative welfare contribution may occur while some others with positive welfare contribution may never happen. But this is not the case for discrete auction markets with maximizing social welfare objective function.

Scharff and Amelin (2016) empirically analysed the trading behaviour on the Elbas intraday market. Their study shows that factors like high share of wind power in Denmark, restricted available transmission capacity from Norway to continental Europe for intraday market and high balancing prices in Finland result in varying trading behaviour in different price zones. They also illustrate that half of the Elbas intraday trades are settled 3 hours before delivery time most likely by wind power producers with short forecast horizon. Finally, it has been concluded that since most of the intraday trades are motivated by intermittent power producers than by conventional power plant outages, continuous trading seems to be a more suitable design for European intraday markets.

It has been debated by Neuhoff et al. (2016) that which intraday market design, continuous or discrete auction, is more suitable for integrating all European intraday markets. They empirically assessed the effect of the additional intraday auction introduced by EPEX in December 2014. This uniform price auction is settled at the begining of continuous intraday session at 3 p.m for the next 96 quarters of the following day. Their observation shows that adding an auction to the current continuous market increases liquidity and market depth with a reduced price volatility along with removing the speed race (which is an important issue in continuous trading). Nevertheless, too infrequent auctions may lead to postponed adjustments during intraday market. Hence, the right frequency of intraday auctions is questioned in this paper as an important design question. Moreover, they conclude that with auctions, intraday transmission capacity can be allocated more efficiently and the value of the scarce capacity can be signified while this is not the case for continuous trading. In the end, to reach all those mentioned benefits, they suggest to substitute continuous trading for frequent batch auctions.

The frequent batch auction idea and its advantages over continuous limit order book in financial exchanges firstly were discussed by Budish et al. (2014) and extended later by Budish et al. (2015). Based on their definition, frequent batch auctions are identical to continuous limit order book with two exceptions: 1. time is considered as discrete, not continuous, 2. instead of serial processing of orders based on their time-price priority, they are processed in batch form using a uniform-price auction. By modifying the market in this way, first the speed race is eliminated. Second, instead of competing on speed (to be the first one processed) price is rivaled.

In line with discussions argued by Neuhoff et al. (2016), we show that on one hand the bilateral trading structure of the continuous intraday market does not allow to have an efficient congestion management approach. On the other hand, the European simplified network modeling at the day-ahead and intraday market creates inefficiencies which results in higher imbalance costs in comparison to the case that all transmission network constraints are considered in a market/markets prior to the real-time.

In this paper, by customizing the coordinated multilateral trading (CMT) approach- which was firstly suggested by Wu and Varaiya (1999)- to the current European structure, we fix both aformentioned sources of inefficiency. In other word, the CMT approach allows to model multilateral trades (instead of just having bilateral trades) which are more beneficial in relieving congestion than just bilateral trades. Moreover, by replacing the information transfering between power exchanges and TSOs from available transmission capacity of cross-border interconnectors (which are imaginary lines between zones) to the power transfer distribution factor of congested lines (which are physical lines between nodes), our model is able to reach to the optimal nodal solution at the end of the intraday market, provided that all the circumstances such as supply and demand functions remain fix at day-ahead and all stages of intraday markets and no uncertainty is modeled. However, we will illustrate that our model can also be extended to cope with varied supply and demand functions as well as uncertainty but there is not any guarantee to reach to the optimal nodal solution in this case.

The rest of the paper is organized as follows. In section 2, different approaches of modeling electricity markets which are equivalent to the optimal nodal model will be reviewed. This literature review is needed to understand the relation between the current European design, CMT approach and optimal nodal model. Section 3 gives a detailed explanation of XBID components and describes the research question and the relevant assumptions. Section 4 reviews the CMT approach and shows its relation to the day-ahead and intraday market by mathematical formulation. Section 5 illustrates out customized CMT approach in a 6-node example. Based on the allocated ATCs at the day-ahead stage and trearting day-ahead result with or without curtailment before intraday market, several cases are discussed. Finally, section 6 concludes the paper and future research is discussed.

2 Literature review

In the nodal pricing approach primarily introduced by Schweppe et al. (1988), nodal prices are the shadow price of power balance equations produced by optimal power flow model. The successful experience of implementing nodal pricing in North America, Australia and New Zealand has proven the efficiency of this powerful transmission pricing tool without encountering significant technical problems. This congestion management tool has been considered by European Commission as one of the plausible approaches for integrating European electricity markets (Brunekreeft et al. (2005)). However, in the 1990's, there was a great debate on the efficiency of this approach. The most important objection to nodal pricing approach raised by Wu and Varaiya (1999) is the intervention of transmission system operator (TSO) in economic or market decisions. In other words, in order to acheive a solution for optimal nodal model, which on the one hand guarantees the security and reliability of the power system and on the other hand promotes economic efficiency, the strategic information about cost and demand functions (private information) of generators and consumers must be revealed to the TSO who is just responsible of technical support of the power system. Hence, the information structure and decision making authority are both centralized in nodal pricing.

Therefore, many attempts have been made to decouple these two distinct dimensions of the power system by delegating economic efficiency responsibilities to power exchange and technical support of the power system to TSO. Nevertheless, they can never converge to a system optimal solution if a proper coordination is not established between them.

Accordingly, the coordination models can be interpreted as various decomposition procedures for nodal pricing wherein the TSO solves different subproblems and subject to the subproblem structure, different information is exchanged back and forth. Overally, these decomposition models, namely information revealed by TSO, can be classified into two groups:

1. Price-directed
2. Resource-directed

The method suggested by Chao and Peck (1996) is a price-directed scheme for explicit congestion pricing. In this method, scarce transmission resources are explicitly priced with power transfer distribution factor such that traders must acquire transmission capacity rights to do a transaction. In optimum, Chao-Peck prices are exactly optimal nodal prices.

Like Chao-Peck price-directed method, the capacity charge approach suggested by Bjørndal et al. (2010) can be categorized as a price-driven decomposition of the optimal nodal model. By relaxing the line capacity constraints through Lagrangian relaxation, they are implicitly managed by means of nodal capacity charges, which result in shifts in the supply and demand curves. In a better word, the social optimum solution is achieved by an iterative process between TSO who is announcing nodal capacity charges and power exchange who is solving the unconstrained optimal dispatch problem by clearing the market with respect to the shifted supply and demand curves. It is important to note that these nodal capacity charges are calculated based on an estimate of the shadow price of the line capacity constraints and power transfer distribution factors.

While in the price-directed approaches, price information is announced by TSO at every iteration, in the coordinated multilateral trade model suggested by Wu and Varaiya (1999), technical information related to the congested lines signifying scarce resource availability (resource in this case means capacity of transmission lines) is announced by the TSO. Hence, the decision making authorities related to economics and technical issues of power system are broken up. However, they go one step further by replacing the power exchange with a new entity called a broker. After receiving supply and demand functions of interested generators and consumers along with relevant signals from the TSO, the broker finds profitable multilateral trades that move towards feasible direction but are not necessarily feasible over all transmission lines. Thus, coordination is established through an iterative process, where on the one side power transfer distribution factors (PTDFs) of infeasible lines resulting from the last trades are publicly announced by the TSO and on the other side by utilizing the new information, the broker finds new profitable trades. This process lasts until no further profitable trade can be found. They also proved that their proposed CMT model will achieve the same economic efficiency and the same level of reliability as the nodal pricing model, meaning that social welfare is maximized with respect to the network constraints, provided that generators tend to maximize profit and consumers maximize their utility. Furthermore, they even state that instead of a broker, groups of generators and consumers with the private terms and conditions of a trade (without revealing their cost and benefit functions) can suggest balanced trades to the TSO. Consequently, no price can be extracted from CMT model and it is not necessary that trades happen at the same time.

The CMT idea introduced in 1990's, has been studied by Qin et al. (2017) for designing an innovative flexible market for smart grids. They mention that the great flexibility of the CMT model along with low communication and control burden on the TSO, makes it an attractive approach for coordinating procedures in the distribution system. Moreover, by generalizing the CMT model from a deterministic one settlement market (day-ahead) into a stochastic two-settelemt market (day-ahead and real-time), they confirm that it is possible to achieve the same solution (maximized expected social welfare) as a stochastic optimal dispatch model. Additionally, the dispatch and prices extracted from their model support competitive equilibrium under uncertainty principal which is called Arrow-Debreu equilibrium.

Since we are suggesting to utilize the CMT approach for a more efficient way of managing congestion in the intraday market, further details, related terminology and the relevant mathematical model of CMT will be given in section 4.2.

3 Problem description and assumptions

The sequence of day-ahead, intraday and balancing markets are cleared for each delivery hour of day d. In the European design, the first two markets are cleared by power exchanges only partly addressing the physical transmission network, while the last one is cleared by TSOs settling the energy imbalances respecting to the day-ahead and intraday schedule by considering the full transmission network. Therefore, decoupling of the nodal pricing model by delegating the market efficiency responsibility to the power exchange and security/reliability to the TSO has been done before. Hence, the main idea of the CMT which is decoupling decision making authorities has been done before in the European electricity market. But something that makes current European design to be different from the CMT model is the information sharing content. Hence, in order to have a better understanding of the difference between these two, it is necessary to know more details especially about intraday market and its integration.

In the XBID project, all orders of each power exchange will be shared in a shared order book (SOB) module such that all market participants of other power exchanges located in other bidding zones can see them provided that enough cross-border capacities called available transmission capacities (ATCs) are available. These capacities are provided by the relevant TSOs in the capacity management module (CMM). CMM provides two ATCs for each cross-border line, one for each direction. Orders submitted for different bidding zones can be matched provided there is enough capacity available. When two orders are being matched the SOB and CMM will be updated immediately. Trades are based on the first-come first-served principle such that the highest bid price and the lowest ask price get served first. Whenever a matching happens, the SOB calculates the required quantity to be transferred between the source and destination zone. Then CMM is responsible to find a routing plan which results in capacity allocations and thus updating ATCs.

The routing model applies the minimum cost flow routing problem to select routes with minimum cost satisfying the flow constraints over cross-border lines. But these cross-border lines are treated as edges of a graph which are not reflecting the physical transmission network. Therefore, the objection to this model is that the externalities created by the loop flows which is the main characteristic of electricity networks have not been reflected by this model. Consequently, like day-ahead market, it is very probable that the trades occuring in the intraday market lead to infeasible flows over physical transmission lines. So, market participants still have to pay high imbalance costs due to the simplified network modeling. Even flow-based market coupling has not been successful in achieving the same solution as the benchmark case of nodal pricing.

Regarding the recent decision of the European commission to integrate intraday markets by continuous trading approach and with respect to the iterative nature of the CMT model, we think that this new environment is the right place to utilize the benefits of the CMT model in the integrated intraday market. In this way, firstly, the responsibilities of power exchanges and TSOs have not been mixed up, secondly, by seeing the physical transmission lines in a market cleared prior to the balancing market, we will be able to reduce the imbalanced costs (in real-time balancing market) due to congested lines resulting from the simplified network in day-ahead stage. Consequently, our suggested model will finally reach to the nodal pricing solution at the end of the intraday market.

The most interesting aspect of our suggested model is that even though at the end of the intraday market nodal optimal solution will be achieved, the network transmission constraints are not directly included in the power exchange model (contrary to the nodal model) and the only information that power exchange needs about transmission network is the power transfer distribution factor of congested lines.

The underlying assumptions are made to develope the model:

- In the day-ahead market, power exchanges have access to the zonal level information. Meaning that it is enough for them to know the overal supply and demand functions at each zone.
- While in our suggested model (contrary to the current intraday design) power exchanges need nodal level information such as supply and demand functions at each node, capacity constraints of each market participants and etc.
- There is just one TSO who checks the feasibility of trades or solves curtailment problem.
- It is allowed for TSO to publicly announce the power transfer distribution factor of congested lines in its region of governance.

4 Mathematical models

4.1 Notation

We adopted the same mathematical formulation as Bjorndal et al. (2016). The model entails I participants either generators with positive or consumers with negative values. For each $i \in I$, there exists solutions x_i for day-ahead and $X_i^{(k)}$ for stage k of intraday market. Iteration $k = 0$ can be considered as day-ahead market stage. Whenever X_i comes without stage superscript k means that we are not talking specifically about day-ahead or intraday stages and we just mean production or consumption quantity of participant i.

The day-ahead cost function of generator i which could be of any type (quadratic, stepwise or piece-wise linear) is shown by $c_i(x_i)$. Consumers can also be considered as generators with negative values. Therefore, to keep conciseness, the benefit function of consumers can be shown as $c_i(x_i)$ with negative value of x_i ($x_i < 0$). Thus, $c_i(x_i)$ can be interpreted as a cost function of all market participants. Similarly, $\tilde{c}_i^{(k)}(X_i^{(k)})$ illustrates the cost and benefit function of generators and consumers at each stage of the intraday market.

C_i^1 represents the set of feasible solutions corresponding to participant i for day-ahead market (which can be the capacity constraint of each market participanyt i), whereas $C_i^{2(k)}$ proportionates to the feasible solutions corresponding to the iteration k of intraday market which is dependent on the decision x_i from the day-ahead market and decisions $X_i^{(1)}, ..., X_i^{(k-1)}$ from previous iterations of intraday market. Therefore, a feasible solution to both day-ahead and intraday markets must satisfy the following constraints:

$$x_i \in C_i^1 \qquad i \in I \tag{1}$$
$$X_i^{(k)} \in C_i^{2(k)}(x_i, X_i^{(1)}, ..., X_i^{(k-1)}) \qquad i \in I \tag{2}$$

Each generator and load i locates in a specific node $n \in N$ as well as a pre-determined zone $z \in Z$. Nodes of the network are connected by a set of physical transmission lines L. Corresponds to each line l, there is a vector of flows $F = (f_l)_{l \in L}$ and capacity limitation cap_l. $PTDF = (ptdf_{l,n})$ is the $L \times N$ power transfer distribution factor matrix; the (l, n)th element in the matrix states that if 1 MW of power is injected at node $n \in N$, how much of it passes through line $l \in L$ and receives at the reference node. If ν_0 and ν_1 show the starting and ending nodes of line l and $f_l > 0$, then it means that power is flowing from ν_0 to ν_1.

For every adjacent zones which are connected by physical transmission lines l, there exists an inter-zonal interconnector $e \in E$ which conveys commercial flows between zones. Likewise the definition of f_l, corresponds to each inter-zonal interconnector e, there is a flow $(f_e)_{e \in E}$. If ω_0 and ω_1 show the starting and ending zones of interconnector e and $f_e > 0$, then it means that commercial flow is flowing from ω_0 to ω_1.

U^1 and U^2 represents network constraints in the day-ahead and intraday markets respectively. More detailed explanation about network constraints are given in sections 4.3 and 4.4.

4.2 Coordinated multilateral trading process

The novel idea mentioned by Wu and Varaiya (1999) and then by Qin et al. (2017) is that instead of creating centralized infrastructure with high level of coordination, a free-market style of meet-and-trade is able to provide an opportunity for all generators and consumers to seek profit on their own, implying that they can conduct the economic function themselves, deciding about price, trading terms and conditions as well as trading quantity. Hence, the direct effect of this mechanism is that price information is private. However, the idea of meet-and-trade without any coordination with TSO could result in flows that violate the transmission line capacity constraints. Consequently, Wu and Varaiya suggested an idea where the TSO and free traders coordinate with as little information sharing as possible such that the reliability of the power system is guaranteed at every step of the trading process, or in a better word, a feasible solution is attained at every step. Therefore, they even neglected the power exchange role and just let market participants coordinate with TSO directly or with the help of a broker. Hence, trade notion is the main constituent of such a kind of market design.

Even if the definition of multilateral, feasible, feasible direction and profitable trades as well as other related terminology have been fully explained in Wu and Varaiya (1999) and expanded to uncertain models by Qin et al. (2017), we will have a short review of them customized to our purposes.

Definition 1 : Multilateral trade : is a trade including more than one party where in a lossless system the sum of generation of generators participating in the trade equals the sum of consumption of involved loads.

For example, a three-lateral trade between two generators located at node i and j respectively and a consumer at node k for α MW to be generated by each generator and 2α MW to be consumed by load is described by injection vector $\tau = (0, ..., \alpha, ..., \alpha, ..., -2\alpha, ..., 0)$. In general, a multilateral trade between m parties (which hereafter can be called m-lateral trade) $I^m \subset I$ of participants, is represented by an injection vector $\tau^m = (X_i^m)$ such that $\sum_{i \in I^m} X_i^m = 0$ and for $i \notin I^m$, $X_i^m = 0$ and X_i^m describes the production or consumption of participant i at trade m.

The building block of continuous trading approach is bilateral trade. But as Wu and Varaiya (1999) proved, it is not possible to relieve congestion over a congested line just by bilateral trades. Hence, having multilateral trades to converge to the optimal nodal solution is a necessity.

System injection vector $\boldsymbol{\tau}$ is the result of the set of all multilateral trades $\boldsymbol{\tau} = \sum_{m=1}^{M} \tau^m$. Each element in N-dimentional vector $\boldsymbol{\tau} = (\tau_n)$ demonstrates the net outflow (if $\tau_n > 0$)/net inflow (if $\tau_n < 0$) of power from/to node n to/from the network.

Thusfar, multilateral trades with the aim of maximizing profit are formed by market participants neglecting the power system reliability constraints. Hence, the TSO has to check the feasibility of the trades and curtail them whenever needed.

By receiving vector $\boldsymbol{\tau}$, TSO has all the required information to calculate vector of flows F:

$$F = PTDF.\boldsymbol{\tau} \tag{3}$$

Flows must satisfy the line capacity constraints:

$$\mid f_l \mid \leqslant cap_l \tag{4}$$

Hence, if at least one violation occurs, the TSO has to curtail the trades to get a feasible flow.

Definition 2 : Uniform curtailment : the simplest way of cutting extra injection is just to accept a portion of the trade τ^m, such that $\gamma^m \tau^m$ is accepted by the TSO for $0 \leqslant \gamma_m < 1$. $\gamma^m = 1$ means that the whole trade τ^m is accepted without curtailment.

After each curtailment, vector τ^m will be updated by replacing it with its relevant curtailed vector $\tau^m \leftarrow \gamma^m \tau^m$ and therefore an updated injection vector $\boldsymbol{\tau}$. Thus updated τ^m and $\boldsymbol{\tau}$ after curtailment are feasible solution satisfying constraints (3) and (4).

Definition 3 : Feasible direction trade : assume that $\boldsymbol{\tau}$ is a feasible solution for equations (3) and (4) and the flows resulting from this injection has the following property:

$$f_l = cap_l \qquad l = l_1, l_2, ..., l_r \tag{5}$$

$$f_l < cap_l \qquad otherwise \tag{6}$$

Meaning that after distributing the flows resulting from injection vector $\boldsymbol{\tau}$ by equation (3), $r \subset L$ lines are congested and others are below their capacity limit. A multilateral trade $\Delta\tau = (\Delta X_i)$ is a feasible direction trade at $\boldsymbol{\tau}$ if:

$$PTDF.\Delta\tau \leqslant 0 \tag{7}$$

for congested lines $l = l_1, l_2, ..., l_r$.

The l'th element of $PTDF.\Delta\tau$ expresses the net power flow of line l resulting from the trade $\Delta\tau$. Therefore, in order to have a trade in the feasible direction, the trade must not increase the net power flowing through the congested lines.

Definition 4 : profitable multilateral trade : $\Delta\tau$ is a profitable multilateral trade at $\boldsymbol{\tau}$ (which is feasible for (3) and (4)) if it can increase the total welfare or with respect to the definition of C (cost function of generators or negative of benefit function for consumers) it reduces the total cost.

Definition 5 : Broker : is a party who arranges the trades. Based on the definition from Wu and Varaiya (1999), a third party entity can facilitate the trades between generators and consumers by finding profitable trades in the feasible direction through the following optimization problem:

$$\text{Maximize}_{\Delta X_i} \qquad \sum_{i \in I^m} c_i(X_i) - c_i(X_i + \Delta X_i) \tag{8}$$

$$\text{subject to:} \qquad \sum_{i \in I^m} \Delta X_i = 0 \tag{9}$$

$$\sum_{n \in N: i \in n \& i \in I^m} ptdf_{l,n}.\Delta X_i \leqslant 0 \qquad l = l_1, l_2, ..., l_r \tag{10}$$

$I^m \subset I$ is the set of generators and consumers participating in trade m. The objective of the broker is to find ΔX_i such that the cost reduction by moving from X_i to $X_i + \Delta X_i$ is maximized respecting to the power balance constraint among I^m members along with feasible direction trade constraints for congested lines.

As we mentioned before, after curtailing the infeasible flows resulting from the broker's solution, the TSO announces the congested lines (lines at maximum flow) and their related PTDFs. Hence, the broker receives new signals for going in the right direction. The broker's decision making problem is very similar to the economic dispatch model. However, instead of modeling full transmission network, broker tries to move toward feasibility regarding to the signals obtained from TSO.

It is obvious that if more parties are involved, the higher the chance of finding a more profitable solution. So the best case is that all generators and consumers are involved in the broker's problem, which then can be identical to the auction operated by the power exchange. However, the structure of the auction is different from other auctions like day-ahead and balancing market in the sense that at each auction (run at each iteration), the power exchange runs the auction to find an extra profitable trade on the basis of the previous profitable trades found in previous auctions.

Therefore, in this paper we assume that the power exchange can play the role of the broker. The main reason behind this conclusion is that the meet-and-trade approach suggested by Wu and Varaiya (1999) and Qin et al. (2017) only demonstrates the possibility of reaching an efficient market outcome in a decentralized setting, under strong assumptions of zero search cost and perfect information. They don't imply that a decentralized market is superior than centralized in practice, as there will be significant search costs for finding the right trading partners unless there is some information platform that collects participants cost/benefit information and shares it to the suitable parties. A good market structure should be some middle ground between a fully centralized market and fully decentralized one, as centralization requires significant communication cost while decentralization leads to a higher search cost.

As the required infrastructure is provided by European power exchanges through the limit order book in which market participants can find their trading partners without paying high search cost of meet-and-trade approach at the same time they have not to reveal their private information like their cost functions to the power exchange. Hence, it seems that this new environment can be considered as a middle ground between fully centralization and decentralization. But since the trades in the current limit order book is beased on continuous trading and as mentioned before this structure is not efficient for managing congestion, in this paper we suggest to utilize this infrastructure for running frequent auctions. This auction can also be designed like batch auctions. Meaning that, in every batch auction a subset of participants are taking part and it has been cleared like auction. So, the clearing result can be interpreted as a multilateral trade among this subset of participants. However, in this paper, we have a simplifying assumption thet every frequent auction is an auction with all participants involving.

Thus, by this assumption that broker could be the power exchange, we conclude that the power exchange and TSO problems are attained by decomposing the nodal pricing model such that power exchange problem is considered as a master while TSO problem is the subproblem and feasible direction trade constraints are linking cuts.

To sum up, the coordinated multilateral trading process customized to our intraday market is briefly depicted in fig.1.

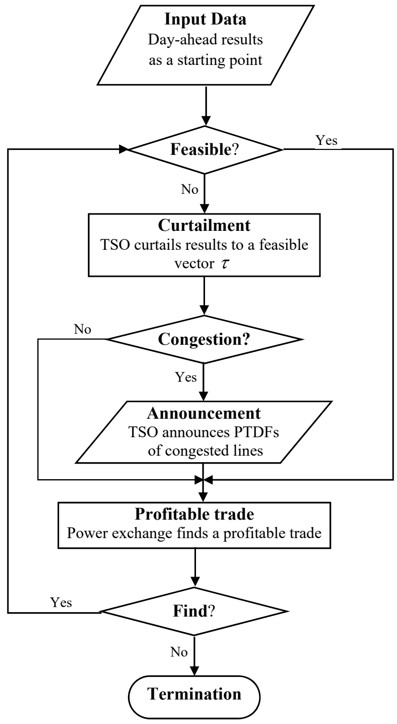

Figure 1: Intraday market structure with CMT approach- approach.A

4.3 Day-ahead market with conventional dispatch

The current European electricity market is a two-settlement system on the basis of day-ahead and balancing market results (I am not sure about intraday market impact on settlement) with deterministic modeling; means that balancing market uncertainties and consequently the relevant bids/offers are not taken into consideration in the day-ahead stage. So, in this paper, the conventional day-ahead market which is consistent with the current European design is considered.

As mentioned in section 4.1, x_i is the vector of day-ahead production of conventional generators G and stochastic generators W as well as day-ahead consumption of flexible loads D. In contrast to the conventional dispatch model mentioned by Morales et al. (2014) where physical network constraints are fully modeled in the day-ahead stage, we assume that our conventional day-ahead dispatch model is cleared with respect to the simplified network constraints compatible to the European zonal model with limited capacities of interconnectors between zones or ATCs.

Hence, the day-ahead market is a pool composed of all fully coordinated power exchanges who receives offers and bids of their related zones as well as the ATCs of the interconnectors corresponding to their related

TSOs. The mathematical formulation for day-ahead market is as follows:

$$\text{Minimize}_{x,f} \qquad \sum_{i \in I} c_i(x_i) \tag{11}$$

$$\text{subject to:} \qquad x_i \in C_i^1, \qquad i \in I \tag{12}$$

$$-\tau_z(f) + \sum_{i \in z} x_i = 0, \qquad : \lambda_z \qquad z \in Z \tag{13}$$

$$\tau_z(f) = \sum_{e:\omega_0(e)=z} f_e - \sum_{e:\omega_1(e)=z} f_e, \qquad z \in Z \tag{14}$$

$$f \in U^1 \tag{15}$$

The constraint (12) reflects the production/consumption capacity constraints of participant i. $\tau_z(f)$ declares the net outflow (if $\tau_z(f) > 0$)/ net inflow (if $\tau_z(f) < 0$) of power from/to zone z to/from other zones. The day-ahead power balance constraint at each zone $z \in Z$ is demonstrated by equation (13), meaning that production minus consumption with positive/negative sign equals net outflow/inflow at each zone. Therefore, the shadow price λ_z of this equation is interpreted as the day-ahead clearing price of zone z. Unlike nodal day-ahead market, just commercial flows which do not reflect physical network constraints are modeled in the zonal day-ahead market. U^1 only shows the inter-zonal trade capacities and is equivalent to the following constraints:

$$-ATC_e \leqslant f_e \leqslant ATC_e \qquad e \in E \tag{16}$$

Thus, the day-ahead dispatch model (11)-(15) can be elucidated as a partly network-constrained auction where the cheapest generators and the consumers with the highest willingness to pay are cleared. Due to the network simplification at day-ahead stage, most probably the day-ahead solution is not satisfied by the physical network constraints and therefore is not a feasible initial trade for intraday market. Two approaches can be adopted here:

- Approach.A: follow exactly the same procedure as CMT approach and curtail the day-ahead solution to get an initial feasible trade before starting intraday market. Hence, intraday market starts with a feasible trade. The advantage of this approach is the guarantee of achieving to the nodal optimal solution at the end of the intraday market. Nevertheless, it can be criticized by saying that curtailing the day-ahead solution is not allowed unless a proportionate payment mechanism is designed for that.
- Approach.B: start intraday market with an infeasible day-ahead solution while appropriate signals (which are PTDFs of overloaded lines) are signified to the power exchange. Therefore, at the first iteration of the intraday market, power exchange tries to find a more profitable trade while flows of overloaded lines at the day-ahead stage not get worsen. Since day-ahead solution is infeasible and first iteration of intraday market just tries to not worsen infeasibility (not try to remove that) the first curtailment model most probably is infeasible unless the capacity of overloaded lines at the day-ahead stage is relaxed and fixed to their day-ahead values. The advantage of this approach is that we have not curtailed the day-ahead solution and at the end of the intraday market we achieve a more profitable and more feasible solution than the day-ahead market. In a better word, the final intraday solution is more profitable and more importantly is feasible over all physical transmission lines except the overloaded lines at the day-ahead stage and the flows through the day-ahead overloaded lines have not get worsen. This solution is exactly similar to the optimal nodal solution with relaxed capacity limitation over day-ahead overloaded lines (in optimal nodal model replace the capacity limit of overloaded lines at day-ahead stage with the flows through that lines resulted from day-ahead solution).

The first approach is exactly based on the flowchart given in figure.1 and its related intraday market and curtailment models are explained at sections 4.4.1 and 4.4.2. The flowchart related to the second approach is depiceted in figure.2.

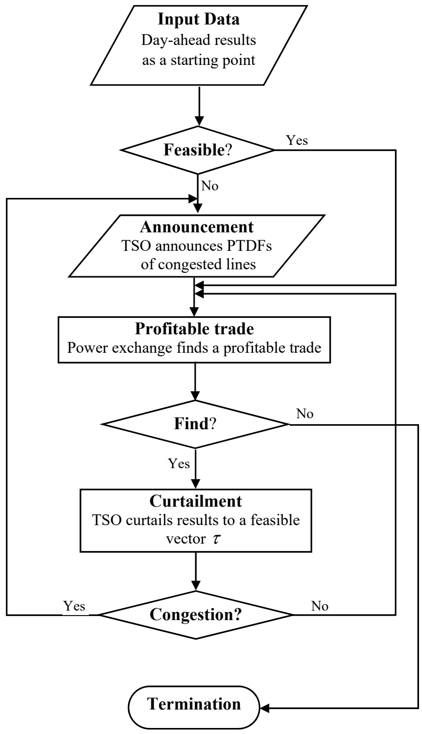

Figure 2: Intraday market structure with CMT approach- approach.B

Following to the approach.A, at first before intraday market opening, day-ahead solution has to be curtailed by the TSO if it is infeasible. Therefore, in the next section the curtailment model deployed by the TSO before and during the intraday market operation will be explained.

Once, the optimal day-ahead results x_i^* are obtained from (11)-(15), the intraday market is responsible to cope with the energy imbalances during hours after day-ahead market closure until 15 minutes before balancing market opening.

4.4 Intraday market auctions by CMT approach

There could be several reasons for participating in the intraday market. Rahimi et al. (2018) had an overal overview of the rationale behind intraday market involvement. These reasons could be unplanned power plant outages, forecast error from intermittent renewable energy sources, load forecast error and etc.

In our suggested model, generators and consumers are able to submit new offers and bids at every stage of the intraday market reflecting their updated situation or new information extracted from previous stages. For the sake of simplicity, we assume that their status is exactly the same as their day-ahead position. Therefore, the same cost and benefit functions c_i as day-ahead market are submitted at each intraday market iteration. Thus, at this paper, the only purpose of intraday market participation is relieving congestion resulted from the day-ahead solution to avoid paying high imbalance costs due to the network constraints violation.

4.4.1 Curtailment model utilized by the TSO

Profitable trades found by the power exchange do not consider power system reliability constraints. Thus, the TSO is responsible to verify that trades meet the reliability constraints. If violation occurs, trade must be curtailed.

Curtailment can be formulated in several ways but the uniform curtailment model mentioned by Wu and Varaiya (1999) and Qin et al. (2017) has specific characteristics that makes it attractive for our purposes:

- It curtails the last trade evenly without any discrimination between market participants
- Since power exchange finds the most profitable trade, by this uniform curtailment model as less possible as the last profitable trade will be curtailed. This results in the less iterations between TSO and power exchange and therefore faster convergence.

At each iteration k of the intraday market, TSO is solving the following uniform curtailment model:

$$\text{Maximize}_{\gamma,f} \quad \gamma \tag{17}$$

$$\text{subject to:} \quad -\tau_n + \sum_{i\in n}(X_i^{(k)} + \gamma^{(k)}\Delta X_i^{(k)}) = 0, \qquad n \in N \tag{18}$$

$$\tau_n = \sum_{l:\nu_0(l)=n} f_l - \sum_{l:\nu_1(l)=n} f_l, \qquad n \in N \tag{19}$$

$$f \in U^2 \tag{20}$$

$$0 \leqslant \gamma^{(k)} \leqslant 1 \tag{21}$$

Here, $\gamma^{(k)}$ is the only variable of the curtailment model that curtails the latest profitable trade ΔX uniformly. $X_i^{(k)}$ and $\Delta X_i^{(k)}$ are parameters in the curtailment model and term $X_i^{(k)} + \gamma^{(k)}\Delta X_i^{(k)}$ means that if the current state of participant i is $X_i^{(k)}$ and power exchange finds the latest profitable trade $\Delta X_i^{(k)}$, then as much possible as the trade $\Delta X_i^{(k)}$ must be accepted such that the power balance constraint (18) as well as network constraints (19)-(20) are satisfied.

τ_n in equation (19) represents the net outflow (if $\tau_n > 0$)/net inflow (if $\tau_n < 0$) of power from/to node n to/from the network. Hence, power balance equation (18) tries to find $\gamma^{(k)}$ such that the net production/net consumption at node n equals to net outflow/inflow. Moreover, U^2 denotes all physical network constraints related to a DC load flow model. Consequently, (20) is equivalent to the following constraints:

$$f_l = \sum_{n\in N} ptdf_{l,n}.\tau_n \qquad l \in L \tag{22}$$

$$-cap_l \leqslant f_l \leqslant cap_l \qquad l \in L \tag{23}$$

Constraint (21) demonstrates that a portion of the latest profitable trade $\Delta X_i^{(k)}$ can be curtailed by the curtailment factor $\gamma^{(k)}$, such that $\gamma^{(k)} = 1$ means the trade $\Delta X_i^{(k)}$ is accepted without curtailment while $\gamma^{(k)} = 0$ shows that $\Delta X_i^{(k)}$ is entirely curtailed.

It should be noted that we have not differentiated between generators and consumers curtailment and assume that all have equal curtailing priority.

After solving curtailment model and finding optimal $\gamma^{(k)}$, generation/consumption quantity X_i of participant i is updated as $X_i^{(k+1)} = X_i^{(k)} + \gamma^{(k)}\Delta X_i^{(k)}$ and $k = k + 1$.

It is assumed that in approach.A, day-ahead model and its curtailment are happening at iteration $k = 0$.

Therefore, the initial values of X_i and ΔX_i are as follows:

$$\Delta X_i^{(0)} = x_i^* \text{ and } X_i^{(0)} = 0 \tag{24}$$

So, at the outset of the intraday market, the state of each participant i is $X_i^{(1)} = \gamma^{(0)}.x_i^*$ which is the day-ahead curtailed quantity.

Whereas in approach.B, just day-ahead is occuring at iteration $k = 0$ and initial values are as (24). But at the begining of the intraday market, the state of each participant i is as before $X_i^{(1)} = x_i^*$.

Whenever curtailment model is running, the PTDFs of the lines which are congested (at their capacity limits) are announced by the TSO. It lets market participants to submit the trades that are in the feasible direction given the current state of the system. Hence, the subset $L_c^{(k)} \subseteq L$ that is:

$$L_c^{(k)} = \{l \in L : |f_l| = cap_l\} \tag{25}$$

is announced by TSO at the end of k's curtailment.

4.4.2 Finding profitable trade by power exchange

By receiving the new information about congested lines (new information $L_c^{(k)}$), power exchange can modify the previous allocated schedules in order to move toward feasibility (for network constraints) by finding profitable deviations from the current schedule. These deviations could be positive or negative. To accomodate a positive deviation, several actions may be taken:

- To increase the power production of flexible generators. It means that they have to resell an extra amount of energy with positive sign of ΔX in the current stage of the intraday market.
- To increase the power consumption of flexible loads, meaning that an extra amount of ΔX would be repurchased in the current stage of the intraday market.
- Extra production of intermittent renewables because of underestimation. Meaning that, by receiving updated forecast, if intermittent generators underestimated their production, they can sell extra amount of ΔX in the current stage of the intraday market.

Likewise, the following actions may be taken for negative deviation:

- To decrease the power production of flexible generators by buying back a negative amount of ΔX.
- To decrease the power consumption by selling a negative amount of ΔX.
- Production reduction of internittent renewables because of overestimation, meaning that if they overestimated their production, they can buy back a negative amount of ΔX.

Hence, the power exchange can solve the following optimization problem to find a profitable trade ΔX in the feasible direction:

$$\text{Maximize}_{\Delta X, f} \qquad \sum_{i \in I} [\tilde{c}_i(X_i^{(k)}) - \tilde{c}_i(X_i^{(k)} + \Delta X_i)] \tag{26}$$

$$\text{subject to:} \qquad X_i^{(k)} + \Delta X_i \in C_i^{(k)}(X_i^{(1)}, ..., X_i^{(k-1)}), \qquad i \in I \tag{27}$$

$$-\tau_n + \sum_{i \in n} \Delta X_i = 0, \qquad n \in N \tag{28}$$

$$\tau_n = \sum_{l:\nu_0(l)=n} f_l - \sum_{l:\nu_1(l)=n} f_l, \qquad n \in N \tag{29}$$

$$\sum_{n \in N} PTDF_{l,n}.\tau_n \leqslant 0 \qquad l \in L_c^{(k)} \tag{30}$$

Where ΔX_i are positive or negative deviations from the current state of the system ($X_i^{(k)}$) and are the variables of the intraday market problem at stage k. At each stage of the intraday market, participant i can submit a new cost or benefit function $\tilde{c}_i$. Hence, the objective of the power exchange is to find a new trade starting from the current state of the system such that the net profit of the trade (or the total cost reduction) is maximized. Constraint (27) imposes the capacity constraints on the new state $X_i^{(k)} + \Delta X_i$, means that the new deviations must be lower or equal than the available capacity of participant i at stage k. Equations (28) and (29) guarantee that all positive or negative deviations of generators and consumers are in balance. By receiving new information about congested lines and their related $PTDF$s, power exchange utilizes the inequalities (30) to move toward feasibility by imposing feasible direction trade constraints. It should be noted that constraints (30) are just imposed on the congested lines $L_c^{(k)}$. Therefore, the line capacity constraints (4) are satisfied by $X_i^{(k)} + \Delta X_i$ for these lines. However, none of these constraints guarantees that the flow resulted from the new state $X_i^{(k)} + \Delta X_i$ is feasible over the other lines and this is the reason for running the curtailment problem afterwards. ΔX_i will be fixed as $\Delta X_i^{(k)} = \Delta X_i$ to be used in the curtailment model at stage k.

As mentioned before, in this paper we assumed that the cost and benefit functions offered by market participants in each iteration of the intraday market are exactly the same as day-ahead market cost and benefit functions, means that $\tilde{c}_i = c_i$. The rationale behind this assumption is that we assume that the position of market participants is not changing after day-ahead and during intraday market and the only aim of participating in the intraday market is just finding optimal solution which is feasible with respect to the physical transmission lines at the end of the intraday market session to avoid imbalances occur due to line capacity violations in balancing market.

However, by this assumption, the main aim of intraday market participation which is correcting day-ahead decisions due to the changes happen after day-ahead market closure has not accomplished yet. One way to fulfill this aim by the approach mentioned in this paper is that we let market participants to submit different cost and benefit functions to the power exchange problem (at each iteration) whenever they see the changes in their position. For instance, if an unplanned outage of a unit of generation happens in the middle of the intraday market operation, then the generator can cancel its day-ahead contract by submitting a new cost function such that the outage quantity can surely be bought back in the current or futures iterations of intraday market.

But what happens to the number of iterations if market participants are allowed to submit differebt offers/bids. Does the number of iterations increases by this new assumption? These are open questions; especially if we replace the full power exchange model with batch auctions, it is not clear that the approach is still converge to the nodal optimal solution with a limited number of iterations.

5 Results and discussion

5.1 Illustrative example

In this section, a small deterministic 6-bus system is used to clarify how our suggested CMT-based intraday market can be implemented. By deterministic example, it is easier to explore how the trading process is managed by CMT model. A deterministic 6-bus example is depicted in figure.3. This system consistes of 2 zones $z \in \{Z_1, Z_2\}$ (which can be interpreted as 2 countries), 6 nodes $n \in \{n_1, ..., n_6\}$, 3 conventional generators $g \in \{G_1, G_2, G_3\}$ placed respectively at nodes 1,2,5. 3 elastic loads $d \in \{D_1, D_2, D_3\}$ located at nodes 3,4,6 and finally 8 lines $l \in \{L_1, ..., L_8\}$. The capacity of the lines is also shown in the figure.3. The Susceptance of all lines and the resulted PTDF matrix is illustrated in table.1.

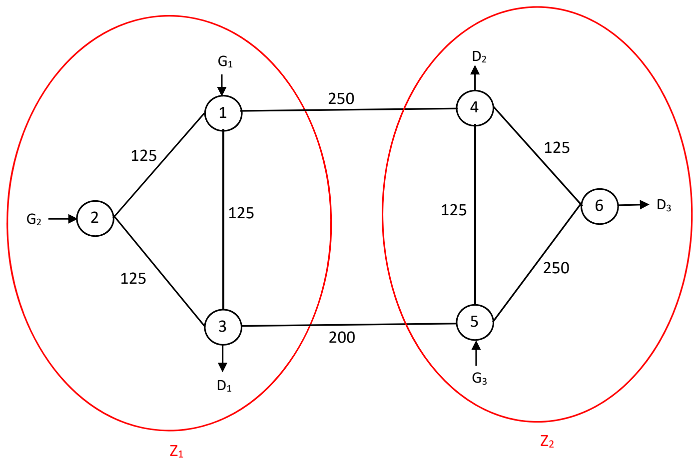

Figure 3: 6-bus example

Table 1: Lines characteristic parameters

Lines	Susceptance	PTDF					
		n_1	n_2	n_3	n_4	n_5	n_6
1-2	1	0.088	-0.530	-0.105	0.030	-0.020	0
1-3	1.5	0.279	-0.011	-0.332	0.094	-0.064	0
1-4	1.6	0.634	0.540	0.437	-0.124	0.084	0
2-3	0.9	0.088	0.470	-0.105	0.030	-0.020	0
3-5	1.1	0.366	0.460	0.563	0.124	-0.084	0
4-5	1.3	0.160	0.095	0.023	0.329	-0.223	0
4-6	0.95	0.474	0.446	0.414	0.547	0.307	0
5-6	1.4	0.526	0.554	0.586	0.453	0.693	0

The related market participants' data is listed below:

- G_1: Is a nuclear power plant with capacity of 450 MW, and a constant marginal cost of 12 €$/MWh$
- G_2: Is a gas power plant with 350 MW capacity and a constant marginal cost of 20 €$/MWh$
- G_3: Is a coal power plant with hard coal fuel. The capacity of this plant is 400 MW and its marginal cost of production equals 17 €$/MWh$

And three loads

- D_1: Is a load with medium willingness to pay of 23 $\text{€}/MWh$ and maximum consumption of 450 MW
- D_2: Is a load with low willingness to pay of 21 $\text{€}/MWh$ and consumption capacity of 400 MW
- D_3: Is a load with high willingness to pay of 30 $\text{€}/MWh$ and maximum consumption of 350 MW

Table 2: 6-bus data

Generators	**Cost**	**Capacity**	**Loads**	**Benefit**	**Capacity**
$\mathbf{G_1}$	12	450	$\mathbf{D_1}$	23	450
$\mathbf{G_2}$	20	350	$\mathbf{D_2}$	21	400
$\mathbf{G_3}$	17	400	$\mathbf{D_3}$	30	350

There is one interconnector e between zones z_1 and z_2. Based on ATC of this interconnector and different approaches mentioned in section 4.3, several cases will be discussed in the subsequent sections.

5.1.1 Case 1 : $ATC = \infty$ and approach.A

5.1.1.1 Day-ahead market with $ATC = \infty$

At first, we assume that there is not any limitation on the amount of power transferring between these two zones, meaning that ATC is set to a very big number. The interpretation is that the uniform pricing approach is used for the whole system with the following mathematical formulation:

$$\text{Minimize}_{x_D,x_G} \quad 12x_{G_1} + 20x_{G_2} + 17x_{G_3} - 23x_{D_1} - 21x_{D_2} - 30x_{D_3} \tag{31}$$

$$\text{subject to:} \quad x_{G_1} + x_{G_2} - x_{D_1} = f_{e_{z_1,z_2}} \tag{32}$$

$$x_{G_3} - x_{D_2} - x_{D_3} = -f_{e_{z_1,z_2}} \tag{33}$$

$$0 \leqslant x_{G_1} \leqslant 450, 0 \leqslant x_{G_2} \leqslant 350, 0 \leqslant x_{G_3} \leqslant 400 \tag{34}$$

$$0 \leqslant x_{D_1} \leqslant 450, 0 \leqslant x_{D_2} \leqslant 400, 0 \leqslant x_{D_3} \leqslant 350 \tag{35}$$

$$f_{e_{z_1,z_2}} \leqslant \infty \tag{36}$$

This day-ahead model is built upon a simplified transmission network that just consider an imaginary interconnector e_{z_1,z_2} between these two zones with unlimited capacity. It is assumed that all generators and all loads can be scheduled at day-ahead stage. Therefore, optimization variables in problem (31)-(36) are the day-ahead production $x_{G_1}, ..., x_{G_3}$ and consumption $x_{D_1}, ..., x_{D_3}$. The objective of the day-ahead market is to minimize the total cost of the system (benefit of loads are considered as negative costs) or maximizing the social welfare. Equations (32) and (33) shows the zonal power balance constraints at each zone z_1 and z_2 and constraints (34) and (35) enforce capacity limits on generation and load. Finally, the constraint (36) imposes interconnectors limitation based on ATC.

By solving this problem, the day-ahead market is settled by the generation quantities $\{x^*_{G_1}, x^*_{G_2}, x^*_{G_3}\} = \{450, 350, 400\}$, the consumption quantities $\{x^*_{D_1}, x^*_{D_2}, x^*_{D_3}\} = \{450, 400, 350\}$ and flow over interconnector e_{z_1,z_2} equals to 350. The social surplus (objective value) resulted from the day-ahead market clearing is 10050.

As shown in figure.4, by calculating the power flowing through the physical transmission lines (which are neglected at the day-ahead market) by TSO, 5 lines $\{L_{1-3}, L_{1-4}, L_{2-3}, L_{4-5}, L_{5-6}\}$ overloaded. This means that the day-ahead result is not a feasible solution with respect to the physical network constraints (black numbers on the lines show the capacity while red numbers show the flow resulted from the day-ahead schedule and green circles around them represent the overloaded lines).

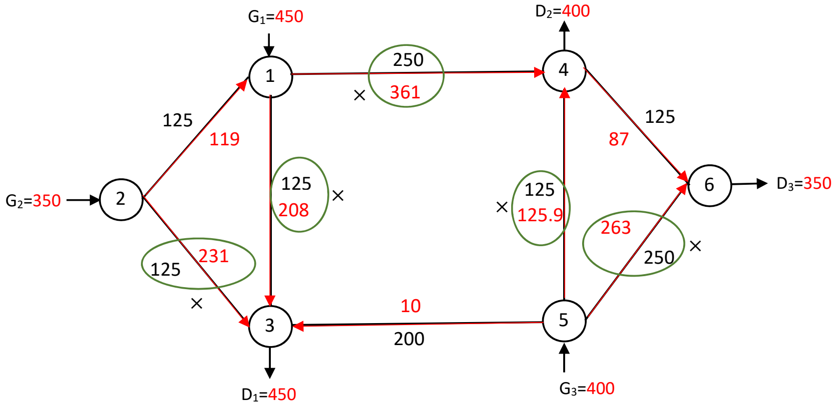

Figure 4: Day-ahead clearing result with $ATC = \infty$, Social surplus=10050

Hence, intraday market will be utilized afterwards to help relieving congestion (overload) resulted from the day-ahead market solution.

5.1.1.2 Intraday market - Approach.A

In approach.A, before begining the intraday market, by the succeeding optimization problem, the TSO has to curtail the day-ahead result if it is not a feasible solution for the transmission network:

Iteration 0 - Curtailment by TSO

$$\text{Maximize}_{\gamma,f} \qquad \gamma \tag{37}$$

$$\begin{aligned}\text{subject to:}\quad & (X_{G_1}^{(0)} + \gamma\Delta X_{G_1}^{(0)}) + (X_{G_2}^{(0)} + \gamma\Delta X_{G_2}^{(0)}) + (X_{G_3}^{(0)} + \gamma\Delta X_{G_3}^{(0)}) \\ & - (X_{D_1}^{(0)} + \gamma\Delta X_{D_1}^{(0)}) - (X_{D_2}^{(0)} + \gamma\Delta X_{D_2}^{(0)}) - (X_{D_3}^{(0)} + \gamma\Delta X_{D_3}^{(0)}) = 0\end{aligned} \tag{38}$$

$$\begin{bmatrix} f_{1-2} \\ f_{1-3} \\ f_{1-4} \\ f_{2-3} \\ f_{3-5} \\ f_{4-5} \\ f_{4-6} \\ f_{5-6} \end{bmatrix} = \begin{bmatrix} 0.088 & -0.530 & -0.105 & 0.030 & -0.020 & 0 \\ 0.279 & -0.011 & -0.332 & 0.094 & -0.064 & 0 \\ 0.634 & 0.540 & 0.437 & -0.124 & 0.084 & 0 \\ 0.088 & 0.470 & -0.105 & 0.030 & -0.020 & 0 \\ 0.366 & 0.460 & 0.563 & 0.124 & -0.084 & 0 \\ 0.160 & 0.095 & 0.023 & 0.329 & -0.223 & 0 \\ 0.474 & 0.446 & 0.414 & 0.547 & 0.307 & 0 \\ 0.526 & 0.554 & 0.586 & 0.453 & 0.693 & 0 \end{bmatrix} \begin{bmatrix} X_{G_1}^{(0)} + \gamma\Delta X_{G_1}^{(0)} \\ X_{G_2}^{(0)} + \gamma\Delta X_{G_2}^{(0)} \\ X_{G_3}^{(0)} + \gamma\Delta X_{G_3}^{(0)} \\ -X_{D_1}^{(0)} - \gamma\Delta X_{D_1}^{(0)} \\ -X_{D_2}^{(0)} - \gamma\Delta X_{D_2}^{(0)} \\ -X_{D_3}^{(0)} - \gamma\Delta X_{D_3}^{(0)} \end{bmatrix} \tag{39}$$

$$\begin{bmatrix} -125 \\ -125 \\ -250 \\ -125 \\ -200 \\ -125 \\ -125 \\ -250 \end{bmatrix} \leqslant \begin{bmatrix} f_{1-2} \\ f_{1-3} \\ f_{1-4} \\ f_{2-3} \\ f_{3-5} \\ f_{4-5} \\ f_{4-6} \\ f_{5-6} \end{bmatrix} \leqslant \begin{bmatrix} 125 \\ 125 \\ 250 \\ 125 \\ 200 \\ 125 \\ 125 \\ 250 \end{bmatrix} \tag{40}$$

All $X^{(0)}$ are parameters equal to zero and $\Delta X_i^{(0)} = x_i^*$. By solving this problem, TSO announces that $\gamma = 0.54$ of the day-ahead schedule must be curtailed. The curtailed quantities and updated flows are displayed in figure.5 by red color. TSO also announces the PTDF of line L_{2-3} which is congested as a public information to all market participants and power exchange. The current updated solution is $X^{(1)} = 0.54X^{(0)}$.

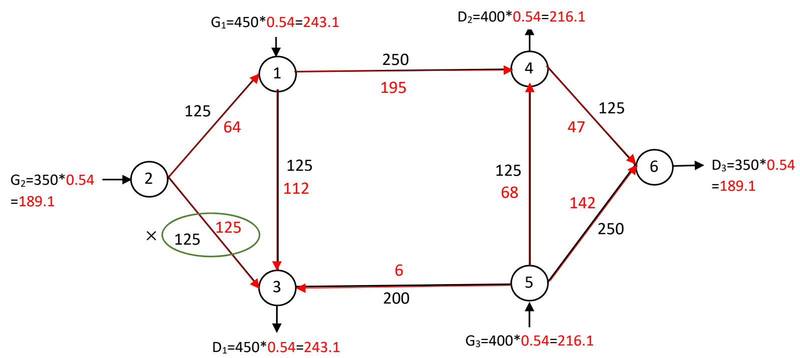

Figure 5: Curtailment of the day-ahead result, $\gamma = 0.54$

Iteration 1 - Finding profitable trade by power exchange

At the outset of the intraday market, by revealing new information about curtailed day-ahead schedule and congested lines, power exchange tries to find a new profitable trade in the feasible direction such that flow over line L_{2-3} not worsen.

Therefore, power exchange solves the following optimization problem to find a new trade ΔX:

$$
\begin{aligned}
\underset{\Delta X_D, \Delta X_G}{\text{Minimize}} \quad & 12X_{G_1}^{(1)} + 20X_{G_2}^{(1)} + 17X_{G_3}^{(1)} - 23X_{D_1}^{(1)} - 21X_{D_2}^{(1)} - 30X_{D_3}^{(1)} \\
& - (12(X_{G_1}^{(1)} + \Delta X_{G_1}) + 20(X_{G_2}^{(1)} + \Delta X_{G_1}) + 17(X_{G_3}^{(1)} + \Delta X_{G_3}) \\
& - 23(X_{D_1}^{(1)} + \Delta X_{D_1}) - 21(X_{D_2}^{(1)} + \Delta X_{D_2}) - 30(X_{D_3}^{(1)} + \Delta X_{D_3})) && (41) \\
\text{subject to:} \quad & \Delta X_{G_1} + \Delta X_{G_2} + \Delta X_{G_3} - \Delta X_{D_1} - \Delta X_{D_2} - \Delta X_{D_3} = 0 && (42) \\
& 0 \leqslant X_{G_1}^{(1)} + \Delta X_{G_1} \leqslant 450,\ 0 \leqslant X_{G_2}^{(1)} + \Delta X_{G_2} \leqslant 350,\ 0 \leqslant X_{G_3}^{(1)} + \Delta X_{G_3} \leqslant 400 && (43) \\
& 0 \leqslant X_{D_1}^{(1)} + \Delta X_{D_1} \leqslant 450,\ 0 \leqslant X_{D_2}^{(1)} + \Delta X_{D_2} \leqslant 400,\ 0 \leqslant X_{D_3}^{(1)} + \Delta X_{D_3} \leqslant 350 && (44) \\
& 0.088\Delta X_{G_1} + 0.47\Delta X_{G_2} - 0.02\Delta X_{G_3} - (-0.105\Delta X_{D_1}) - 0.03\Delta X_{D_2} - 0\Delta X_{D_3} \leqslant 0 && (45)
\end{aligned}
$$

Inequality (45) shows the feasible direction trade constraint for line L_{2-3}. The new profitable trade and its resulted flows are depicted in figure.6.

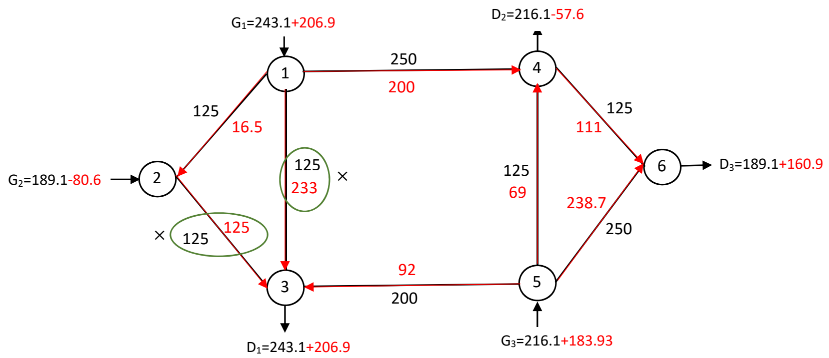

Figure 6: First intraday auction to find profitable trade - social welfare=4380

By adding the feasible direction trade constraint on line L_{2-3}, overflow of this line was hindered but there is not any limitation on other lines. Hence, line L_{1-3} is also overloaded. We still have not reached to a feasible solution. So, this new profitable trade has to be curtailed by mathematical formulation similar to (37)-(40).

Iteration 1 - Curtailment by TSO

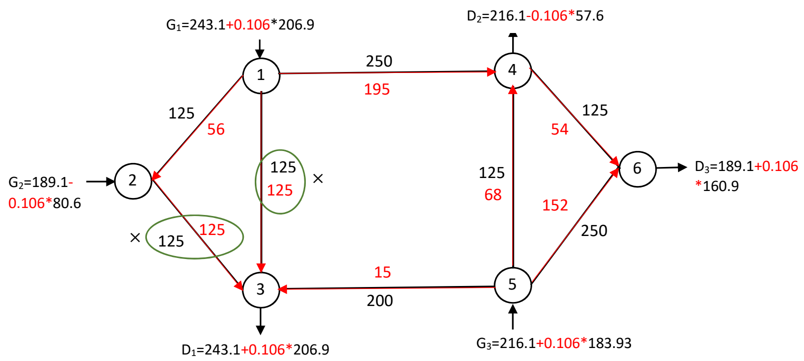

Figure 7: Curtailment of first intraday auction - $\gamma = 0.106$

Iteration 2 - Finding profitable trade by power exchange

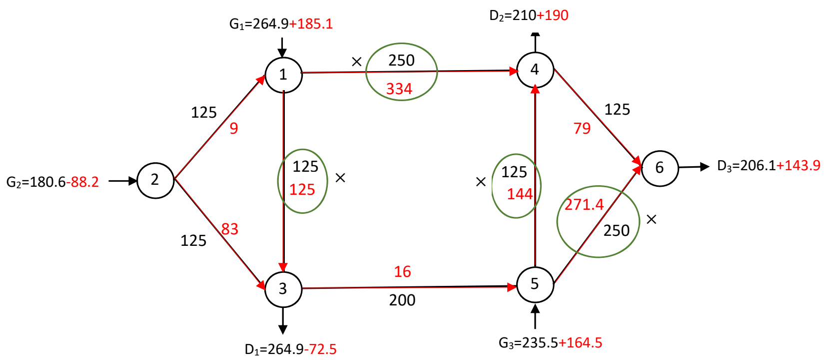

Figure 8: Second intraday auction to find profitable trade - social welfare=3386

Iteration 2 - Curtailment by TSO

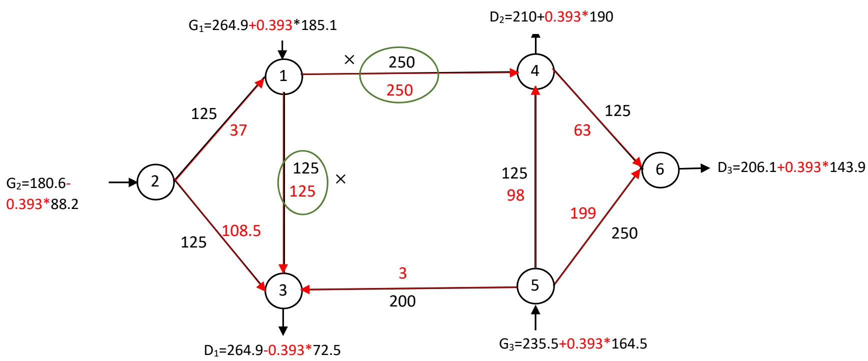

Figure 9: Curtailment of second intraday auction - $\gamma = 0.393$

Iteration 3 - Finding profitable trade by power exchange

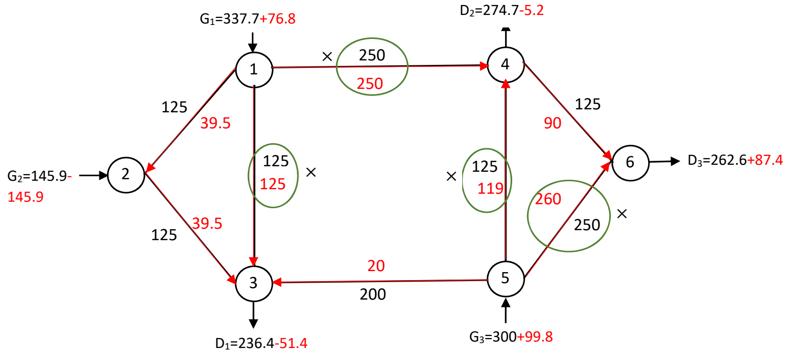

Figure 10: Third intraday auction to find profitable trade - social welfare=1628

Iteration 3 - Curtailment by TSO

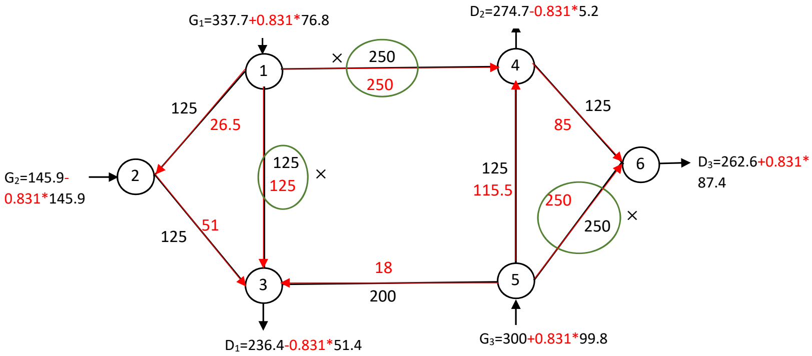

Figure 11: Curtailment of third intraday auction - $\gamma = 0.831$

Iteration 4 - Finding profitable trade by power exchange

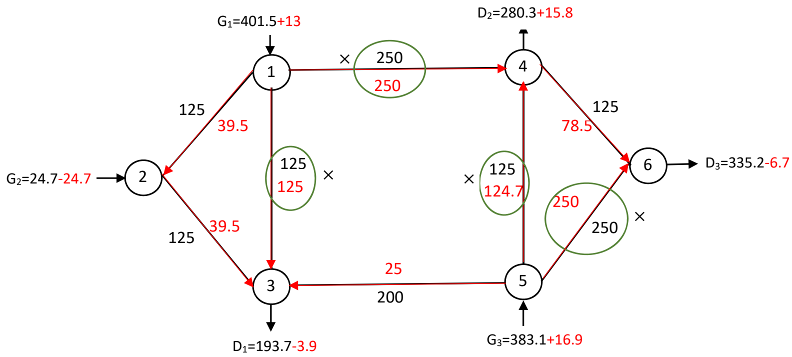

Figure 12: Fourth intraday auction to find profitable trade - social welfare=92

Iteration 4 - Curtailment by TSO

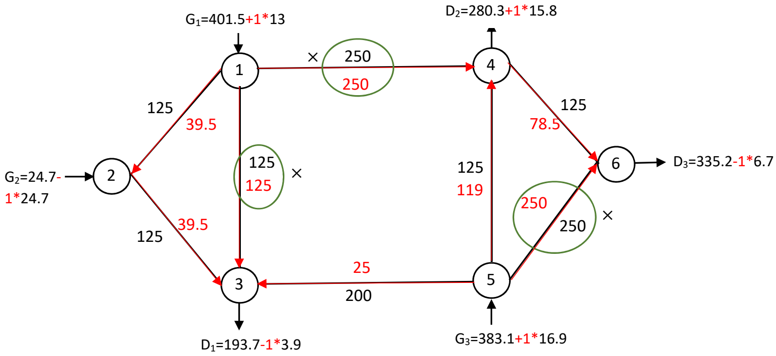

Figure 13: Curtailment of Fourth intraday auction - $\gamma = 1$

Iteration 5 - Finding profitable trade by power exchange

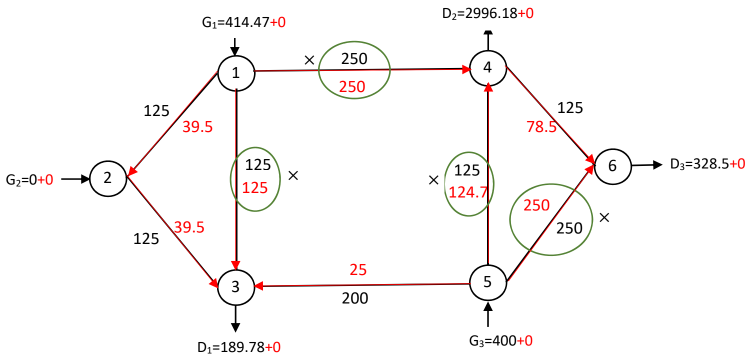

Figure 14: Fifth intraday auction to find profitable trade - social welfare=0

In the last auction of the intraday market no further profitable trade can be found by the power exchange. So, figure.14 shows that after 5 iterations the final solution which is both profitable and feasible is attained.

5.1.2 Case 2 : $ATC = 0$ and approach.A

5.1.2.1 Day-ahead market with $ATC = 0$

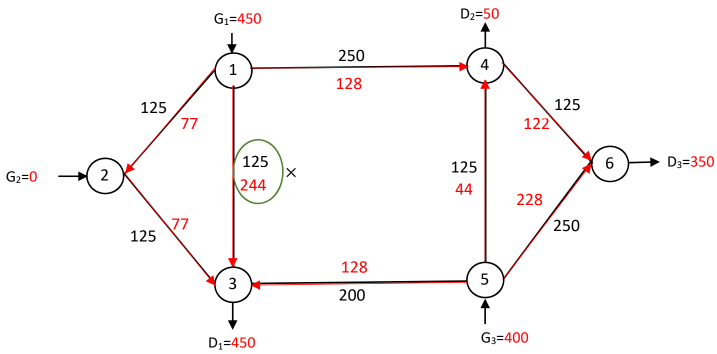

Figure 15: Day-ahead clearing result with $ATC = 0$, Social surplus=9700

5.1.2.2 Intraday market - Approach.A

In approach.A, before begining the intraday market, the TSO has to curtail the day-ahead result if it is not a feasible solution for the transmission network:

Iteration 0 - Curtailment by TSO

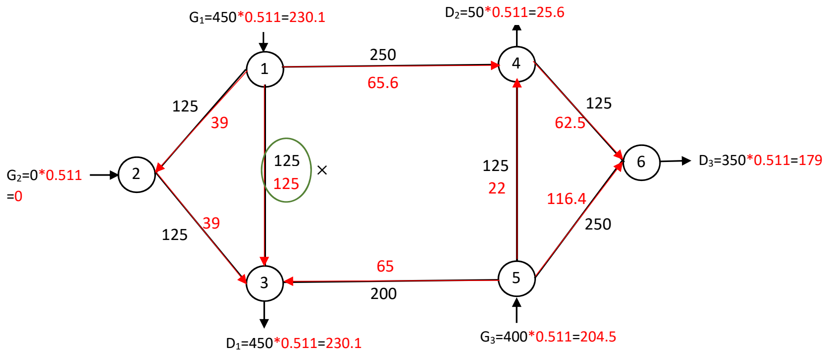

Figure 16: Curtailment of the day-ahead result, $\gamma = 0.511$

Iteration 1 - Finding profitable trade by power exchange

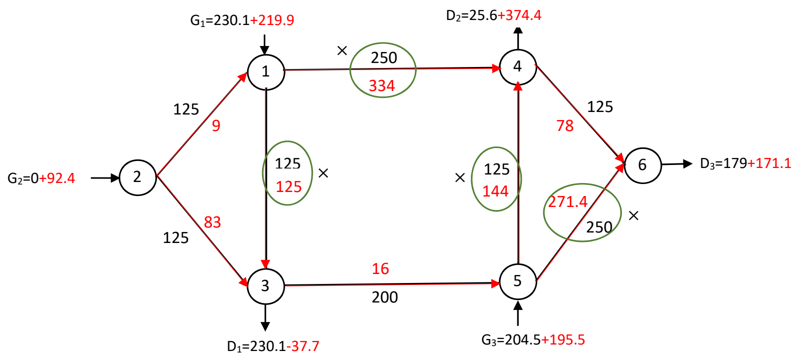

Figure 17: First intraday auction to find profitable trade - social welfare=4318

Iteration 1 - Curtailment by TSO

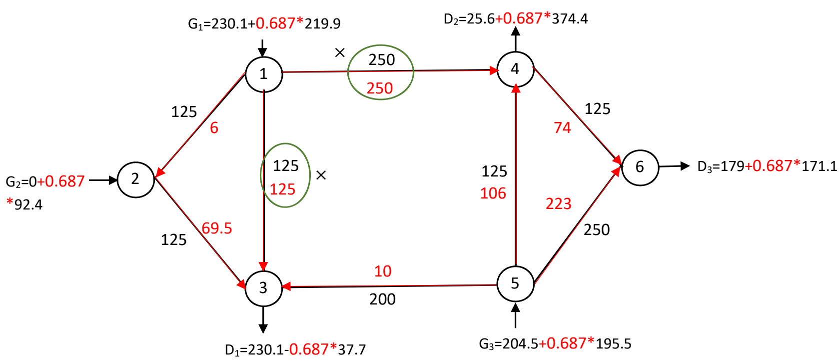

Figure 18: Curtailment of first intraday auction - $\gamma = 0.687$

Iteration 2 - Finding profitable trade by power exchange

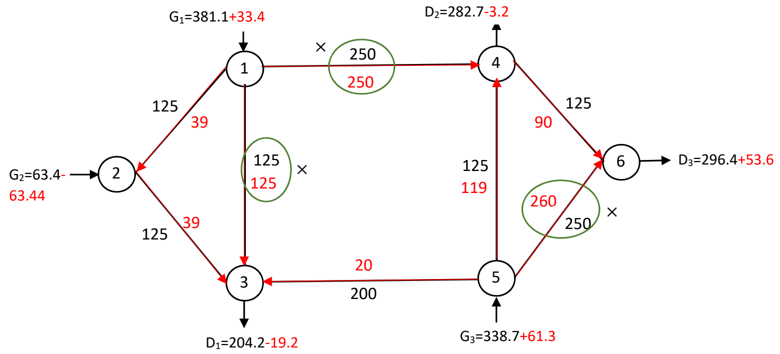

Figure 19: Second intraday auction to find profitable trade - social welfare=926

Iteration 2 - Curtailment by TSO

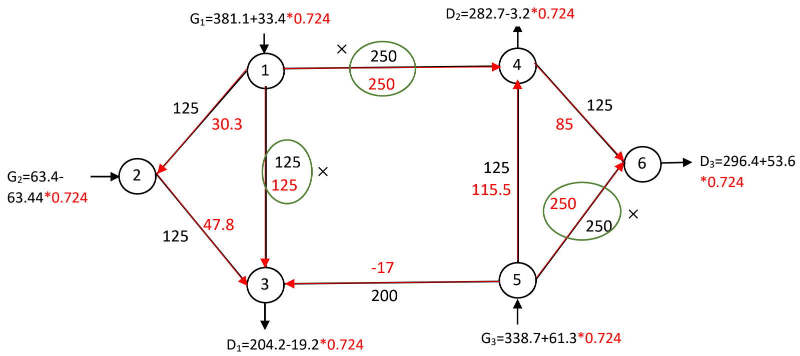

Figure 20: Curtailment of second intraday auction - $\gamma = 0.724$

Iteration 3 - Finding profitable trade by power exchange

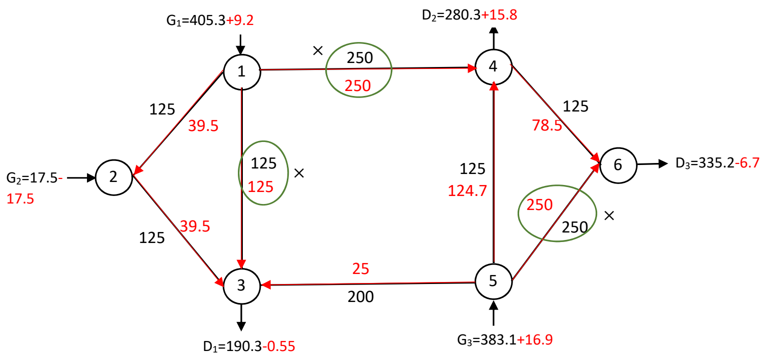

Figure 21: Third intraday auction to find profitable trade - social welfare=72

Iteration 3 - Curtailment by TSO

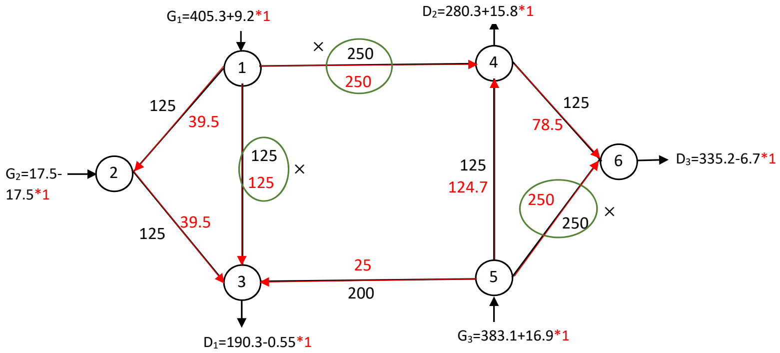

Figure 22: Curtailment of third intraday auction - $\gamma = 1$

Iteration 4 - Finding profitable trade by power exchange

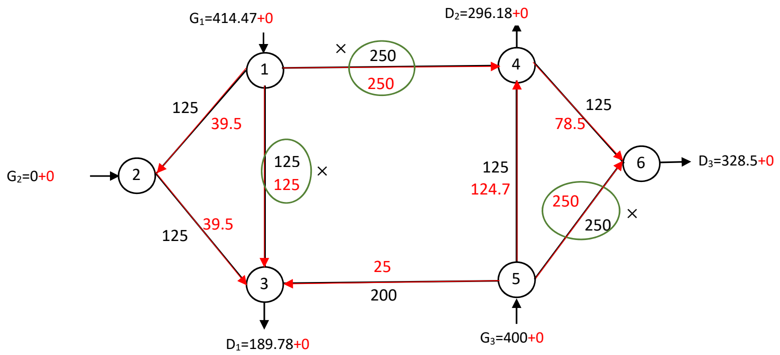

Figure 23: Fourth intraday auction to find profitable trade - social welfare=0

After 4 iterations both profitable and feasible solution is attained.

5.1.3 Benchmark case : Optimal nodal model at day-ahead market

In order to understand how much efficient is our suggested model, the results can be compared with the benchmark case of optimal nodal model firstly suggested by Schweppe et al. (1988). Optimal nodal solution result is depicted in figure.24. With respect to the assumptions we mentioned in section 3, very interesting result is attained; "Irrespective of what ATC values were adopted in the day-ahead market, at the end of the intraday market, optimal nodal solution is achieved". So, one question could be whether do we still need the day-ahead market specially with the structure mentioned in section 4.3, because it seems that intraday market can be initiated with any starting point and still remains efficient.

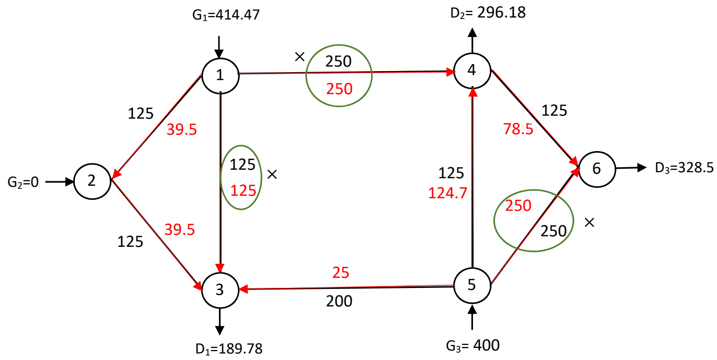

Figure 24: Optimal nodal result - social surplus=8666.5

5.1.4 Case 3 : $ATC = \infty$ and approach.B

5.1.4.1 Day-ahead market with $ATC = \infty$

The solution is exactly similar to section 5.1.1.1 and figure.4. The PTDF of line L_{1-3} is announced by the TSO and in the next iteration without curtailment, power exchange tries to find a more profitable solution such that the flow through line L_{1-3} not get worsen.

5.1.4.2 Intraday market - Approach.B

Iteration 1 - Finding profitable trade by power exchange

No further profitable trade is found. So by this approach it is not possible to improve the flows on overflowed lines.

5.1.5 Case 4 : $ATC = 0$ and approach.B

5.1.5.1 Day-ahead market with $ATC = 0$

The solution is exactly similar to section 5.1.2 and figure.15.

5.1.5.2 Intraday market - Approach.B

Iteration 1 - Finding profitable trade by power exchange

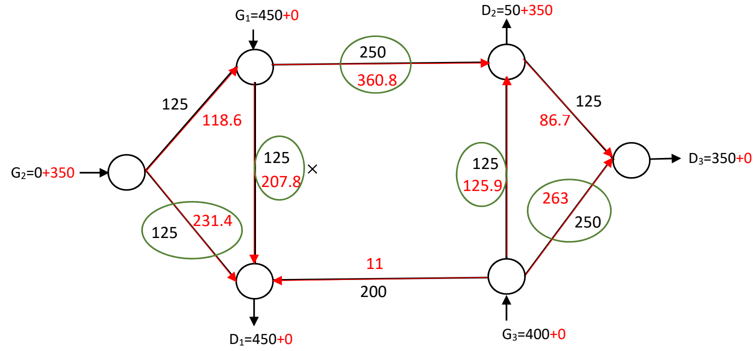

Figure 25: First intraday auction to find profitable trade - social welfare=350

Iteration 1 - Curtailment by TSO

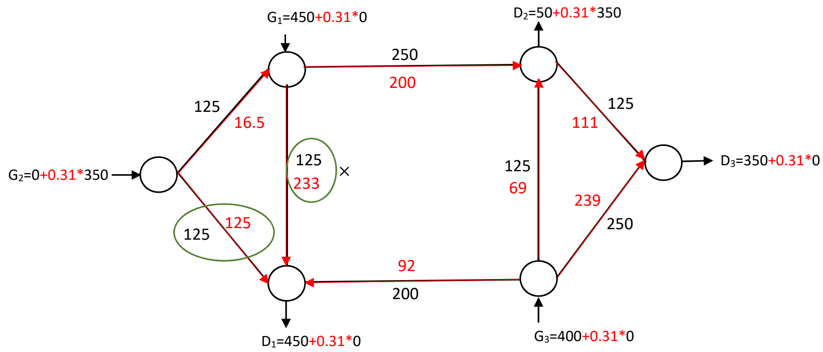

Figure 26: Curtailment of first intraday auction - $\gamma = 0.31$

Iteration 2 - Finding profitable trade by power exchange

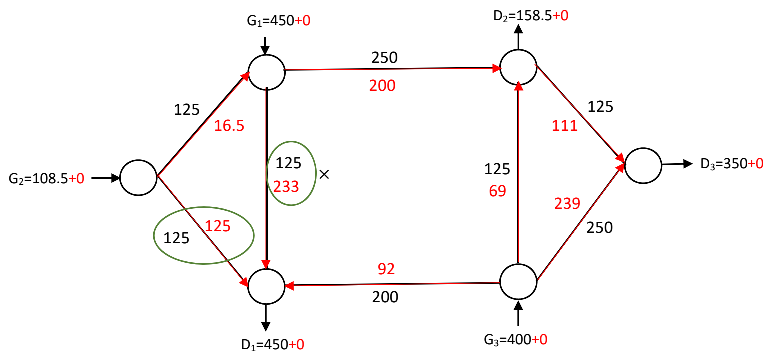

Figure 27: Second intraday auction to find profitable trade - social welfare=0

No further profitable trade could be found. Hence, this solution is the most profitable and feasible solution through all physical transmission lines except for line L_{1-3} which overloaded at day-ahead market. The advantage of approach.B is that even if the network is still infeasible for this line but by CMT approach the final flow through this line (233) is not worsen than flow resulted from the day-ahead solution which is 244.

5.1.6 Case 5 : Benchmark case : optimal nodal model at day-ahead market with relaxed capacities of overloaded lines in section 5.1.2.1

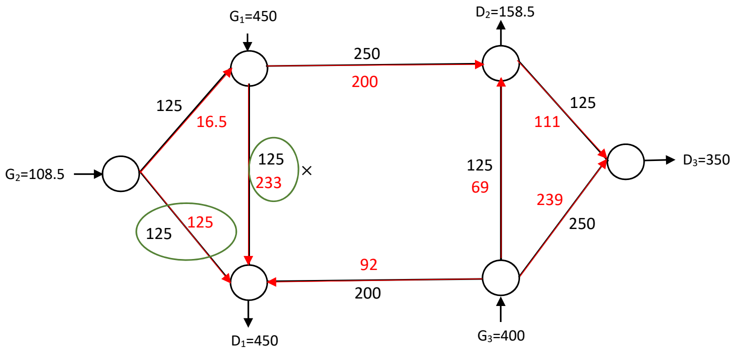

Figure 28: Optimal nodal result with relaxed capacity for line L_{1-3}

6 Conclusion

In designing an efficient electricity market, modeling transmission network capacity constraints has been always debated be market designers because of the complexities created by the loop flow characteristic of the electricity. That is why different market designs have been selected for managing network constraints.

The advantage of the CMT model over nodal pricing is that like optimal nodal model supply and demand are considered at the nodal level while contrary to that transmission capacity constraints are not directly modeled in the market model (power exchange problem). Hence, this method incorporates all the advantages of the nodal pricing without mixing up the power exchange and TSO roles. Moreover, since the nature of the information used for coordination between power exchanges and TSos is not complicated, it may facilitate coordination in areas with seperate power exchanges like European markets.

With respect to the recent decisions on the European intraday markets, our suggestion is that the coordinated multilateral trade approach can be an efficient procedure for managing transmission constraints in the integrated European intraday markets. The European power exchanges run their day-ahead model as before based on the Euphemia algorithm which needs the zonal level data. While the main important requirement for our customized CMT approach is access to the nodal level data. This means that if in the intraday market power exchanges have access to the nodal level supply and demand functions, then the coordination between power exchanges and TSo can be done very easily just by transfering technical information.

In this paper, we assume that supply and demand functions are not changing from day-ahead to each intraday market stage and the only purpose of intraday market participation is to reduce high imbalance costs due to the network simplification at day-ahead stage. Even by this simplifying assumption further research is required to understand whether the number of iterations (which are equivalent to the number of intraday auctions) are tractable for massive networks.Furthermore, in order to capture the other main aim of intraday market which is fascilating large-scale integration of intermittent generation, it would be interesting to slightly perturbe data in each step by varying supply and demand functions (different asks/bids at each stage). Thus, further investigation is necessary to discover the impact of varying orders at each stage on the number of iterations.

Finally, the last influential item on the number of iterations is replacing our assumed full auction (at each stage) with batch auctions. As in each batch auction a subset of participants (which in large networks and frequent auction, it can be very small subset) are taking part, more iterations are expected.

In all these aforementioned cases, we are eager to know the possibility of reaching to the optimal nodal solution by a tractable number of iterations.

At the end, in the real case of more TSOs where each of them just have access to their own network more studies are necessary to understand how curtailment model must be decomposed into each TSO curtailment problem and which kinds of mechanisms are needed to fascilitate the cooperation among them.

References

Bjorndal, E., Bjorndal, M. H., Midthun, K. T., and Zakeri, G. (2016). Congestion management in a stochastic dispatch model for electricity markets. Technical report, NHH Dept. of Business and Management Science Discussion Paper No. 2016/12. Available at SSRN: https://ssrn.com/abstract=2829365.

Bjørndal, M., Jörnsten, K., and Rud, L. (2010). Capacity charges: A price adjustment process for managing congestion in electricity transmission networks. In *Energy, Natural Resources and Environmental Economics*, pages 267–292. Springer.

Brunekreeft, G., Neuhoff, K., and Newbery, D. (2005). Electricity transmission: An overview of the current debate. *Utilities Policy*, 13(2):73–93.

Budish, E., Cramton, P., and Shim, J. (2014). Implementation details for frequent batch auctions: Slowing down markets to the blink of an eye. *American Economic Review*, 104(5):418–24.

Budish, E., Cramton, P., and Shim, J. (2015). The high-frequency trading arms race: Frequent batch auctions as a market design response. *The Quarterly Journal of Economics*, 130(4):1547–1621.

Chao, H.-P. and Peck, S. (1996). A market mechanism for electric power transmission. *Journal of regulatory economics*, 10(1):25–59.

Hagemann, S. (2013). Price determinants in the german intraday market for electricity: an empirical analysis. Technical report, EWL working paper.

Henriot, A. (2012). Market design with wind: managing low-predictability in intraday markets. *EUI working papers RSCAS 2012/63*.

Jafari, A. M., Zareipour, H., Schellenberg, A., and Amjady, N. (2014). The value of intra-day markets in power systems with high wind power penetration. *IEEE Transactions on Power Systems*, 29(3):1121–1132.

Morales, J. M., Zugno, M., Pineda, S., and Pinson, P. (2014). Electricity market clearing with improved scheduling of stochastic production. *European Journal of Operational Research*, 235(3):765–774.

Neuhoff, K., Ritter, N., Salah-Abou-El-Enien, A., and Vassilopoulos, P. (2016). Intraday markets for power: Discretizing the continuous trading? Technical report, E.

Nordpool (2016). Nord pool intraday user guide.

Qin, J., Rajagopal, R., and Varaiya, P. P. (2017). Flexible market for smart grid: coordinated trading of contingent contracts. *IEEE Transactions on Control of Network Systems*.

Rahimi, S., Bjorndal, M., Bjorndal, E., and Andersson, J. (2018). Congestion management in continuous trading intraday market. *CMS 2018 - Conference on Computational Management Science*.

Scharff, R. and Amelin, M. (2016). Trading behaviour on the continuous intraday market elbas. *Energy Policy*, 88:544–557.

Schweppe, F. C., Caramanis, M. C., Tabors, R. D., and Bohn, R. E. (1988). *Spot pricing of electricity*. Springer Science & Business Media.

Weber, A. and Schröder, S. (2011). Efficiency of continuous double auctions in the electricity market. In *Energy Market (EEM), 2011 8th International Conference on the European*, pages 87–92. IEEE.

Weber, C. (2010). Adequate intraday market design to enable the integration of wind energy into the european power systems. *Energy Policy*, 38(7):3155–3163.

Wu, F. F. and Varaiya, P. (1999). Coordinated multilateral trades for electric power networks: theory and implementation1. *International Journal of Electrical Power & Energy Systems*, 21(2):75–102.