Partial Benders Decomposition Strategies: An Application to Two-Stage Stochastic Network Design

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In the context of stochastic programming, the Benders algorithm [1] is an essential methodology that is readily applied to the problems addressed in the field, see [2]. Stochastic programming deals with optimization problems where a subset of parameters involve a level of uncertainty (i.e., stochastic parameters). In a stochastic program, decisions are defined in stages according to when the stochastic parameters become known. In a two-stage model, one distinguishes the decisions that need to be made before any information is known (i.e., the first stage or *a priori* decisions) from the decisions that are taken once the values of the stochastic parameters are observed (i.e., the second stage or *recourse* decisions). The pursued objective is then to find an a priori solution which minimizes its associated cost plus the expected recourse cost.

In this presentation, we focus on two-stage stochastic network design problems, where in the first stage a network is designed and then used in the second stage to flow products between origin and destination nodes. The Benders strategy, coined the L-Shaped algorithm when applied to stochastic programs [3], enables such problems to be decomposed according to the realizations of the random event that set the values of the stochastic parameters included in the model (e.g., the demand volumes between origin and destination nodes). A finite set of representative scenarios is usually used to approximate the possible outcomes for the values of the stochastic parameters. Using such a set, the stochastic program can be formulated in an extensive form by duplicating the second stage decisions for each scenario, see [2]. The large-scale nature of such models is due, in a large proportion, to the number of scenarios used to properly formulate the uncertainty present. Therefore, Benders decomposition greatly simplifies the solution of these problems. However, this strategy also comes with important drawbacks that need to be addressed to produce an overall efficient solution procedure.

Once applied to the considered model, the decomposition strategy produces a master problem (defined on the network design decisions) and scenario subproblems (defined on the flow decisions associated to each scenario). The master problem represents a relaxation of the original two-stage model. The L-Shaped algorithm then proceeds by iteratively solving the master and scenario subproblems to respectively guide the search process and generate violated feasibility and optimality cuts. Feasibility cuts induce the designed networks to be feasible in the second stage, while optimality cuts define the values of the expected recourse cost (i.e., the expected flow costs) associated to feasible networks. The main drawback when applying the L-Shaped algorithm is that the initial relaxation produces a considerably weaker formulation for the obtained master problem. The feasibility and optimality cuts provide the formulation of the second stage of the stochastic problem. Once they are relaxed, the master problem looses all relevant information concerning the recourse decisions. In turn, this leads to various computational problems such as instability issues with respect to the cuts that are generated; an erratic progression of the bounds generated by the L-Shaped algorithm; and an overall slow convergence of the solution process.

In order to overcome these challenges, we define a novel strategy that is based on the idea of retaining a subset of scenario subproblems in the master formulation. We refer to this approach as partial Benders decomposition. By doing so, the formulation of the master problem is automatically improved given that a smaller part of the original problem is relaxed at the beginning of the solution process. We develop two general strategies (representation and covering) to implement partial decomposition that aim to reduce the number of cuts added by the L-Shaped algorithm in order to converge. These strategies are based on the general principle of identifying scenario subproblems to retain that present desirable characteristics. Extensive numerical experiments were conducted on stochastic network design instances available from the literature. The analysis of the results obtained clearly show that partial decomposition outperforms the classical approach and that specific strategies are more efficient depending on the characteristics of the problems.

References

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