Nonlinear Tax Incidence and Optimal Taxation in General Equilibrium*

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Abstract

We study the incidence and the optimal design of nonlinear income taxes in a Mirrleesian economy with a continuum of endogenous wages. We characterize analytically the incidence of any tax reform by showing that one can mathematically formalize this problem as an integral equation. For a CES production function, we show theoretically and numerically that the general equilibrium forces raise the revenue gains from increasing the progressivity of the U.S. tax schedule. This result is reinforced in the case of a Translog technology where closer skill types are stronger substitutes. We then characterize the optimum tax schedule, and derive a simple closed-form expression for the top tax rate. The U-shape of optimal marginal tax rates is more pronounced than in partial equilibrium. The joint analysis of tax incidence and optimal taxation reveals that the economic insights obtained for the optimum may be reversed when considering reforms of a suboptimal tax code.

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Introduction

In this paper we study the incidence and the optimal design of nonlinear income taxes in a general equilibrium Mirrlees (1971) economy. There is an aggregate production function with a continuum of labor inputs and imperfect substitutability between skills. Hence the wage, or marginal product, of each skill type is endogenous.

We connect two classical strands of the public finance literature that have so far been somewhat disconnected: the tax incidence literature (Harberger, 1962; Kotlikoff and Summers, 1987; Fullerton and Metcalf, 2002), and the literature on optimal nonlinear income taxation (Mirrlees, 1971; Stiglitz, 1982; Diamond, 1998; Saez, 2001). The objective of the tax incidence analysis (in our context with one sector and a continuum of labor inputs) is to characterize the first-order effects of locally reforming any given, potentially suboptimal, tax system on the distribution of individual wages, labor supplies, and utilities, as well as on government revenue and social welfare. A characterization of optimal taxes is then obtained by imposing that no such tax reform has a positive impact on social welfare. We show in particular that the very forces that drive the shape of the optimum tax schedule may lead to reverse policy recommendations when the aim is to a reform suboptimal tax schedule, e.g., the current US tax code.

We start by focusing on the incidence of general tax reforms. We extend the existing literature by considering arbitrarily nonlinear taxes and a continuum of endogenous wages. In partial equilibrium, the effects of a tax change on the labor supply of a given agent can be straightforwardly derived as a function of the elasticity of labor supply of that agent (Saez, 2001). The key difficulty in general equilibrium is that this direct partial equilibrium effect impacts the wage, and thus the labor supply, of every other individual. This in turn feeds back into the wage distribution, which further affects labor supply decisions, and so on. Solving for the fixed point in the labor supply adjustment of each agent is the key step in the analysis of tax incidence and the primary technical challenge of our paper.

We first show that this a priori complex problem of deriving the effects of tax reforms on individual labor supply can be mathematically formalized as solving an integral equation. The mathematical tools of the theory of integral equations allow us to derive an analytical solution to this problem for a general production function, which furthermore has a clear economic meaning. Specifically, this solution can be represented a series; its first term is the partial equilibrium impact of the reform, and each of its subsequent terms captures a successive round of cross-wage feedback effects in general equilibrium.

We proceed to the analysis of the two most commonly used production functions in the literature, CES and Translog.[1] In the CES case, we show that the solution to the integral equation takes a particularly simple closed form as a function of the labor supply elasticities, the income distribution, and the elasticity of substitution between skills in production, which is new to general equilibrium setting. In the Translog case, which allows for a general pattern of elasticities of substitution, we specify a functional form that formalizes the idea that workers with closer productivities are more substitutable. For this "distance-dependent" specification, we analytically derive an arbitrarily close approximation of the solution to the integral equation, and hence of the incidence of tax reforms. In particular, the second order approximation implies that the general equilibrium effects on agents' labor supplies can be summarized by a simple affine function of the cross-wage elasticity with a single "average" type.

We then focus on a particularly interesting baseline tax system, the "constant rate of progressivity" tax schedule, that closely approximates the current U.S. tax code (Heathcote, Storesletten, and Violante, 2014). We show two theoretical results for a CES technology. First, the general equilibrium effects of any local (linear or nonlinear) tax reform on aggregate revenue are equal to zero if the baseline tax schedule is linear. Second, and more generally, if the baseline tax system is strictly progressive (as in the U.S.), our incidence formula shows that the revenue gains of raising the marginal tax rates on low incomes are lower than in partial equilibrium, and higher for high incomes. In other words, starting from the U.S. tax code, the general equilibrium forces raise the revenue gains from increasing the progressivity of the tax schedule.

At first sight this result may seem to be at odds with the familiar insight of Stiglitz (1982) in the two-income model, which we generalize to a continuum of incomes in our section on optimal taxation. Indeed, those results say that the optimal tax rates should be lower at the top, and higher at the bottom, of the income distribution, relative to the partial equilibrium benchmark. That is, the optimal tax schedule should be more regressive when the general equilibrium forces are taken into account. The reason for this difference is that we consider here reforms of the current (potentially suboptimal) U.S. tax code, with low marginal tax rates at the bottom. Instead, the results about the optimum tax schedule use as a benchmark the optimal partial equi-

[1] See Bucci and Ushchev (2014) for a careful study of various production functions with a continuum of inputs and constant or variable elasticities of substitution.

librium tax schedule, which features high marginal tax rates at the bottom (Diamond, 1998; Saez, 2001). Our insight is therefore that results about the optimum tax schedule may actually be reversed when considering reforms of a suboptimal tax schedule. This "trickle up result" for government revenue underlines the importance of our tax reform approach, and leads us to conclude that one should be cautious, in practice, when applying the results of the general equilibrium optimal tax theory.

Our second set of results concerns the derivation of optimal taxes in the general equilibrium setting. For a general production function, we provide two complementary proofs of the optimal tax formula. The first uses a mechanism design approach as in Mirrlees (1971) (or Stiglitz (1982) in general equilibrium). The second approach is based on variational arguments in the spirit of Saez (2001), and is an application of our tax incidence analysis discussed above. The key step that allows us to reduce the complexity of the optimal taxation problem consists of equating to zero the social welfare effects of a specific combination of tax reforms, namely, a perturbation of the marginal tax rate at a given income level with a counteracting reform that cancels out the induced general equilibrium effects on labor supply. While this analysis of course delivers the same formula as the mechanism design approach, it provides distinct economic insights about the key economic forces underlying the design of optimal taxes.

We then focus on the case of the CES production function, and extend two of the most influential results from the Mirrleesian literature to our framework with endogenous wages. The first is the optimal top tax rate formula of Saez (2001). We derive a particularly simple closed-form generalization of this result in terms of one additional sufficient statistic, namely, the (finite) elasticity of substitution between skills. Using the values of this parameter estimated in the empirical literature immediately leads to clear policy recommendations for the optimal tax rate on high incomes in the presence of "trickle-down" effects on wages. The second key result is the familiar U-shaped pattern of optimal marginal tax rates first obtained by Diamond (1998). We show that the general equilibrium forces not only confirm this pattern, but make it even more pronounced, with a stronger dip in the bulk of the income distribution. This has an interesting and unexpected implication: where marginal tax rates are increasing (resp., decreasing) in the partial equilibrium model, they now increase (resp., decrease) faster, implying a more progressive (resp., regressive) pattern of optimal tax rates above (resp., below) a certain income level. This qualifies the regressivity implication of the two-type analysis of Stiglitz (1982), already described above, and highlights the importance of studying these forces within the workhorse model of taxation.

Our final set of results is a quantitative analysis of both the tax incidence and the optimal taxes in the general equilibrium setting. We first calibrate our model to the U.S. tax code.[2] To meaningfully compare the general equilibrium optimal taxes to the partial equilibrium optimum, we further construct a policy-relevant benchmark. Specifically, we compare our policy recommendations to those one would obtain by applying the analysis of Diamond (1998), calibrating the model to the same income distribution, and making the same assumptions about the utility function. The simulations of optimal policies confirm our theoretical insights for the CES production function and show their quantitative significance. Moreover, when the production function is Translog and the elasticity of substitution is distance-dependent, we show that our key results, both on tax incidence and optimal taxation, are not only confirmed but reinforced: the revenue gains from raising the progressivity of the U.S. tax code, and the U-shaped pattern of optimal marginal tax rates, are more pronounced than in the CES case. We finally compute, for a Rawlsian social planner, the welfare gains from setting taxes optimally, relative to the partial equilibrium optimum benchmark. These gains are quantitatively significant. Both for CES and Translog production functions, with reasonable values of the elasticities of substitution, they are up to 3.5% gains in consumption equivalent.

Related Literature. This paper is related to the literature on tax incidence (see, e.g., Harberger (1962) and Shoven and Whalley (1984) for the seminal papers, Hines (2009) for emphasizing the importance of general equilibrium in taxation, and Kotlikoff and Summers (1987) and Fullerton and Metcalf (2002)) for comprehensive surveys. Our paper extends this framework to an economy with a continuum of (labor) inputs with arbitrary nonlinear tax schedules. Thus, we study tax incidence in the workhorse model of optimal nonlinear labor income taxation of (Mirrlees, 1971; Diamond, 1998), thereby contributing to the unification of the two major strands in the taxation literature (tax incidence and optimal taxation).

The optimal taxation problem in general equilibrium with arbitrary nonlinear tax instruments has originally been studied by Stiglitz (1982) in a model with two types.

[2] The calibration is based on Heathcote, Storesletten, and Violante (2014) who show that a two-parameter tax function with contrant rate of progressivity can well approximate the U.S. tax code. See also Guner, Kaygusuz, and Ventura (2014) and Antras, de Gortari, and Itskhoki (2016).

The key result of Stiglitz (1982) is that at the optimum tax system, general equilibrium forces lead to a more regressive tax schedule. In the recent optimal taxation literature, there are two strands that relate to our work. First, a series of important contributions by Scheuer (2014); Rothschild and Scheuer (2013, 2014); Scheuer and Werning (2015), Chen and Rothschild (2015), Ales, Kurnaz, and Sleet (2015) and Ales and Sleet (2016) form the modern analysis of optimal nonlinear taxes in general equilibrium. Specifically, Rothschild and Scheuer (2013, 2014) generalize Stiglitz (1982) to a setting with N sectors and a continuum of (infinitely substitutable) skills in each sector, leading to a multidimensional screening problem. Ales, Kurnaz, and Sleet (2015) and Ales and Sleet (2016) microfound the production function by incorporating an assignment model into the Mirrlees framework and study the implications of technological change and CEO-firm matching for optimal taxation. Our model is simpler than those of Rothschild and Scheuer (2013, 2014) and Ales, Kurnaz, and Sleet (2015). In particular, different types earn different wages (there is no overlap in the wage distributions of different types, as opposed to the framework of Rothschild and Scheuer (2013, 2014)), and the production function is exogenous (in contrast to Ales, Kurnaz, and Sleet (2015)). Our simpler setting, on the other hand, allows us to get a sharper characterization of the effects of taxes on individual and aggregate welfare, and to directly compare our results to the (Mirrlees, 1971) benchmark. Finally, our setting is distinct from those of Scheuer and Werning (2015, 2016), whose modeling of the technology is such that the general equilibrium effects cancel out at the optimum tax schedule, so that the formula of Mirrlees (1971) extends to more general production functions. We discuss in detail the difference between our framework and theirs in Section 3.2 below.

The second strand in the literature characterizes optimal government policy, within restricted classes of nonlinear tax schedules, in general equilibrium extensions of the continuous-type Mirrleesian framework. Heathcote, Storesletten, and Violante (2014) study optimal tax progressivity in a model with tax schedules of the CRP form in general equilibrium.[3] Itskhoki (2008) and Antras, de Gortari, and Itskhoki (2016) characterize the impact of distortionary redistribution of the gains from trade in an open economy using a linear or CRP tax schedule, allowing for imperfect substitutability between the tasks performed by different agents. Their production functions are CES with a continuum of skills or tasks, as in several parts of our paper.

[3] See also Heathcote and Tsujiyama (2016) for a detailed analysis of the performance of various restricted functional forms to approximate the fully optimal taxes.

Moreover, one of our key theoretical results, as well as our quantitiative analysis, use the CRP schedule as an approximation of the current U.S. tax system. On the one hand, our model is simpler than their framework as we abstract from idiosyncratic risk and take skills as given, and we study a static and closed economy. On the other hand, for most of our theoretical analysis we do not restrict ourselves to a particular functional form for taxes. Our papers share, however, one important goal: to derive simple closed form expressions for the effects of tax reforms in general equilibrium Mirrleesian environments.

Our study of tax incidence is based on a variational, or "tax reform" approach, originally pioneered by Piketty (1997) and Saez (2001), and extended to several other contexts by, e.g., Kleven, Kreiner, and Saez (2009) (for the case of multidimensional screening), and Golosov, Tsyvinski, and Werquin (2014) (for dynamic stochastic models). In this paper we extend this approach to the general equilibrium framework with endogenous wages. Our resulting optimal tax formula coincides with the characterization we also obtain using the traditional mechanism design approach, which is similar to those derived by Rothschild and Scheuer (2013, 2014) and Ales, Kurnaz, and Sleet (2015). However, using the variational approach uncovers different fundamental economic forces and provides an alternative, arguably clearer, economic intuition for the characterization of the optimum.[4]

Our paper revisits the key policy-relevant insights obtained in the partial equilibrium model by Diamond (1998) and Saez (2001), namely the closed-form formula for the optimal top tax rate and the U-shape of optimal marginal tax rates. Our generalization of the optimal top tax rate to the case of endogenous wages is related to Piketty, Saez, and Stantcheva (2014). They extend the Saez (2001) top tax formula to a setting with a compensation bargaining channel using a variational approach, and show that it increases optimal top taxes as bargaining is a zero sum game. More generally, Rothschild and Scheuer (2016) study optimal taxation in the presence of rent-seeking. In this paper we abstract from such considerations and assume that individuals are paid their marginal productivity. Finally, our quantitative analysis is in the spirit of those of Feldstein (1973) and Allen (1982), who specified the production function to simple functional forms, but focused on the case of linear taxes.

[4] Rothstein (2010) studies the desirability of EITC-type tax reforms in a model with heterogenous labor inputs and nonlinear taxation. He only considers own-wage effects, however, and no cross-wage effects. Further he treats intensive margin labor supply responses as occurring along linearized budget constraints.

This paper is organized as follows. Section 1 describes our framework and defines the key structural elasticity variables. In Section 2 we analyze the tax incidence problem with a continuum of wages and nonlinear income taxes. In Section 3 we derive optimal taxes in general equilibrium. Finally, we present quantitative results in Section 4. The proofs of all the formulas and results of this paper are gathered in Appendix B. Appendix C contains additional details and robustness checks for our numerical simulations.

1 Environment

1.1 Equilibrium

Individuals have preferences over consumption c and labor supply l given by $U(c,l) = u(c - v(l))$,[5] where u and v are twice continuously differentiable and satisfy $u', v' > 0$, $u'' \leq 0$, $v'' > 0$. There is a continuum of types (productivities) $\theta \in \Theta = \left[\underline{\theta}, \overline{\theta}\right] \subset \mathbb{R}_+$, distributed according to the p.d.f. $f_\theta(\cdot)$ and c.d.f. $F_\theta(\cdot)$. Each type is composed of a mass 1 of identical agents.

An individual of type θ earns a wage $w(\theta)$, that she takes as given. She chooses her labor supply $l(\theta)$ and earns taxable income $y(\theta) = w(\theta)l(\theta)$. Her consumption is equal to $y(\theta) - T(y(\theta))$, where $T : \mathbb{R}_+ \to \mathbb{R}$ is the income tax schedule imposed by the government, which is twice continuously differentiable. The optimal labor supply choice $l(\theta)$ is the solution to the first-order condition of the utility-maximization problem:

$$v'(l(\theta)) = \left[1 - T'(w(\theta)\, l(\theta))\right] w(\theta)\,. \tag{1}$$

We denote by $\mathcal{U}(\theta)$ the indirect utility and by $L(\theta) \equiv l(\theta)\, f_\theta(\theta)$ the total amount of labor supplied by individuals of type θ.

There is a continuum of mass 1 of identical firms that produce output using the labor of all types θ. We represent the continuum of labor inputs $\mathscr{L} = \{L(\theta)\}_{\theta\in\Theta}$ as a finite, non-negative Borel measure on the compact metric space $(\Theta, \mathcal{B}(\Theta))$.[6]

[5] Note that U implies no income effects on labor supply. This simplifying assumption is commonly made in the optimal taxation literature. See, e.g., Diamond (1998).

[6] Thus, for any Borelian set $B \in \mathcal{B}(\Theta)$ (e.g., an interval in Θ), $\mathscr{L}(B)$ is the total amount of labor supplied by individuals with productivity $\theta \in B$. Denote by $\mathcal{M}$ the space of such measures $\mathscr{L}$. This construction follows Hart (1979) and Fradera (1986); see also Parenti, Thisse, and Ushchev (2014) and Scheuer and Werning (2016).

We then define the production function as $\mathscr{F}(\mathscr{L}) = \mathscr{F}\left(\{L(\theta)\}_{\theta\in\Theta}\right)$ and write the representative firm's profit-maximization problem given the wage schedule $\{w(\theta)\}_{\theta\in\Theta}$ as

$$\max_{\mathscr{L}} \left[\mathscr{F}(\mathscr{L}) - \int_{\Theta} w(\theta) L(\theta) d\theta\right].$$

We assume that the production function $\mathscr{F}$ has constant returns to scale. In equilibrium, firms earn no profits and the wage $w(\theta)$ is equal to the marginal productivity of type-θ labor. Heuristically, we have $w(\theta) = \frac{\partial}{\partial L(\theta)}\mathscr{F}\left(\{L(\theta')\}_{\theta'\in\Theta}\right)$. Formally, $w(\theta)$ is equal to the Gateaux derivative of the production function when the labor effort schedule $\mathscr{L}$ is perturbed in the direction of the Dirac measure at θ, δ_θ:

$$w(\theta) = \lim_{\mu\to 0} \frac{1}{\mu}\left\{\mathscr{F}(\mathscr{L} + \mu\delta_\theta) - \mathscr{F}(\mathscr{L})\right\}. \tag{2}$$

It follows from definition (2) that the wage $w(\theta)$ can be represented as a functional $\omega : \Theta \times \mathbb{R}_+ \times \mathcal{M} \to \mathbb{R}_+$ that has three arguments: the individual's type $\theta \in \Theta$, her labor supply $L(\theta) \in \mathbb{R}_+$, and the measure $\mathscr{L} \in \mathcal{M}$ that describes all agents' labor effort:

$$w(\theta) = \omega(\theta, L(\theta), \mathscr{L}). \tag{3}$$

We make the following assumption throughout the paper:[7]

Assumption 1. *The wage and earnings functions $\theta \mapsto w(\theta)$ and $\theta \mapsto y(\theta)$ are continuously differentiable and strictly increasing.*

We can then define the densities of wages and incomes, respectively, as $f_w(w(\theta)) = (w'(\theta))^{-1} f_\theta(\theta)$ and $f_y(y(\theta)) = (y'(\theta))^{-1} f_\theta(\theta)$, with corresponding c.d.f. $F_w(\cdot)$ and $F_y(\cdot)$.

1.2 Social welfare

The government's objective is weighted utilitarian. The schedule of Pareto weights is represented by the p.d.f. $\tilde{f}_\theta(\cdot)$. If $\tilde{f}_\theta(\theta) = f_\theta(\theta)$ for all $\theta \in \Theta$, the planner is utilitarian. Social welfare is then defined by:

$$\mathscr{G} \equiv \int_{\Theta} u\left[w(\theta) l(\theta) - T(w(\theta) l(\theta)) - v(l(\theta))\right] \tilde{f}_\theta(\theta) d\theta. \tag{4}$$

[7] We verify numerically that this assumption is satisfied.

Denote by λ the marginal value of public funds.[8] We define the social marginal welfare weight (see, e.g., Saez and Stantcheva (2016)) associated with individuals of type θ as

$$g_\theta(\theta) = \frac{u'\left[c(\theta) - v(l(\theta))\right] \tilde{f}_\theta(\theta)}{\lambda f_\theta(\theta)}. \tag{5}$$

We finally denote by $\tilde{f}_y(y(\theta)) = (y'(\theta))^{-1} \tilde{f}_\theta(\theta)$ and $g_y(y(\theta)) = g_\theta(\theta)$ the corresponding schedules of Pareto weights and marginal social welfare weights on incomes.

1.3 Elasticity concepts

Partial equilibrium labor supply elasticities. Consider an individual with type θ, income $y = y(\theta)$, and marginal tax rate $\tau(\theta) = T'(y(\theta))$. We first define the labor supply elasticity with respect to the net-of-tax rate along the linearized budget constraint, keeping the wage constant, as:[9,10]

$$\varepsilon_{l,1-\tau}(\theta) = \left.\frac{\partial \ln l(\theta)}{\partial \ln(1-\tau(\theta))}\right|_{w(\theta)} = \frac{v'(l(\theta))}{l(\theta)\, v''(l(\theta))}. \tag{6}$$

Second, we define the elasticity along the nonlinear budget constraint as the total labor supply response to a perturbation of the net-of-tax rate, keeping wages constant.[11] This elasticity accounts for the fact that if the baseline tax schedule is nonlinear, a change in individual labor supply induces a change in the marginal tax rate, and hence a further labor supply adjustment. Solving for this fixed point, we obtain:

$$\tilde{\varepsilon}_{l,1-\tau}(\theta) = \frac{1 - T'(y(\theta))}{1 - T'(y(\theta)) + \varepsilon_{l,1-\tau}(\theta)\, y(\theta)\, T''(y(\theta))} \varepsilon_{l,1-\tau}(\theta). \tag{7}$$

Finally, we define the labor supply elasticity with respect to an exogenous perturbation in the wage along the nonlinear budget constraint, still ignoring the induced general equilibrium effects. A change in the wage rate affects the wedge

[8] At this stage, we do not take a stand on how the marginal value of public funds λ is determined. It can be defined exogenously as the social value of marginally increasing government revenue to fund a public good, or endogenously by imposing that all the perturbations of the tax system that we consider are revenue neutral, e.g., by redistributing lump-sum any excess revenue.

[9] See, e.g. Saez (2001), Golosov, Tsyvinski, and Werquin (2014). Note that Assumption 1 allows us to write interchangeably $\varepsilon_{l,1-\tau}(\theta)$ or $\varepsilon_{l,1-\tau}(y(\theta))$ depending on the context, and similarly for all the elasticities that we define.

[10] The formulas of this section are derived in Appendix B.2.1.

[11] See, e.g., Jacquet and Lehmann (2016), Scheuer and Werning (2016).

$(1-\tau(\theta))\,w(\theta)$ directly as in the case of an exogenous perturbation in the net-of-tax rate, and indirectly through its effect on the marginal tax rate $\tau(\theta) = T'(w(\theta)\,l(\theta))$. Therefore

$$\tilde{\varepsilon}_{l,w}(\theta) = \frac{1 - T'(y(\theta)) - y(\theta)\,T''(y(\theta))}{1 - T'(y(\theta)) + \varepsilon_{l,1-\tau}(\theta)\,y(\theta)\,T''(y(\theta))}\varepsilon_{l,1-\tau}(\theta)\,. \tag{8}$$

Wage elasticities. We now define the elasticity of the wage of type θ, $w(\theta)$, with respect to the labor supply of type θ', $L(\theta')$. Heuristically, $\gamma(\theta,\theta') = \frac{\partial \ln w(\theta)}{\partial \ln L(\theta')} = L(\theta')\,\mathscr{F}''_{\theta,\theta'}/\mathscr{F}'_{\theta}$. Formally, $\gamma(\theta,\theta')$ is defined as the Gateaux derivative of the functional $\omega(\theta, L(\theta), \mathscr{L})$ when $\mathscr{L} = \{L(x)\}_{x\in\Theta}$ is perturbed in the Dirac direction $\delta_{\theta'}$. Note that the function $\theta \mapsto \gamma(\theta,\theta')$ generally "jumps" when $\theta = \theta'$, i.e., when we consider the impact of the labor supply of type-θ' agents on their own wage.[12] We thus define the cross-wage elasticity, for $\theta \neq \theta'$, as $\bar{\gamma}(\theta,\theta') = \gamma(\theta,\theta')$, and the own-wage elasticity $\bar{\bar{\gamma}}(\theta',\theta')$ as the difference between $\gamma(\theta',\theta')$ and $\lim_{\theta\to\theta'}\bar{\gamma}(\theta,\theta')$.

Definition 1. Define the cross-wage elasticity as

$$\bar{\gamma}(\theta,\theta') = L(\theta') \times \lim_{\mu\to 0}\frac{1}{\mu}\left\{\ln\omega(\theta, L(\theta), \mathscr{L} + \mu\delta_{\theta'}) - \ln\omega(\theta, L(\theta), \mathscr{L})\right\}, \tag{9}$$

and the own-wage elasticity as

$$\bar{\bar{\gamma}}(\theta,\theta) = \frac{\partial \ln\omega(\theta, L(\theta), \mathscr{L})}{\partial \ln L(\theta)}. \tag{10}$$

The wage elasticity $\gamma(\theta,\theta')$, for any $(\theta,\theta') \in \Theta^2$, is then given by

$$\gamma(\theta,\theta') = \bar{\gamma}(\theta,\theta') + \bar{\bar{\gamma}}(\theta',\theta')\,\delta_{\theta'}(\theta)\,. \tag{11}$$

We assume throughout that the function $\theta \mapsto \bar{\gamma}(\theta,\theta')$ is continuously differentiable.

General equilibrium labor supply elasticities. If wages are endogenous, the partial equilibrium elasticities (6) to (8) do not fully capture the labor supply response to a tax change. At a first step and for notational simplicity, we now introduce an elasticity concept that takes into account the endogeneity of the wage $w(\theta)$ to its own

[12] This is because the labor supply $L(\theta')$ appears only through $\mathscr{L}$ in the arguments of $\omega(\theta, L(\theta), \mathscr{L})$ if $\theta' \neq \theta$, while it also appears directly as the second argument of the functional if $\theta' = \theta$. See Section 1.4 for examples.

labor supply $L(\theta)$, keeping everyone else's labor supply fixed:

$$\tilde{E}_{l,1-\tau}(\theta) = \frac{\tilde{\varepsilon}_{l,1-\tau}(\theta)}{1-\bar{\bar{\gamma}}(\theta,\theta)\,\tilde{\varepsilon}_{l,w}(\theta)}, \text{ and } \tilde{E}_{l,w}(\theta) = \frac{\tilde{\varepsilon}_{l,w}(\theta)}{1-\bar{\bar{\gamma}}(\theta,\theta)\,\tilde{\varepsilon}_{l,w}(\theta)}. \tag{12}$$

Intuitively, a percentage increase in the labor supply of type-θ individuals by $\tilde{\varepsilon}_{l,1-\tau}(\theta)$, triggered by an increase in their net-of-tax rate, lowers their own wage by $\bar{\bar{\gamma}}(\theta,\theta)$, which in turn decreases their labor supply by $\bar{\bar{\gamma}}(\theta,\theta)\,\tilde{\varepsilon}_{l,w}(\theta)$. Solving for this fixed point yields the expression for $\tilde{E}_{l,1-\tau}(\theta)$ in (12).

1.4 Examples

In order to illustrate the wage and elasticity concepts introduced so far, we now analyze the two most common production functions used in the literature, namely CES and Translog.[13]

Example 1. (CES technology.) The production function has constant elasticity of substitution (CES) if

$$\mathscr{F}\left(\{L(\theta)\}_{\theta\in\Theta}\right) = \left[\int_{\Theta} a(\theta)\,(L(\theta))^{\rho}\,d\theta\right]^{1/\rho}, \tag{13}$$

for some constant $\rho \in (-\infty, 1]$. The wage schedule is given by

$$w(\theta) = a(\theta)\,(L(\theta))^{\rho-1}\left[\int_{\Theta} a(x)\,(L(x))^{\rho}\,dx\right]^{\frac{1}{\rho}-1}, \tag{14}$$

which highlights its dependence on the three variables $\theta \in \Theta$, $L(\theta) \in \mathbb{R}_+$, and $\mathscr{L} = \{L(\theta')\}_{\theta'\in\Theta} \in \mathcal{M}$. The cross- and own-wage elasticities are given by

$$\bar{\gamma}(\theta,\theta') = (1-\rho)\frac{a(\theta')\,L(\theta')^{\rho}}{\int_{\Theta} a(x)\,L(x)^{\rho}\,dx}, \text{ and } \bar{\bar{\gamma}}(\theta,\theta) = \rho - 1. \tag{15}$$

Note that $\bar{\bar{\gamma}}(\theta,\theta) < 0$ is constant. Moreover, $\bar{\gamma}(\theta,\theta') > 0$ does not depend on θ, implying that a change in the labor supply of type θ' has the same effect (in percentage terms) on the wage of every type $\theta \neq \theta'$. Denoting by $\sigma(\theta,\theta') = -\left[\frac{\partial \ln(w(\theta)/w(\theta'))}{\partial \ln(L(\theta)/L(\theta'))}\right]^{-1}$ the elasticity of substitution between any two labor inputs, we have $\sigma(\theta,\theta') = \frac{1}{1-\rho} \equiv \sigma$

[13] The formulas of this section are proved in Appendix B.2.2.

for all $(\theta, \theta') \in \Theta^2$. The cases $\sigma = 1$ and $\sigma = 0$ correspond respectively to the Cobb-Douglas and Leontieff production functions, and $\sigma = \infty$ is the partial equilibrium case where wages are exogenous.

Our second main example is the Translog production function, which can be used as a second-order approximation to any production function (see details in Christensen, Jorgenson, and Lau (1973)).

Example 2. (Translog technology.) The production function is transcendental logarithmic (Translog) if

$$\begin{aligned} \ln \mathscr{F}\left(\{L(\theta)\}_{\theta \in \Theta}\right) = a_0 + \int_{\Theta} a(\theta) \ln L(\theta)\, d\theta + \ldots \\ \frac{1}{2}\int_{\Theta} \bar{\bar{\beta}}(\theta, \theta) (\ln L(\theta))^2\, d\theta + \frac{1}{2}\int_{\Theta \times \Theta} \bar{\beta}(\theta, \theta') (\ln L(\theta)) (\ln L(\theta'))\, d\theta d\theta', \end{aligned} \tag{16}$$

where for all θ, θ', $\int_{\Theta} a(\theta')\, d\theta' = 1$, $\bar{\beta}(\theta, \theta') = \bar{\beta}(\theta', \theta)$, and $\bar{\bar{\beta}}(\theta, \theta) = -\int_{\Theta} \bar{\beta}(\theta, \theta')\, d\theta'$. These restrictions ensure that the technology has constant returns to scale. When $\bar{\beta}(\theta, \theta') = 0$ for all θ, θ', the production function is Cobb-Douglas. The wage schedule is given by

$$w(\theta) = \frac{\mathscr{F}(\mathscr{L})}{L(\theta)} \left\{ a(\theta) + \bar{\bar{\beta}}(\theta, \theta) \ln L(\theta) + \int_{\Theta} \bar{\beta}(\theta, \theta') \ln L(\theta')\, d\theta' \right\}. \tag{17}$$

The cross- and own-wage elasticities are given by

$$\bar{\gamma}(\theta, \theta') = \chi(\theta') + \frac{\bar{\beta}(\theta, \theta')}{\chi(\theta)}, \text{ and } \bar{\bar{\gamma}}(\theta, \theta) = -1 + \frac{\bar{\bar{\beta}}(\theta, \theta)}{\chi(\theta)}, \tag{18}$$

where $\chi(\theta) = \frac{w(\theta)L(\theta)}{\mathscr{F}(\mathscr{L})}$ denotes the type-θ labor share of output. Whenever we specify a Translog technology in the sequel, we assume the functional form:

$$\bar{\beta}(\theta, \theta') = \alpha(\theta)\, \alpha(\theta') \left[c - \exp\left(-\frac{1}{2s^2} (\theta - \theta')^2 \right) \right], \tag{19}$$

where c, s are constants and α is a given function. Formula (18) then shows that the cross-wage elasticities $\bar{\gamma}(\theta, \theta')$ are increasing in the distance between the types θ and θ'. This captures the economic intuition that two individuals are stronger substitutes the closer their productivity.[14]

[14]The expression for $\sigma(\theta, \theta')$ is derived in Appendix B.2.2. Teulings (1995) obtains this property

2 General tax incidence analysis

In this section we derive the first-order effects of arbitrary local perturbations ("tax reforms") of a given baseline, potentially suboptimal, tax schedule on: (i) individual labor supplies, wages, and utilities; and (ii) government revenue and social welfare. We analyze this incidence problem for a general production function in Sections 2.1 and 2.2, and for specific technologies (CES and Translog) in Sections 2.3 and 2.4.

2.1 Effects of tax reforms on labor supply

As in partial equilibrium (Saez, 2001), analyzing the incidence effects of tax reforms relies crucially on solving for each individual's change in labor supply in terms of behavioral elasticities. This problem is, however, much more involved in general equilibrium. In the former setting, without income effects, a change in the tax rate of a given individual, say θ, induces only a change in the labor effort of that type (measured by the elasticity (7)). In the latter setting, instead, this labor supply response of type θ affects the wage, and hence the labor supply, of every other agent $\theta' \neq \theta$. This in turn feeds back into the wage of θ, which further impacts her labor supply, and so on. Representing the total effect of this infinite chain reaction triggered by arbitrarily non-linear tax reforms is thus a priori a complex task.[15]

The key step towards the general characterization of the economic incidence of taxes, and our first main theoretical contribution, consists of showing that this problem can be mathematically formulated as solving an integral equation.[16] Formally,

in an assignment model. More generally, the results of this paper go through if we interpret our production function as the outcome of an task-assignment model, in which the assignment is invariant to taxes. See Ales, Kurnaz, and Sleet (2015) and Scheuer and Werning (2015).

[15] One way to side-step this problem would be to define, for each specific tax reform, a "policy elasticity" (as in Hendren (2015), Piketty and Saez (2013)), equal to each individual's total labor supply response to the corresponding reform. It would then be straightforward to derive the effects of a particular tax reform on, say, social welfare, as a function of its policy elasticity variable (see Section 2.2). However, the key difficulty of the incidence problem consists precisely of expressing this total labor supply response in terms of the structural elasticity parameters introduced in Section 1.3. Doing so allows us to uncover the fundamental economic forces underlying the incidence of taxation. It also allows us to represent the effects of any tax reform in terms of a common set of structural elasticity parameters, without requiring the case-by-case evaluation of the policy elasticity associated with each specific reform that one might consider implementing in practice.

[16] The general theory of linear integral equations is exposed in, e.g., Tricomi (1985), Kress (2014), and, for a concise introduction, Zemyan (2012). Moreover, simple closed-form solutions can be derived in many cases (see Polyanin and Manzhirov (2008)). Finally, numerical techniques are widely available and can be easily implemented (see, e.g., Press (2007) and Section 2.6 in Zemyan

consider an arbitrary non-linear tax reform of a baseline tax schedule $T(\cdot)$. This tax reform can be represented by a continuously differentiable function $h(\cdot)$ on $\mathbb{R}_+$, so that the perturbed tax schedule is $T(\cdot) + \mu h(\cdot)$, where $\mu \in \mathbb{R}$. Our aim is to compute the first-order effect of this perturbation h on individual labor supply, when the magnitude of the tax change is small, i.e., as $\mu \to 0$. This is formally expressed by the Gateaux derivative of the labor supply functional $T \mapsto l(\theta; T)$ in the direction h, that is,[17]

$$\mathrm{d}\hat{l}(\theta, h) = \lim_{\mu \to 0} \frac{1}{\mu} \left\{ \ln l(\theta; T + \mu h) - \ln l(\theta; T) \right\}. \tag{20}$$

This expression captures the total change in the labor supply of type θ in response to the tax reform h, taking into account all of the general equilibrium effects due to the endogeneity of wages.

Lemma 1. *The impact of a perturbation h of the baseline tax schedule T on individual labor supplies, $\mathrm{d}\hat{l}(\cdot, h)$, is the solution to the integral equation*

$$\mathrm{d}\hat{l}(\theta, h) = -\tilde{E}_{l,1-\tau}(\theta) \frac{h'(y(\theta))}{1 - T'(y(\theta))} + \tilde{E}_{l,w}(\theta) \int_{\Theta} \bar{\gamma}(\theta, \theta') \, \mathrm{d}\hat{l}(\theta', h) \, d\theta', \tag{21}$$

for all $\theta \in \Theta$.

Proof. See Appendix B.3.1. □

Formula (21) is a Fredholm integral equation of the second kind. Its unknown, which appears under the integral sign, is the function $\theta \mapsto \mathrm{d}\hat{l}(\theta, h)$. Its kernel is $K_1(\theta, \theta') = \tilde{E}_{l,w}(\theta) \bar{\gamma}(\theta, \theta')$. We now provide its intuition.

Due to the reform the net-of-tax rate of individual θ changes, in percentage terms, by $-\frac{h'(y(\theta))}{1-T'(y(\theta))}$. By definition of the elasticity (7), this tax reform induces a direct percentage change in labor supply $l(\theta)$ equal to $-\tilde{\varepsilon}_{l,1-\tau}(\theta) \frac{h'}{1-T'}$. This is the expression we would obtain in partial equilibrium.[18] In general equilibrium, type-θ labor supply is also impacted indirectly by the change in all other individuals' labor supplies. Specifically, the change in labor supply of each type θ', $\mathrm{d}\hat{l}(\theta', h)$, triggers a change in the wage of type θ equal to $\gamma(\theta, \theta') \times \mathrm{d}\hat{l}(\theta', h)$, and thus a further adjustment in her labor supply equal to $\tilde{\varepsilon}_{l,w}(\theta) \gamma(\theta, \theta') \mathrm{d}\hat{l}(\theta', h)$. Summing these effects over $\theta' \in \Theta$, and

(2012)), leading to straightforward quantitative evaluations of the incidence of arbitrary tax reforms (see Section 4).

[17]The notation $\mathrm{d}\hat{l}(\theta, h)$ ignores for simplicity the dependence of the Gateaux derivative on the baseline tax schedule T.

[18]See, e.g., p. 217 in Saez (2001).

using equation (11) to disentangle the own- and cross-wage effects in the resulting integral, leads to formula (21).[19]

The next step of our analysis of tax incidence consists of characterizing the solution to (21).

Proposition 1. *Assume that the condition* $\int_{\Theta^2} |K_1(\theta,\theta')|^2 \, d\theta d\theta' < 1$ *holds, where* $K_1(\theta,\theta') = \tilde{E}_{l,w}(\theta)\,\bar{\gamma}(\theta,\theta')$.[20] *The unique solution to the integral equation (21) is given by*

$$\mathrm{d}\hat{l}(\theta,h) = -\tilde{E}_{l,1-\tau}(\theta)\frac{h'(y(\theta))}{1-T'(y(\theta))} - \int_{\Theta} \mathcal{R}(\theta,\theta')\,\tilde{E}_{l,1-\tau}(\theta')\frac{h'(y(\theta'))}{1-T'(y(\theta'))}d\theta', \quad (22)$$

where for all $\theta,\theta' \in \Theta$, *the resolvent* $\mathcal{R}(\theta,\theta')$ *is given by the Neumann series of iterated kernels* $\{K_n(\theta,\theta')\}_{n\geq 1}$,

$$\mathcal{R}(\theta,\theta') = \sum_{n=1}^{\infty} K_n(\theta,\theta'), \ \text{ with } \ K_{n+1}(\theta,\theta') = \int_{\Theta} K_n(\theta,\theta'')\,K_1(\theta'',\theta')\,d\theta''. \quad (23)$$

Proof. See Appendix B.3.2. □

We now show that the abstract mathematical representation (22) of the general solution to the integral equation (21) has a clear economic interpretation. The first term in the right hand side of (22), $-\tilde{E}_{l,1-\tau}\frac{h'}{1-T'}$, is the direct (partial equilibrium) effect of the reform on labor supply $l(\theta)$, weighted by the own-wage general equilibrium effects, as already described for equation (21). The second term, that depends on the series (23), accounts for the infinite sequence of cross-wage effects in general equilibrium.

Specifically, consider the first iterated kernel ($n = 1$) in the series (23), which writes

$$\int_{\Theta} K_1(\theta,\theta')\,\tilde{E}_{l,1-\tau}(\theta')\frac{h'(y(\theta'))}{1-T'(y(\theta'))}d\theta'. \quad (24)$$

This integral expresses the fact that for any θ', the percentage change in the labor supply of θ' due to the tax reform (including the own-wage effects), $d\ln l(\theta') =$

[19] This last step ensures that the kernel of the integral equation features only the cross-wage effects $\bar{\gamma}(\theta,\theta')$, and is thus smooth. The labor supply elasticities $\tilde{\varepsilon}_{l,1-\tau}(\theta)$ and $\tilde{\varepsilon}_{l,w}(\theta)$ are then weighted by the own-wage effects $\bar{\bar{\gamma}}(\theta,\theta)\,\tilde{\varepsilon}_{l,w}(\theta)$, to yield $\tilde{E}_{l,1-\tau}(\theta)$ and $\tilde{E}_{l,w}(\theta)$ defined in (11).

[20] We discuss this technical condition in the Appendix, and it can be verified numerically. It ensures that the infinite series (23) converges. When it is not satisfied, we can more generally express the solution to (21) with the same representation (22) but with a more complex resolvent (see Section 2.4 in Zemyan (2012)).

$\tilde{E}_{l,1-\tau}(\theta') \frac{h'(y(\theta'))}{1-T'(y(\theta'))}$, induces a change in the wage of type θ equal to $\bar{\gamma}(\theta, \theta') \, d \ln l(\theta')$, and hence a labor supply change $\tilde{E}_{l,w}(\theta) \bar{\gamma}(\theta, \theta') \, d \ln l(\theta') = K_1(\theta, \theta') \, d \ln l(\theta')$. The integral (24) thus accounts for the effects of each type θ' on the labor supply of θ through direct cross-wage effects.

The second iterated kernel ($n = 2$) in (23) accounts for the effects of each type θ' on the labor supply of θ, indirectly through the behavior of third parties θ''. This term writes

$$\int_{\Theta} \left\{ \int_{\Theta} K_1(\theta, \theta'') \, K_1(\theta'', \theta') \, d\theta'' \right\} \tilde{E}_{l,1-\tau}(\theta') \frac{h'(y(\theta'))}{1-T'(y(\theta'))} d\theta'. \tag{25}$$

For any θ', the percentage change in the labor supply of θ' due to the tax reform, $d \ln l(\theta') = \tilde{E}_{l,1-\tau}(\theta') \frac{h'(y(\theta'))}{1-T'(y(\theta'))}$, induces a change in the wage of any other type θ'' given by $\bar{\gamma}(\theta'', \theta') \, d \ln l(\theta')$, and hence a change in the labor supply of θ'' given by $d \ln l(\theta'') = \tilde{E}_{l,w}(\theta'') \bar{\gamma}(\theta'', \theta') \, d \ln l(\theta') = K_1(\theta'', \theta') \, d \ln l(\theta')$. This in turn affects the labor supply of type θ by the amount $\tilde{E}_{l,w}(\theta) \bar{\gamma}(\theta, \theta'') \, d \ln l(\theta'') = K_1(\theta, \theta'') \, d \ln l(\theta'')$. Summing over types θ' and θ'' leads to the expression (25).

An inductive reasoning shows similarly that the terms $n \geq 3$ account for the effects of each θ' on the labor supply of θ through n successive stages of cross-wage effects, e.g., for $n = 3$, $\theta' \to \theta'' \to \theta''' \to \theta$.

2.2 Effects of tax reforms on wages and welfare

Once the labor supply response $\mathrm{d}\hat{l}(\theta, h)$ is characterized (Lemma 1 and Proposition 1), we can easily derive the incidence of a tax reform h on individual wages and utilities, and on government revenue and social welfare. Corollaries 1 and 2 express the effects of h as a function of this labor supply response. The corresponding Gateaux derivatives $\mathrm{d}\hat{w}(\theta, h)$, $\mathrm{d}\mathscr{U}(\theta, h)$, $\mathrm{d}\mathscr{R}(T, h)$, $\mathrm{d}\mathscr{W}(T, h)$ are defined similarly to $\mathrm{d}\hat{l}(\theta, h)$ in (20).

Corollary 1. *The first-order effects of a perturbation h of the baseline tax schedule T on wages are given by*

$$\mathrm{d}\hat{w}(\theta, h) = \frac{\tilde{\varepsilon}_{l,1-\tau}(\theta)}{\tilde{\varepsilon}_{l,w}(\theta)} \frac{h'(y(\theta))}{1-T'(y(\theta))} + \frac{\mathrm{d}\hat{l}(\theta, h)}{\tilde{\varepsilon}_{l,w}(\theta)}. \tag{26}$$

The effects on individual utilities are given by

$$\mathrm{d}\mathcal{U}(\theta,h)=\left[-h\left(y\left(\theta\right)\right)+\left(1-T'\left(y\left(\theta\right)\right)\right)y\left(\theta\right)\mathrm{d}\hat{w}\left(\theta,h\right)\right]u'\left(\theta\right). \tag{27}$$

Proof. See Appendix B.3.3. ☐

Multiplying both sides of equation (26) by $\tilde{\varepsilon}_{l,w}(\theta)$ simply gives the adjustment of type-θ labor supply, $\mathrm{d}\hat{l}(\theta,h)$, as the sum of the partial equilibrium response, $-\tilde{\varepsilon}_{l,1-\tau}\frac{h'}{1-T'}$, and the general equilibrium effect induced by the wage change, $\tilde{\varepsilon}_{l,w}\mathrm{d}\hat{w}(\theta,h)$. In equation (27), the first term on the right hand side, $-u'(\theta)h(y(\theta))$, is due to the fact that the tax increase $h(y(\theta))$ makes the individual poorer and hence reduces her utility. The second term accounts for the change in consumption, and hence welfare, due to the wage adjustment $\mathrm{d}\hat{w}(\theta,h)$.[21]

Corollary 2. *The first-order effect of a perturbation h of the baseline tax schedule T on tax revenue is given by*

$$\mathrm{d}\mathscr{R}(T,h)=\int_{\mathbb{R}}h(y)f_y(y)\,dy+\int_{\mathbb{R}}T'(y)\left[\mathrm{d}\hat{l}(y,h)+\mathrm{d}\hat{w}(y,h)\right]yf_y(y)\,dy. \tag{28}$$

The effect on social welfare, measured in terms of public funds, is given by

$$\mathrm{d}\mathscr{W}(T,h)=\mathrm{d}\mathscr{R}(T,h)+\int_{\mathbb{R}}\left[-h(y)+\left(1-T'(y)\right)y\mathrm{d}\hat{w}(y,h)\right]g_y(y)f_y(y)\,dy. \tag{29}$$

Proof. See Appendix B.3.3. ☐

The first term in the right hand side of (28) is the mechanical (or statutory) effect of the tax reform $h(\cdot)$, i.e., the change in government revenue if individual behavior remained constant. The second term is the behavioral effect of the reform. The labor supply and wage adjustments $\mathrm{d}\hat{l}(y,h)$ and $\mathrm{d}\hat{w}(y,h)$ both induce a change in government revenue proportional to the marginal tax rate $T'(y)$. Summing these effects over all individuals using the density of incomes $f_y(y)$ yields (28). Finally, equation (29) expresses the effect of the tax reform on social welfare as the change in tax revenue $\mathrm{d}\mathscr{R}(T,h)$, plus the sum of changes in individual utilities (27), weighted by the Pareto weights $\tilde{f}_y(y)$ and normalized by the shadow value of public funds λ, leading to the marginal social welfare weights $g_y(y)$ defined in (5).

[21] Note that by the envelope theorem, the change in labor supply induced directly by the tax reform and indirectly by the implied wage change has a zero first-order impact on the individual's utility.

2.3 CES production function

The solution to the integral equation (22), and hence the incidence effects of tax reforms, can be simplified by specifying functional forms for the production function. We obtain the simplest expressions when the kernel of the integral equation, $K_1(\theta,\theta') = \tilde{E}_{l,w}(\theta)\,\bar{\gamma}(\theta,\theta')$, is multiplicatively separable between θ and θ', i.e., of the form $\kappa_1(\theta)\,\kappa_2(\theta')$. This occurs in particular for a CES technology (Example 1), since in this case the cross-wage effects $\bar{\gamma}(\theta,\theta') \equiv \kappa_2(\theta')$ depend only on θ'.

Proposition 2. *Suppose that the production function is CES. The solution to the integral equation (21) is then given by:*

$$\mathrm{d}\hat{l}(\theta,h) = -\,\tilde{E}_{l,1-\tau}(\theta)\,\frac{h'(y(\theta))}{1-T'(y(\theta))} - \tilde{E}_{l,w}(\theta)\,\frac{\int_\Theta \bar{\gamma}(\theta,\theta')\,\tilde{E}_{l,1-\tau}(\theta')\,\frac{h'(y(\theta'))}{1-T'(y(\theta'))}d\theta'}{1-\int_\Theta \tilde{E}_{l,w}(\theta')\,\bar{\gamma}(\theta',\theta')\,d\theta'}. \tag{30}$$

Proof. See Appendix B.3.4. Note that equation (30) requires $\int_\Theta \tilde{E}_{l,w}(\theta')\,\bar{\gamma}(\theta',\theta')\,d\theta' \neq 1$, which is generically satisfied. In fact, suppose for simplicity that the baseline tax schedule T is linear and the disutility of labor is isoelastic, so that $\tilde{E}_{l,w}(\theta') = \frac{\varepsilon}{1+\varepsilon/\sigma}$. We then have

$$\int_\Theta \tilde{E}_{l,w}(\theta')\,\bar{\gamma}(\theta',\theta')\,d\theta' = \frac{\varepsilon}{1+\varepsilon/\sigma} \times \frac{1}{\sigma} = \frac{\varepsilon}{\sigma+\varepsilon},$$

which is always strictly below 1 (unless the production function is Leontieff, i.e., $\sigma = 0$). $\square$

Equation (30) shows that in the CES case, the integral term in (21), or the cumulative sum in (22), which accounts for all of the general equilibrium effects on type-θ labor supply, reduces to the first iterated kernel (24), i.e., the direct impact of type-θ' labor supply on $l(\theta)$, appropriately discounted by the denominator in (30). To see this, note that we can express the integral in (21) by multiplying each side of this equation by $\kappa_2(\theta)$ and integrating:

$$\begin{aligned}\int_\Theta \kappa_2(\theta)\,\mathrm{d}\hat{l}(\theta,h)\,d\theta = &-\int_\Theta \kappa_2(\theta)\,\tilde{E}_{l,1-\tau}(\theta)\,\frac{h'(y(\theta))}{1-T'(y(\theta))}d\theta \\ &+\left[\int_\Theta \kappa_1(\theta)\,\kappa_2(\theta)\,d\theta\right]\left[\int_\Theta \kappa_2(\theta')\,\mathrm{d}\hat{l}(\theta',h)\,d\theta'\right].\end{aligned}$$

Solving for $\int_\Theta \kappa_2(\theta)\,\mathrm{d}\hat{l}(\theta,h)\,d\theta$ and substituting in equation (21) yields expression (30).

One benefit of the tax incidence analysis of Proposition 2 over the more restrictive optimal taxation approach (see Section 3) is that it gives us in simple closed form the general equilibrium welfare effects of any local tax reform, around any, potentially suboptimal, baseline tax system. A particularly interesting baseline tax system is the CRP ("constant rate of progressivity") tax schedule (see, e.g., Bénabou (2002); Heathcote, Storesletten, and Violante (2014); Antras, de Gortari, and Itskhoki (2016)) defined as:

$$T(y) = y - \frac{1-\tau}{1-p} y^{1-p}, \tag{31}$$

for $p < 1$. Formally, the parameter p is the elasticity of the net-of-tax rate $1 - T'(y)$ with respect to income y. The tax schedule is linear (resp., progressive, regressive), i.e., the marginal tax rates $T'(y)$ and the average tax rates $T(y)/y$ are constant (resp., increasing, decreasing), if $p = 0$ (resp., $p > 0$, $p < 0$). In addition to being particularly tractable,[22] the CRP tax schedule approximates very closely the U.S tax and transfer system, thus allowing us to obtain sharp and policy-relevant insights.[23]

The following corollary provides a theoretical formula that characterizes the effects of any nonlinear tax reform of the U.S tax code in general equilibrium. We focus without loss of generality on the most natural class of nonlinear tax reforms, which consists of perturbing the marginal tax rate at only one income level $y^* \in \mathbb{R}_+$.[24] Formally, the perturbation is represented by the Dirac delta function $h'(y) = \delta_{y^*}(y)$, so that the total tax liability increases by a uniform lump-sum amount above income y^*, i.e., $h(y)$ is the step function $\mathbb{I}_{\{y \geq y^*\}}$.[25]

Corollary 3. *Suppose that the production function is CES with elasticity of substitution σ, that the disutility of labor is isoelastic, and that the baseline tax schedule is*

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[22]In particular, what allows to simplify (30) to derive (32) is that, assuming that the disutility of labor is isoelastic with parameter ε, the elasticities $\tilde{\varepsilon}_{l,1-\tau}(y)$ and $\tilde{\varepsilon}_{l,w}(y)$ (as well as $\tilde{E}_{l,1-\tau}(y)$) are then constant, respectively equal to $\frac{\varepsilon}{1+p\varepsilon}$ and $\frac{(1-p)\varepsilon}{1+p\varepsilon}$.

[23]Note in particular that all of the formulas derived in this section, including equation (32), are closed-form expressions in which all of the variables (taxes, elasticities, density of incomes) are empirically observable, since they are evaluated given the current tax system.

[24]This class of tax reforms has been introduced by Saez (2001). Any other perturbation can be expressed as an appropriately weighted sum of such elementary perturbations. For instance, the effect of a linear tax reform, which consists of an equal change in the marginal tax rate of every individual, is equal to the sum of the effects of identical such perturbations at each income level. See Golosov, Tsyvinski, and Werquin (2014) for details.

[25]Note that the function $\mathbb{I}_{\{y \geq y^*\}}$ is not differentiable. We can nevertheless use our theory to analyze this reform by applying (30) to a sequence of perturbations $\{h'_n\}_{n \geq 1}$ that converges to δ_{y^*}, which we formally do in the Appendix.

CRP. The effect on social welfare of the perturbation $h(y) = \mathbb{I}_{\{y \geq y^*\}}$ is then given by:

$$\begin{aligned}\frac{\mathrm{d}\mathscr{W}(T,h)}{1-F_y(y^*)} =& 1 - \int_{y^*}^{\infty} g_y(y) \frac{f_y(y)}{1-F_y(y^*)} dy - \frac{y^* f_y(y^*)}{1-F_y(y^*)} \tilde{\varepsilon}_{l,1-\tau} \frac{T'(y^*)}{1-T'(y^*)} \\ & - \frac{y^* f_y(y^*)}{1-F_y(y^*)} \sigma^{-1} \tilde{E}_{l,1-\tau} \int_{\mathbb{R}_+} \frac{\eta(y) - \eta(y^*)}{1-T'(y^*)} \frac{y f_y(y)}{\int_{\mathbb{R}_+} y' f_y(y') dy'} dy,\end{aligned} \tag{32}$$

where we denote $\eta(y) \equiv T'(y)(1+\tilde{\varepsilon}_{l,w}) + (1-T'(y)) g_y(y)$.

In particular, if the baseline tax schedule is linear, the effect of any nonlinear perturbation on government revenue is the same as in partial equilibrium.

Proof. See Appendix B.3.5. □

The first line of formula (32) is the partial equilibrium effect of the perturbation on social welfare (see Saez (2001)) that one would compute if one assumed that the wage distribution is exogenous. The mechanical effect of the tax increase is normalized to 1. The second term on the right hand side is the welfare effect, equal to the sum of the marginal social welfare weights $g_y(y)$ of individuals with income $y \geq y^*$, whose consumption is reduced because their tax liability is higher. The third term is the behavioral effect, which reduces government revenue due to the individual labor supply adjustments, by $dT(y^*) = T'(y^*) dy^* \propto T'(y^*) \tilde{\varepsilon}_{l,1-\tau}$. This term is multiplied by the hazard rate of the income distribution, which measures the fraction of individuals whose labor supply is distorted, $f_y(y^*)$, relative to those who pay the higher absolute tax, $1 - F_y(y^*)$.

The second line of (32) describes the additional effects of the tax reform generated by the endogeneity of wages. First consider only the tax revenue change, i.e., set $g_y(y) = 0$ for all y. We can show that this novel term is equal to[26]

$$-\left(y'(\theta^*)\right)^{-1} \tilde{E}_{l,1-\tau} (1+\tilde{\varepsilon}_{l,w}) \int_{\mathbb{R}_+} \frac{T'(y)}{1-T'(y^*)} \gamma(y,y^*) \frac{y f_y(y)}{1-F_y(y^*)} dy.$$

This expression is intuitive. A change in the net-of tax rate at y^* induces a labor supply change at y^* equal to the (constant) elasticity $\tilde{E}_{l,1-\tau}(y^*)$. This in turn alters

[26] We obtain (32) from this expression by separating the cross- and own-wage effects using (11) and the facts that $\bar{\bar{\gamma}}(y^*, y^*) = \sigma^{-1}$ and $\bar{\gamma}(y, y^*)$ does not depend on y. In the Appendix we derive yet another way of writing this equation: using (31), we easily obtain that the integral in (32) (with $g_y(y) = 0$) is equal to $(1+\tilde{\varepsilon}_{l,w}) \left(1 - \frac{\mathbb{E}c/c^*}{\mathbb{E}y/y^*}\right)$, where $c(y) \equiv y - T(y)$ denotes individual disposable income.

the wage of individuals with income y by $\gamma(y, y^*)\,\tilde{E}_{l,1-\tau}(y^*)$.[27] This finally induces a change in income y given by $(1+\tilde{\varepsilon}_{l,w}(y))\,\gamma(y, y^*)\,\tilde{E}_{l,1-\tau}(y^*)$, of which a share $T'(y)$ goes to the government. Thus, under the assumptions of Corollary 3, the general equilibrium effects boil down to only one round of feedback effects (the first iterated kernel in (23)). Moreover, this expression shows that if the baseline tax schedule is linear, the impact of any nonlinear tax reform on government revenue is identical to that in partial equilibrium. This is a consequence of Euler's homogeneous function theorem, which reads $\int_{\mathbb{R}_+}\gamma(y, y^*)\,y f_y(y)\,dy = 0$, implying that the general equilibrium effects cancel out in the aggregate. Finally, the social welfare effects of the perturbation are accounted for by the marginal social welfare weights $g_y(y)$ in the definition of $\eta(y)$, multiplied by the share $1 - T'(y)$ of the wage change that accrues to individual consumption.

To understand the direction in which these novel forces drive the effects of tax reforms when the baseline tax schedule is nonlinear, consider the key term in the second line of equation (32), still focusing on government revenue:

$$-\int_{\mathbb{R}}\frac{T'(y)-T'(y^*)}{1-T'(y^*)}\,y f_y(y)\,dy.$$

Given that $T'(y) = 1-(1-\tau)\,y^{-p}$, this term is strictly increasing in y^*, first negative (for $y < y^*$) then positive (for $y > y^*$) for a progressive baseline tax schedule ($p > 0$). This implies that the tax revenue gains of raising the marginal tax rate at a given income level y^* are lower than in partial equilibrium for low y^*, and higher for high y^*. In other words, starting from the U.S. tax code (represented by the CRP tax schedule (31)), general equilibrium forces raise the benefits of increasing the progressivity of the tax schedule. We illustrate this "trickle-up" effect on government revenue (or Rawlsian welfare) in Figure 2 in Section 4.

At first sight this result may seem to be at odds with the familiar insight of Stiglitz (1982) in the two-income model, which we generalize to a continuum of incomes in Section 3. Indeed, these results say that the optimal tax rates should be lower at the top, and higher at the bottom, of the income distribution, relative to the partial equilibrium benchmark. That is, the optimal tax schedule should be more regressive when the general equilibrium forces are taken into account. We explain in greater detail this difference in Section 4, but we can already understand its reason. This

[27] The term $y'(\theta^*)$ comes from changing variables from types to incomes in the integrals.

is because we consider here reforms of the current (suboptimal) U.S. tax code, with low marginal tax rates at the bottom. Instead, the results about the optimum tax schedule use as a benchmark the optimal partial equilibrium tax schedule, which features high marginal tax rates at the bottom (see Diamond (1998), Saez (2001)). Thus, starting from a regressive tax code (at the bottom at least) leads to the opposite signs for $T'(y) - T'(y^*)$ in (32), implying gains from raising tax rates at the bottom and lowering them at the top. Intuitively, raising the marginal tax rates at the top increases (resp., decreases) wages for high (resp., low) incomes. Since a fraction $T'(y)$ of this income change accrues to the government, and a fraction $(1 - T'(y))$ accrues to the individuals, it follows that tax revenue (and Rawlsian welfare) increases by a larger amount when the tax rates are low at the bottom and high at the top. The key take-away from this section is thus that insights about the optimum tax schedule may actually be reversed when considering reforms of the current U.S. tax code.

2.4 Translog production function

Suppose now that the production function is Translog, as defined in Example 2, with the specific functional form (19). We use the fact that simple closed-forms for the solution $\mathrm{d}\hat{l}(\theta, h)$ to the integral equation (21) can be obtained when its kernel is the sum of functions that are multiplicatively separable in θ and θ', i.e., of the form $K_1(\theta, \theta') = \sum_{i=1}^{n} \kappa_{i,1}(\theta) \kappa_{i,2}(\theta')$. In the Appendix, we show that a Taylor expansion at any order of the right hand side of (19) as $\theta \to \theta'$ yields this property, and hence a simple approximate solution for $\mathrm{d}\hat{l}(\theta, h)$ at an arbitrary degree of precision. Here, we focus on the second order approximation.

Proposition 3. *Suppose that the production function is Translog with $\bar{\beta}(\theta, \theta')$ given by (19). A second-order Taylor approximation of the kernel of the integral equation (21) implies that its solution can be approximated by*

$$\mathrm{d}\hat{l}(\theta, h) \approx -\tilde{E}_{l,1-\tau}(\theta) \frac{h'(y(\theta))}{1 - T'(y(\theta))} - \tilde{E}_{l,w}(\theta) \left[c_1 + \frac{c_2}{\chi(\theta)} + c_3 \bar{\gamma}\left(\theta, \tilde{\theta}\right) \right], \tag{33}$$

where the constants $(c_n)_{n \geq 3}$ and $\tilde{\theta}$, as well as a bound on the error in the approximation, are given in the Appendix.

Proof. See Appendix B.3.6, where we derive more generally an approximation of the solution at any order N. □

In equation (33), the general equilibrium term is the product of the labor supply elasticity $\tilde{E}_{l,w}(\theta)$ with an affine function of two variables: the inverse of the labor share $\chi(\theta)$, and the cross-wage elasticity $\bar{\gamma}\left(\theta,\tilde{\theta}\right)$ with an "average" type $\tilde{\theta}$. In the Cobb-Douglas case where $\bar{\beta}(\theta,\theta') = 0$ for all θ, θ', we already showed in (30) that the corresponding term is given by $c_1\tilde{E}_{l,w}(\theta)$. The term $\bar{\gamma}\left(\theta,\tilde{\theta}\right)$ shows that, when the distance between types affects their substitutability, the whole sequence of general equilibrium effects on θ can be summarized by a single round of cross-wage effects from one average type $\tilde{\theta}$. This simple and a priori surprising aggregation result is due to our Taylor approximation which implies that in this case $\bar{\beta}(\theta,\theta')$ is linearly increasing in the distance $(\theta-\theta')^2$.

3 Optimal income taxation

In this section, we study the optimal taxation problem. The government maximizes social welfare subject to a resource constraint and the condition that wages and labor supply form an equilibrium:

$$\max_{T(\cdot)} \quad \int_\Theta u\left[w(\theta)\,l(\theta) - T\left(w(\theta)\,l(\theta)\right) - v\left(l(\theta)\right)\right]\tilde{f}_\theta(\theta)\,d\theta \tag{34}$$

$$\text{s.t.} \quad \int_\Theta \left[w(\theta)\,l(\theta) - T\left(w(\theta)\,l(\theta)\right)\right] f_\theta(\theta)\,d\theta \leq \mathscr{F}(\mathscr{L}) \tag{35}$$

$$\text{and} \quad (1)\,,\ (2)\,. \tag{36}$$

We characterize the solution to this problem using a mechanism design approach in Section 3.1, and a variational approach in Section 3.2.

3.1 Mechanism design approach

We first study the government problem (34)-(36) using mechanism-design arguments to derive the optimal informationally-constrained efficient consumption and labor supply allocations $\{c(\theta), l(\theta)\}_{\theta\in\Theta}$, subject to feasibility and incentive compatibility of these allocations.

3.1.1 Government's problem

It is useful to change variables and optimize over $\{V(\theta), l(\theta)\}_{\theta\in\Theta}$, where $V(\theta) \equiv c(\theta) - v(l(\theta))$. The mechanism design problem then reads

$$\max_{V(\cdot),l(\cdot)} \quad \int_\Theta u(V(\theta))\tilde{f}_\theta(\theta)d\theta \tag{37}$$

$$\text{s.t.} \quad \int_\Theta [V(\theta) + v(l(\theta))] f_\theta(\theta)d\theta \leq \mathscr{F}(\mathscr{L}) \tag{38}$$

$$\text{and} \quad V(\theta) \geq V(\theta') + v(l(\theta')) - v\left(l(\theta')\frac{w(\theta')}{w(\theta)}\right), \ \forall(\theta,\theta') \in \Theta^2. \tag{39}$$

The incentive compatibility constraint (39) of type θ can be expressed as a standard envelope condition $V'(\theta) = v'(l(\theta))\, l(\theta)\frac{w'(\theta)}{w(\theta)}$, along with the monotonicity constraints $w'(\theta) > 0$ and $y'(\theta) \geq 0$.[28] An issue with this envelope condition is that $w'(\theta)$ is not only a function of the control variable $l(\theta)$, but also of its derivative $l'(\theta)$.[29] We thus define $b(\theta) = l'(\theta)$ and maximize (37) subject to (38), the envelope condition

$$V'(\theta) = v'(l(\theta))\, l(\theta)\frac{\omega_1[\theta, l(\theta)f_\theta(\theta), \mathscr{L}] + [l(\theta)f'_\theta(\theta) + b(\theta)f_\theta(\theta)]\,\omega_2[\theta, l(\theta)f_\theta(\theta), \mathscr{L}]}{w(\theta)}, \tag{40}$$

and

$$l'(\theta) = b(\theta). \tag{41}$$

This is now a well-defined optimal control problem with two state variables, $V(\theta)$ and $l(\theta)$, and one control variable, $b(\theta)$ (see Seierstad and Sydsaeter (1986)).

3.1.2 Optimal tax schedule

We now characterize the solution to the government problem (37), (38), (40), (41).

Proposition 4. *For any $\theta \in \Theta$, the optimal marginal tax rate $\tau(\theta)$ of type θ satisfies*

$$\frac{\tau(\theta)}{1-\tau(\theta)} = \left(1 + \frac{1}{\varepsilon_{l,1-\tau}(\theta)}\right)\frac{\mu(\theta)}{\lambda w(\theta) f_w(w(\theta))} - \frac{\int_\Theta [\mu(x)v'(l(x))l(x)]'\,\gamma(x,\theta)\,dx}{\lambda(1-\tau(\theta))y(\theta)f_\theta(\theta)}, \tag{42}$$

[28] As is standard in the literature, we assume that these monotonicity conditions are satisfied and verify them ex-post in our numerical simulations.

[29] From (3), we have $w'(\theta) = \omega_1(\theta, L(\theta), \mathscr{L}) + \omega_2(\theta, L(\theta), \mathscr{L})L'(\theta)$, where $L'(\theta) = l'(\theta)\, f_\theta(\theta) + l(\theta)\, f'_\theta(\theta)$.

where $\mu(\theta) = \lambda \int_{\theta}^{\bar{\theta}} (1 - g_\theta(x)) f_\theta(x)dx$ *is the Lagrange multiplier on the envelope condition (40) of type* θ*.*

Proof. See Appendix B.4.1. □

The first term on the right hand side of (42) is the formula for optimal taxes we would obtain in partial equilibrium (see Diamond (1998)). The second term captures the effect of a variation in type-θ labor supply on each incentive constraint.

To gain intuition, consider first a model with two types, as in Stiglitz (1982). In this case, a decrease in the tax on the high type increases her labor supply, which in turn decreases her wage rate; conversely a higher tax on the low type raises her wage. This compression of the pre-tax wage distribution in general equilibrium is beneficial as it relaxes the downward incentive constraint (39) of the high type. Therefore, optimal taxes are more regressive in general equilibrium: the optimal marginal tax rate on the high type is negative (rather than zero) and it is higher for the low type than in partial equilibrium.

Now suppose that there is a discrete set of types, $\Theta = \{\theta_i\}_{i=1,..,N}$. An increase in the wage $w(\theta_i)$ reduces the gap between $w(\theta_{i+1})$ and $w(\theta_i)$, and therefore relaxes the downward incentive constraint (39) of type θ_{i+1}. Denoting by $\mu(\theta_{i+1})$ the Lagrange multiplier on this constraint, this perturbation has a welfare impact equal to

$$\mu(\theta_{i+1})v'\left(l(\theta_i)\frac{w(\theta_i)}{w(\theta_{i+1})}\right)\frac{l(\theta_i)}{w(\theta_{i+1})}dw(\theta_i) > 0. \tag{43}$$

On the other hand, the perturbation increases the gap between $w(\theta_i)$ and $w(\theta_{i-1})$, and therefore tightens the downward incentive constraint of type θ_i, which has a welfare impact equal to

$$-\mu(\theta_i)v'\left(l(\theta_{i-1})\frac{w(\theta_{i-1})}{w(\theta_i)}\right)l(\theta_{i-1})\frac{w(\theta_{i-1})}{w(\theta_i)^2}dw(\theta_i) < 0. \tag{44}$$

Whether the perturbation increases or decreases welfare depends on whether (43) or (44) is larger in magnitude. First, this depends on the relative size of the Lagrange multipliers, $\mu(\theta_{i+1}) - \mu(\theta_i)$, that is, on which incentive constraint binds more strongly. In the continuous-type limit $\Theta = \left[\underline{\theta}, \bar{\theta}\right]$, this yields the term $\mu'(x)$ in (42). Second, it depends on the relative change in the values of deviating, captured by the difference between $v'\left(l(\theta_{i-1})\frac{w(\theta_{i-1})}{w(\theta_i)}\right)l(\theta_{i-1})\frac{w(\theta_{i-1})}{w(\theta_i)^2}$ and $v'\left(l(\theta_i)\frac{w(\theta_i)}{w(\theta_{i+1})}\right)\frac{l(\theta_i)}{w(\theta_{i+1})}$. In the continuous-type limit, this yields the term $[v'(l(x))l(x)]'$ in (42).

The optimal tax formula (42) is similar to those derived by Rothschild and Scheuer (2013, 2014) and Ales, Kurnaz, and Sleet (2015). Rothschild and Scheuer (2013) provide a condition in a two-sector model under which general equilibrium forces lead to a more regressive tax schedule. As we show in Section 3.3, we can also derive results about the shape of marginal tax rates if the production function is CES. In the next section, we characterize the solution to the government problem using a variational approach, i.e., by optimizing directly over tax schedules. This allows us to shed a new light on the economic forces underlying the design of optimal taxes.

3.2 Variational approach

In this section we propose an alternative characterization of the optimal tax schedule, using the variational techniques introduced in Section 2. Consider as in Saez (2001) a tax reform $h_1(y)$ that consists of an increase in the marginal tax rate at income level y^*. This raises the tax bill of incomes $y \geq y^*$ by a uniform lump-sum amount. Formally, this perturbation is the step function $h_1(y) = \mathbb{I}_{\{y \geq y^*\}}$, so that $h_1'(y) = \delta_{y^*}(y)$ is the Dirac delta function at y^*. Denote by θ^* the type that earns $y(\theta^*) = y^*$ in the baseline equilibrium.

Now, consider a counteracting perturbation $h_2(y)$ such that the general equilibrium effects of $h_1 + h_2$ on labor supply (given by (22)) are the same as the partial equilibrium effects of h_1. This implies in particular that the labor supply of all types $\theta \neq \theta^*$ stays constant.[30] An optimality condition is then obtained by imposing that the tax reform $h_1 + h_2$ has no first-order effect on social welfare.[31]

The key step consists in using our key integral equation (21) to derive the counteracting perturbation h_2 that cancels out the general equilibrium effects on labor supply. We do so formally in Appendix B.4.2. Here we provide a sketch of the derivation. In order to leave the labor supply of individual $\theta \neq \theta^*$ unchanged, i.e., $\mathrm{d}\hat{l}(\theta, h_1 + h_2) = 0$, we must ensure that the net-of-tax-wage $(1 - T'(y(\theta)))\, w(\theta)$ re-

[30]The labor supply response to this perturbation is given by $d \ln l(\theta) = -\frac{\tilde{\varepsilon}_{l,1-\tau}(y^*)}{1-T'(y^*)} \mathbb{I}_{\{\theta = \theta^*\}}$.

[31]Note that our tax reform approach in Section 2 immediately delivers a characterization of the optimum tax schedule, by equating the welfare effects of any tax reform h to zero in (29). The reason we analyze instead a combination of two perturbations in this section is that, setting to zero the sum of the effects of h_1 and h_2 leads to a substantially simpler formula for the optimum than we would obtain from the simple analysis of h_1, as all of the general equilibrium effects on labor supply that we analyzed in Section 2.1 are by construction equal to zero. This also implies that the numerical computation of the optimum is much simpler, as we do not have to solve the integral equation (21) for each h_1-reform at each iteration.

mains unchanged, i.e., that the tax change exactly compensates the wage change:

$$d \ln \left(1 - T'\left(y\left(\theta\right)\right)\right) = -d \ln w\left(\theta\right). \tag{45}$$

To find the reform h_2 that satisfies (45), we first compute the right hand side. The combination of perturbations $h_1 + h_2$ induces a change in $\ln w\left(\theta\right)$ given by

$$d \ln w\left(\theta\right) = \int_\Theta \gamma\left(\theta, \theta'\right) d \ln l\left(\theta'\right) d\theta' = -\left(y'\left(\theta^*\right)\right)^{-1} \gamma\left(\theta, \theta^*\right) \frac{\tilde{\varepsilon}_{l,1-\tau}\left(\theta^*\right)}{1 - T'\left(y\left(\theta^*\right)\right)}, \tag{46}$$

where the second equality follows from the fact that, by construction, the labor supply of all agents $\theta' \neq \theta^*$ remains unchanged.[32] If the baseline tax schedule T were linear, the exogenous perturbation h_2 would impact the net-of-tax rate at $y\left(\theta\right)$ simply by $-h_2'\left(y\left(\theta\right)\right)$, so that (45) would require $\frac{h_2'(y(\theta))}{1-T'(y(\theta))} = -\left(y'\left(\theta^*\right)\right)^{-1} \gamma\left(\theta, \theta^*\right) \frac{\tilde{\varepsilon}_{l,1-\tau}(\theta^*)}{1-T'(y(\theta^*))}$. Instead, for a nonlinear tax schedule, the change in income $l\left(\theta\right) dw\left(\theta\right)$ induced by the perturbation h_2 also triggers an indirect endogenous marginal tax rate adjustment. The relation between $d \ln \left(1 - T'\right)$ and h_2' is thus given by

$$d \ln \left(1 - T'\left(y\left(\theta\right)\right)\right) = -\frac{h_2'\left(y\left(\theta\right)\right) + T''\left(y\left(\theta\right)\right) l\left(\theta\right) dw\left(\theta\right)}{1 - T'\left(y\left(\theta\right)\right)}. \tag{47}$$

Equations (45), (46), and (47) then imply that the counteracting perturbation is given by:[33]

$$h_2'\left(y\right) = -\frac{\tilde{\varepsilon}_{l,1-\tau}\left(y^*\right)}{1 - T'\left(y^*\right)} \left(y'\left(\theta^*\right)\right)^{-1} \left(1 - T'\left(y\right) - yT''\left(y\right)\right) \gamma\left(y, y^*\right). \tag{48}$$

We can now derive the welfare effect of $h_1 + h_2$ and equate it to zero to characterize the optimal tax schedule.

[32] The term $\left(y'\left(\theta^*\right)\right)^{-1}$ results from a change of variables, from incomes to types, in the integral.

[33] This counteracting tax perturbation is able to undo all of the general equilibrium effects on labor supply thanks to Assumption 1, i.e., agents who earn the same income y have the same wage and identical wage elasticities $\bar{\gamma}\left(y, y^*\right)$. In a multidimensional screening model with heterogeneity conditional on income, as in Rothschild and Scheuer (2014), the perturbation h_2 would not be a flexible enough tool to cancel out everyone's labor supply response to h_1.

Proposition 5. *The optimal tax schedule satisfies*[34]

$$\begin{aligned}\frac{T'(y^*)}{1-T'(y^*)} =& \frac{1}{\tilde{\varepsilon}_{l,1-\tau}(y^*)}\left(1-\bar{g}_y(y^*)\right)\left(\frac{1-F_y(y^*)}{y^* f_y(y^*)}\right) \\ &- \int_{\mathbb{R}_+}\left[\left(1-\bar{g}_y(y)\right)\left(\frac{1-F_y(y)}{y^* f_y(y^*)}\right)\left(\frac{1-T'(y)}{1-T'(y^*)}\right) y\right]' \frac{\gamma(y,y^*)}{y'(\theta^*)} dy,\end{aligned} \tag{49}$$

where $\bar{g}_y(y) \equiv \int_y^\infty g_y(y') \frac{f_y(y')}{1-F_y(y)} dy'$ *is the average marginal social welfare weight above income* y*.*

Proof. See Appendix B.4.2 and B.4.3. In Appendix B.4.4, we prove that formula (49) is equivalent to the expression (42) derived in the mechanism design problem. □

The first line of (49) is the partial equilibrium formula for optimal tax rates (Saez (2001)): the marginal tax rate at income y^* is proportional to the inverse elasticity of labor supply $\tilde{\varepsilon}_{l,1-\tau}(y^*)$ and to the hazard rate of the income distribution $\frac{1-F_y(y^*)}{y^* f_y(y^*)}$, and is decreasing in the average marginal social welfare weight $\bar{g}(y^*)$.[35]

We now provide a heuristic proof of the second line of (49). Recall that the perturbation $h_1 + h_2$ has two distinct effects on individuals $\theta \neq \theta^*$: first, their wage changes by (46); second, the labor supply effects of this wage change are neutralized by raising their marginal tax rate by (48).

Effects of the wage change. The change (say, the decrease) in θ's wage by $dw(\theta)$ induces a change in income $y(\theta)$ by $l(\theta)\, dw(\theta)$, which in turn impacts individual welfare and government revenue. First, individual θ loses a share $1 - T'(y(\theta))$ of her income change $w(\theta)\, l(\theta)\, d\ln w(\theta)$, which affects her utility and hence aggregate welfare by

$$\begin{aligned}&\int_\Theta g_y(y(\theta))\left[\left(1-T'(y(\theta))\right) y(\theta)\right] \times d\ln w(\theta)\, f_\theta(\theta)\, d\theta \\ =& -\frac{\tilde{\varepsilon}_{l,1-\tau}(y^*)}{1-T'(y^*)}\left(y'(\theta^*)\right)^{-1}\int_{\mathbb{R}_+} g_y(y)\left(1-T'(y)\right)\gamma(y,y^*)\, y f_y(y)\, dy,\end{aligned}$$

[34] We show in Appendix B.4.5 that a simple transformation of this equation allows us to rewrite it as an integral equation in $1 - T'(\cdot)$ that can be analyzed, both theoretically and numerically, using the same techniques as in Section 2.

[35] The intuition for each of these terms is the same as in formula (32). Note that the density of incomes f_y (as well as the labor supply elasticity) in formula (49) is now evaluated at the optimum tax code.

by construction of the marginal social welfare weights and equation (46). The other share $T'(y(\theta))$ accrues to the government, whose revenue changes by

$$-\frac{\tilde{\varepsilon}_{l,1-\tau}(y^*)}{1-T'(y^*)}\left(y'(\theta^*)\right)^{-1}\int_{\mathbb{R}_+} T'(y)\,\gamma(y,y^*)\,y f_y(y)\,dy.$$

Since the production function has constant returns to scale, Euler's homogeneous function theorem reads $\int_{\mathbb{R}_+}\gamma(y,y^*)\,y f_y(y)\,dy = 0$ for all y^*. We can thus replace $T'(y)$ by $1-T'(y)$ in the integrand of the previous equation. Summing the welfare and revenue effects, we obtain that social welfare changes by

$$\frac{\tilde{\varepsilon}_{l,1-\tau}(y^*)}{1-T'(y^*)}\left(y'(\theta^*)\right)^{-1}\int_{\mathbb{R}_+}\left(1-g_y(y)\right)\left(1-T'(y)\right)\gamma(y,y^*)\,y f_y(y)\,dy. \tag{50}$$

Effects of the counteracting marginal tax rate change. The compensating marginal tax rate decrease at income $y(\theta)$, $h_2'(y(\theta))$, which ensures that the labor supply of individual θ does not change, uniformly decreases the tax bill of individuals with income above $y(\theta)$. This lowers government revenue by

$$\begin{aligned} & h_2'(y(\theta))\left(1-F(y(\theta))\right) \\ = & -\frac{\tilde{\varepsilon}_{l,1-\tau}(y^*)}{1-T'(y^*)}\left(y'(\theta^*)\right)^{-1}\left(1-T'(y)-yT''(y)\right)\gamma(y,y^*)\left(1-F(y(\theta))\right). \end{aligned} \tag{51}$$

This revenue gain is valued $\left(1-\bar{g}(y(\theta))\right)h_2'(y(\theta))\left(1-F(y(\theta))\right)$ by the planner, where $\bar{g}(y)$ is the average marginal social welfare weight on individuals with income above y. Summing over incomes $y(\theta)$ gives the change in social welfare from the counteracting perturbation.

Collecting (50) and (51) implies that the general equilibrium forces impact social welfare by

$$\begin{aligned} \frac{\tilde{\varepsilon}_{l,1-\tau}(y^*)}{1-T'(y^*)}\left(y'(\theta^*)\right)^{-1}\int_{\mathbb{R}_+}\Big[& \left(1-g_y(y)\right)\left(1-T'(y)\right)y f_y(y) \\ & -\left(1-\bar{g}_y(y)\right)\left(1-T'(y)-yT''(y)\right)\left(1-F(y)\right)\Big]\gamma(y,y^*)\,dy. \end{aligned}$$

Note finally that the term in brackets is the derivative of the function $\left(1-\bar{g}_y(y)\right)\times\left(1-T'(y)\right)y\times\left(1-F(y)\right)$, leading to formula (49).

We end this section by discussing the relationship between our paper and Scheuer and Werning (2015, 2016). They analyze a general equilibrium extension of Mirrlees (1971) and prove a neutrality result: in their model, the optimal tax formula is the same as in partial equilibrium, even though they consider a more general production function than Mirrlees (1971).[36] The key modeling difference between our framework and theirs is the following. In theirs, all the agents produce the same input with different productivities θ. Denoting by $\lambda(\theta) = \theta l(\theta)$ the agent's production of that input (i.e., her efficiency units of labor), the aggregate production function then maps the distribution of λ into output. In equilibrium, a nonlinear price (earnings) schedule $p(\cdot)$ emerges such that an agent who produces λ units earns income $p(\lambda)$, irrespective of her underlying productivity θ. Hence, when an (atomistic) individual θ provides more effort $l(\theta)$, her income moves along the non-linear schedule $l \mapsto p(\theta \times l)$; e.g., in their superstars model with a convex equilibrium earnings schedule, her income increases faster than linearly. By contrast, in our framework, different values of θ index different inputs in the aggregate production function; for each of these inputs, there is one specific price (wage) $w(\theta)$, and hence a linear earnings schedule $l \mapsto w(\theta) \times l$. Therefore, when an individual θ provides more effort $l(\theta)$, her income increases linearly, as her wage remains constant (since her sector θ doesn't change). In their framework, Scheuer and Werning (2015, 2016) show that the general equilibrium effects exactly cancel out at the optimum tax schedule, even though they would of course be non-zero in the characterization of the incidence effects of tax reforms around a suboptimal tax code. In our framework, as in those of Stiglitz (1982); Rothschild and Scheuer (2014); Ales et al. (2015), these general equilibrium forces are also present at the optimum.

3.3 CES production function

3.3.1 Optimal marginal tax rates

We show in the Appendix that formula (49) dramatically simplifies in the case of a CES production function. Using definition (11) to disentangle the own-wage effects $\bar{\bar{\gamma}}(y^*, y^*)$ and the cross-wage effects $\bar{\gamma}(y, y^*)$ in the integrand, we prove the following corollary:

[36] The policy implications can nevertheless be different. For instance, in Scheuer and Werning (2015), the relevant earnings elasticity in the formula written in terms of the observed income distribution is higher due to the superstar effects.

Corollary 4. *Assume that the production function is CES with elasticity of substitution $\sigma > 0$. Then the optimal marginal tax rate at income y^* is given by*

$$\frac{T'(y^*)}{1-T'(y^*)} = \frac{1}{\tilde{E}_{l,1-\tau}(y^*)} \left(1-\bar{g}_y(y^*)\right)\left(\frac{1-F_y(y^*)}{y^* f_y(y^*)}\right) + \frac{g_y(y^*)-1}{\sigma}. \tag{52}$$

Proof. See Appendix B.4.5. □

Equation (52) highlights the two main differences between the partial equilibrium and the general equilibrium optimal taxes. The first novel effect is driven by the own-wage effects $\bar{\bar{\gamma}}(y^*, y^*)$, and is captured by the inverse of the elasticity $\tilde{E}_{l,1-\tau}(y^*)$, replacing $\tilde{\varepsilon}_{l,1-\tau}(y^*)$ in the partial equilibrium formula. This general equilibrium correction satisfies $\tilde{E}_{l,1-\tau}(y^*) < \tilde{\varepsilon}_{l,1-\tau}(y^*)$ and hence tends to raise optimal marginal tax rates. This is because increasing the marginal tax rate at y^* leads individuals θ^* to lower their own labor supply, which raises their wage and thus dampens the initial behavioral response.

The second novel effect, which works in the opposite direction, is driven by the cross-wage effects $\bar{\gamma}(y, y^*)$, and is captured by the term $\frac{1-g(y^*)}{\sigma}$ in (52). The lower labor supply at type θ^* lowers the wage, and hence the labor supply, of all the other types $\theta \neq \theta^*$. Suppose that the government values the welfare of individuals θ^* less than average, i.e., $g(y^*) < 1$.[37] This negative externality on θ induced by the behavior of θ^* implies that the cost of raising the marginal tax rate at y^* is higher than in partial equilibrium, which tends to lower the optimal tax rate. Conversely, the government gains by raising the optimal tax rates of individuals y^* whose welfare is valued more than average, i.e., $g(y^*) > 1$. This induces these agents to work less and earn a higher wage, which makes them strictly better off, at the expense of the other individuals in the economy, whose wage decreases.

Formula (52) generalizes the partial equilibrium optimal tax formula of Diamond (1998) and Saez (2001) to a CES production function, and the general equilibrium analysis of Stiglitz (1982) to the workhorse framework of taxation, with a continuum of types and arbitrary nonlinear taxes. It allows us to go beyond the purely qualitative insights obtained in the two-type framework and make operational the theory of optimal tax design in general equilibrium. In particular, we show in the next two subsections how the key results obtained in partial equilibrium are affected, namely,

[37] Since lump-sum transfers are available to the government, the average marginal social welfare weight in the economy is equal to 1.

the characterization of the optimal top tax rate and the U-shape of marginal tax rates.

3.3.2 Top tax rate

In this subsection we derive the implications of equation (52) for the optimal top tax rate.

Corollary 5. *Assume that the production function is CES with elasticity of substitution $\sigma > 0$ and that the elasticity of labor supply is constant and equal to ε. Assume moreover that in the data, incomes are Pareto distributed at the tail with coefficient α, and that the top marginal tax rate that applies to these incomes is constant. Assume finally that the marginal social welfare weights at the top converge to $\bar{g}$. Then the optimal top tax rate satisfies*

$$\frac{\tau_{top}}{1-\tau_{top}} = \frac{1-\bar{g}}{\alpha\varepsilon} + \frac{1-\bar{g}}{\alpha\sigma} - \frac{1-\bar{g}}{\sigma}. \tag{53}$$

In particular, we have $\frac{\tau_{top}}{1-\tau_{top}} < \frac{1-\bar{g}}{\alpha\varepsilon}$, where $\frac{1-\bar{g}}{\alpha\varepsilon}$ is the optimal top tax rate in the partial equilibrium model.

Proof. See Appendix B.4.6. The non-trivial part of the proof consists in showing that for a CES production function, if the income distribution has a Pareto tail in the data, then it has the same Pareto tail at the optimum, even though the wage distribution is endogenous. □

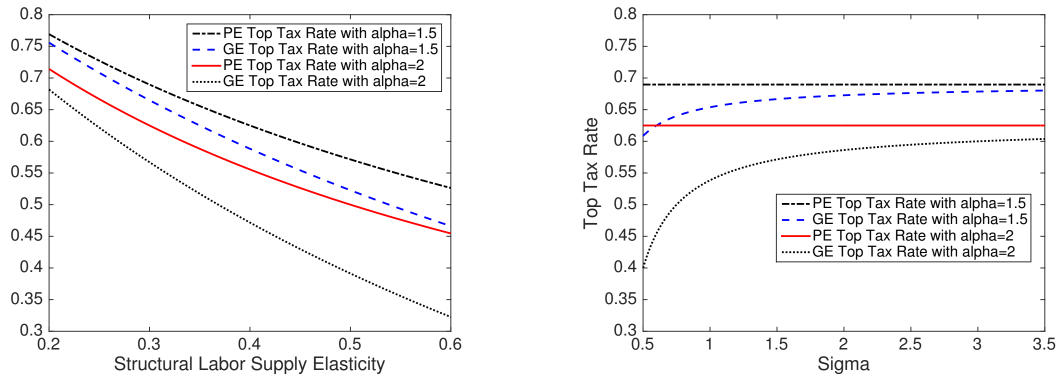
Formula (53) generalizes the familiar top tax rate result of Saez (2001) to a CES production function. There is one new sufficient statistic, the elasticity of substitution between skills in production σ, that is no longer restricted to be infinite. This proposition implies a strictly lower top marginal tax rate than in partial equilibrium.[38] Back-of-the envelope calculations of the optimal top tax rate illustrate the power of this formula. Suppose that $\bar{g} = 0$, $\alpha = 2$, $\varepsilon = 1/2$, and $\sigma = 1.5$.[39] We immediately

[38] Note that perhaps surprisingly, the own-wage effects imply that the general equilibrium correction is stronger (so that the top tax rate should be reduced by a larger amount, relative to the partial equilibrium benchmark) when inequality at the top is lower, i.e. when α is higher, and when the average welfare weight $\bar{g}$ at the top is lower. Moreover, since the solution for τ_{top} is concavely increasing in the right hand side of (53), this formula implies that the optimal top tax rate is more sensitive to the labor supply elasticity ε than in partial equilibrium.

[39] These values are meant to be only illustrative, but they are in the ballpark of those estimated in the empirical literature. See our calibration in Section 4 below.

obtain that the optimal tax rate on top incomes is equal to $\tau_{top,PE} = 50\%$ in partial equilibrium, and falls to $\tau_{top} = 40\%$ once general equilibrium forces are taken into account. Figure 1 shows more comprehensive comparative statics.[40]

Figure 1: Top tax rate: $\sigma = 1.4$ (left panel) and $\varepsilon = 0.33$ (right panel)



3.3.3 U-shape of optimal marginal tax rates

We finally analyze the impact of general equilibrium on the familiar U-shape of optimal tax rates derived by Diamond (1998) in the partial equilibrium framework.

Corollary 6. *Assume that the production function is CES with elasticity of substitution $\sigma > 0$ and that the social planner is Rawlsian, i.e. $g_y(y(\theta)) = 0$ for all $\theta > \underline{\theta}$. If the partial equilibrium optimal tax formula*[41]

$$\frac{1}{\tilde{E}_{l,1-\tau}(\theta)} \frac{1 - F_y(y(\theta))}{y(\theta) f_y(y(\theta))}$$

implies a U-shaped pattern of marginal tax rates, then the additional term $-1/\sigma$ leads to a general equilibrium correction for $T'(\cdot)$ that is also U-shaped.

Proof. In this case the cross wage term $(g_y(y^*) - 1)/\sigma$ in (52) is constant and negative for all $\theta > \underline{\theta}$. Given that the marginal tax rate is concavely increasing in the right hand side of ((52)), the result immediately follows. □

Corollary 6 follows straightforwardly from equation (52) and is a simple but economically important result as it implies that the general equilibrium forces tend to

[40] See also Green and Phillips (2015) who study quantitatively the size of the optimal top tax rate in a two-sector model.

[41] Note that this "partial equilibrium" formula is computed given the wage distribution at the optimum general equilibrium tax schedule.

reinforce the U-shape of the optimal marginal tax rates. This implies in particular a more pronounced dip, so that the marginal tax rate on intermediate incomes falls by more than that on top incomes. Therefore, while the two-type model of Stiglitz (1982) suggests that optimal tax rates should be more regressive (i.e., higher at the bottom and lower at the top) than in partial equilibrium, we see here that this insight must be qualified once we extend it to an environment with a continuum of incomes: taxes should be more regressive for incomes below the bottom of the U, but more progressive in the region where optimal marginal tax rates were already increasing. We illustrate this result quantitatively in Section 4.3.

4 Numerical simulations

In this section we provide a quantitative analysis of our results. We parametrize our model in Section 4.1. We then analyze the incidence of tax reforms around the U.S. tax code in Section 4.2, and compute the optimal tax schedule in Section 4.3. We reconcile these two sets of results in Section 4.4 and show how our results extend to a Translog production function in Section 4.5.

4.1 Calibration

We assume that preferences are quasilinear with isoelastic disutility of labor, $U(c,l) = c - l^{1+\frac{1}{\varepsilon}} / \left(1+\frac{1}{\varepsilon}\right)$, with $\varepsilon = 0.33$ in our benchmark calibration (Chetty, Guren, Manoli, and Weber, 2011). To match the U.S. yearly earnings distribution, we assume that $f_y(\cdot)$ is log-normal with mean 10 and variance 0.95 up to income $y = \$150,000$, above which we append a Pareto distribution with coefficient $\alpha = 1.5$ (Diamond and Saez, 2011) (see Appendix C.1 for details). We take a CRP tax schedule defined in (31) with $p = 0.151$ and $\tau = -3$ (Heathcote, Storesletten, and Violante, 2014). We can then calibrate the resulting wage distribution from the agents' first-order conditions (1) (Saez, 2001). Assuming a CES production function as a benchmark with values for the elasticity of substitution $\sigma \in \{0.6, 1.4, 3.1\}$, taken respectively from Dustmann, Frattini, and Preston (2013), Katz and Murphy (1992) and Borjas (2003), and Card and Lemieux (2001),[42] we finally obtain the values of $a(\theta)$ in (13) using the values of

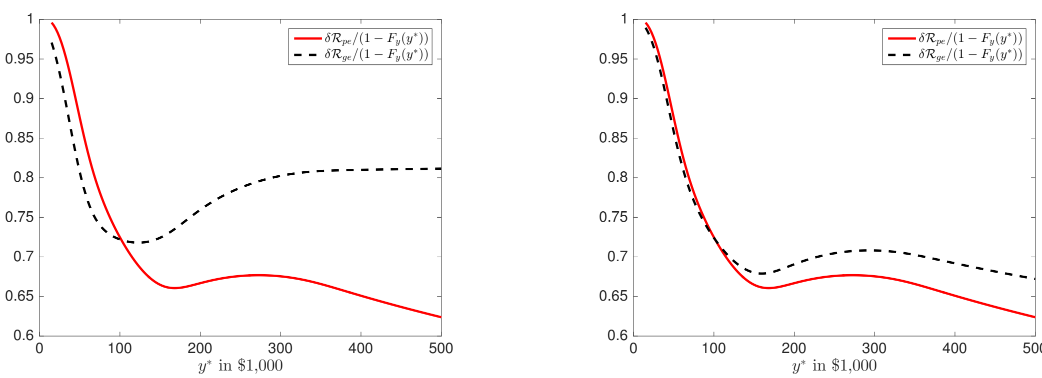
[42]Dustmann, Frattini, and Preston (2013) is the most relevant study for our analysis, as they do not classify workers according to their education level (as opposed to Katz and Murphy (1992), Card and Lemieux (2001), Borjas (2003)) but according to their position in the wage distribution,

$l(\theta)$ and $w(\theta)$ at each income level.

4.2 Tax incidence

We first study the incidence of tax reforms on government revenue, based on the theoretical analysis of Section 2. The reforms that we consider are those that increase the marginal tax rate at a single income level y^* (Saez, 2001). We thus plot the values of $\mathrm{d}\mathscr{R}(y^*)/(1-F_y(y^*))$, as defined in (28), as a function of the income y^* where tax rates are perturbed.[43] Figure 2 illustrates the result for $\sigma = 0.6$ and $\sigma = 3.1$ (blacked dashed curves), and for the partial equilibrium case $\sigma = \infty$ (red bold curves). In line with the result of Corollary 3, we observe that raising the marginal tax rates for intermediate and high incomes (starting from about \$100,000) is more desirable, in terms of government revenue, when the general equilibrium effects are taken into account, while the opposite holds for low income levels. Hence the partial equilibrium model significantly underestimates the revenue benefits (i.e., Rawlsian welfare with $\underline{\theta} = 0$) of increasing the progressivity of the tax code.[44]

Figure 2: Tax incidence: $\sigma = 0.6$ (Dustmann, Frattini, and Preston (2013), left panel) and $\sigma = 3.1$ (Card and Lemieux (2001), right panel)



i.e., by type θ.

[43] A value of 0.7, say, at a given income level y^*, means that for each additional dollar of tax revenue mechanically levied by the statutory increase in the marginal tax rate at y^*, the government effectively gains only 70 cents, while 30 cents are lost through the behavioral responses of indviduals to the tax reform. That is, the marginal excess burden of this tax reform is 30%.

[44] In Appendix C.3 we extend this analysis to the welfare effects for a utilitirian planner.

4.3 Optimal taxation

Throughout this section we consider a Rawlsian social objective, which consists of maximizing the utility of agents with zero income, i.e., the lump-sum component of the tax schedule. In Appendix C.3 we consider the case of a utilitarian planner; the results are very similar.

4.3.1 Partial equilibrium benchmark

We start by constructing a policy-relevant partial equilibrium benchmark to analyze how the normative prescriptions for optimal taxes are affected by the general equilibrium effects. We define the marginal tax rates that a partial equilibrium planner would set from Diamond (1998), using the same data to calibrate the model, and making the same assumptions about the utility function, but wrongly assuming that the wage distribution is exogenous:

$$\frac{\tau_{PE}(\theta)}{1-\tau_{PE}(\theta)} = \left(1+\frac{1}{\varepsilon}\right)\frac{1-F_w^d(w_d(\theta))}{f_w^d(w_d(\theta))w_d(\theta)}, \tag{54}$$

where $F_w^d(w_d(\theta))$ is the wage distribution in the data, i.e., obtained from the observed income distribution and the first-order conditions (1).[45]

4.3.2 Simulation results

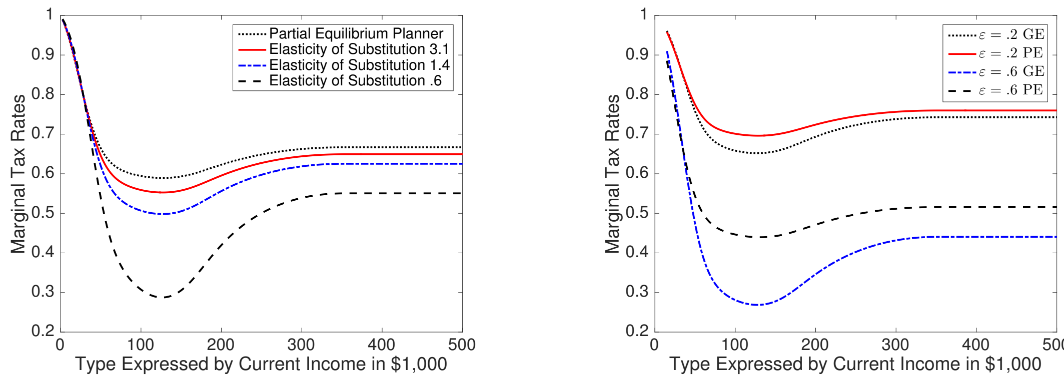
The role of the elasticity of substitution. The right panel of Figure 3 plots the optimal marginal tax rates as a function of types[46] for three different values of the elasticity of substitution, and for the partial equilibrium planner defined in (54). The latter schedule has a familiar U-shape (Diamond, 1998; Saez, 2001). In line with our theoretical results of Section 3.3, the top tax rate is lower in general equilibrium and decreasing with σ. Moreover, the optimal marginal tax rates are reduced by an even larger amount at income levels close to the bottom of the U (around \$100,000),

[45] An alternative would consist of defining a self-confirming policy equilibrium as suggested by Rothschild and Scheuer (2013, 2016), where the resulting wage distribution would be consistent with the planner's beliefs.

[46] The scale on the horizontal axis on the left panel is measured in income; e.g., the value of the optimal marginal tax rate at the notch \$100,000 is that of a type θ who earns an income $y(\theta) = \$100,000$ in the calibration to the U.S data. The income that this type earns in the optimal allocation is generally different (see the right panel). In Appendix C.2, we plot the optimal tax schedules as a function of income at the optimum.

and are higher at low income levels (below \$40,000).[47] This implies that the U-shape obtained in partial equilibrium is reinforced by the general equilibrium considerations.

Figure 3: Optimal marginal tax rates as a function of σ and ε



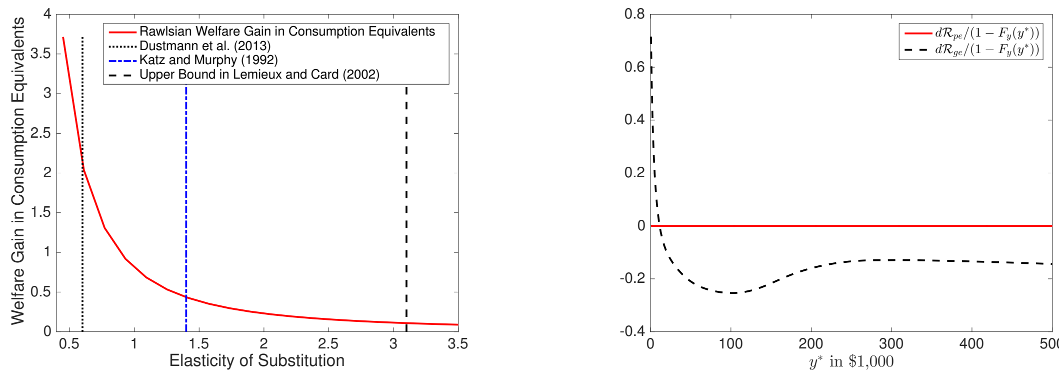
The role of the elasticity of labor supply. The right panel of Figure 3 plots the optimal marginal tax rates for two different values of the elasticity of labor supply ε with $\sigma = 1.4$, and compares them with the partial equilibrium optimum (54). In line with our theoretical results of Section 3.3, we observe that (i) the magnitude of the correction due to general equilibrium forces is increasing in the structural labor supply elasticity; and (ii) optimal marginal tax rates in general equilibrium are more sensitive to the value of the elasticity.

Welfare gains. The left panel of Figure 4 plots the welfare gains of moving from the optimal partial equilibrium taxes to the optimal general equilibrium taxes, as a function of the elasticity of substitution σ. These gains are expressed in consumption equivalent, which (given our welfare criterion) corresponds to a uniform increase in the lump sum transfer. Naturally these gains are decreasing in σ and converge to zero as $\sigma \to \infty$. For low values of σ, they can be as high as 3.5 percent, and they remain nontrivial for the whole range of plausible values of the elasticity.[48]

[47] Since the partial equilibrium tax rates are already very high at those low income levels, the general equilibrium corrections are quantitatively very small (at most 1.8 percentage points).

[48] For our specifications of the Translog production function, these gains are in the same ballpark, see Appendix C.5.

Figure 4: Welfare gains (left panel) and Tax incidence for PE optimum (right panel)



Proposition 6 and Figure 10 in Appendix B.4.7 propose a theoretical and quantitative "general equilibrium wedge accounting" decomposition of the difference between the partial equilibrium benchmark (54) and the general equilibrium optimum (52). It highlights the roles of the three key general equilibrium corrections coming from the own-wage effects, the cross-wage effects, and the endogeneity of the hazard rates of the wage distribution.

4.4 Reconciling the tax incidence and optimal taxation results

As we already discussed in Section 2.3, the results of Sections 4.2 and 4.3 may seem contradictory at first sight. On the one hand, Corollary 3 and Figure 2 show that the general equilibrium forces make it desirable, for a Rawlsian planner, to increase the marginal tax rates on high incomes, relative to the partial equilibrium benchmark, if we start from the current U.S. tax system. On the other hand, Corollary 5 and Figure 3 implies that the optimal tax rates on high incomes are strictly lower, for high incomes, than in the partial equilibrium optimum (54).

To reconcile these two sets of results, we compute in the right panel of Figure 4 the incidence of the tax reforms $h(\cdot) = \delta_{y^*}(\cdot)$, starting from the optimal partial equilibrium tax schedule (54).[49] This graph shows that when the low-income marginal tax rates are high (as in the partial equilibrium optimum) rather than low (as in the U.S. tax code), the general equilibrium forces call for lower tax rates for intermediate and high incomes, and higher marginal tax rates for low incomes. Importantly, this

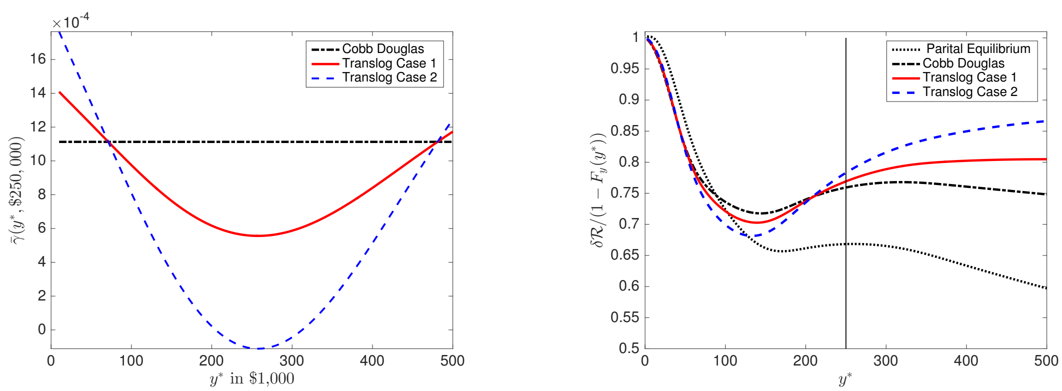
[49]In this figure, we set $\sigma = 1.4$ and $\varepsilon = 0.33$. The red bold line plots the effects of the tax reform on government revenue in partial equilibrium (see (54)). These effects are uniformly equal to zero by construction.

graph helps understanding the more pronounced U-shaped in the optimum through the lens of the tax incidence analysis. Starting from the partial equilibrium optimum, the gains from perturbing the marginal tax rates are U-shaped and negative, except at the very bottom of the income distribution.

4.5 Translog Production Function

We now consider the case of a Translog production function, where the elasticities of substitution are distance-dependent (see Section 1.4). Our primary goal is to compare the policy implications of this production function[50] to those obtained with a CES technology. We start by plotting in Figure 5 the values of the wage elasticities $\bar{\gamma}(y, y^*)$ for $y^* = \$250,000$. The dashed-dotted black line is the benchmark Cobb-Douglas case, for which $\bar{\beta}(\theta, \theta^*) = 0$ for all θ: it shows that a change in the labor supply of θ^* has the same impact, in percentage terms, on the wage of every agent $\theta \neq \theta^*$. The bold red and dashed blue curves show the wage elasticities $\bar{\gamma}(y, y^*)$ for two different Translog specifications (19). We chose the function $\alpha(\theta)$ such that for all y^*, $\bar{\gamma}(y^*, y^*)$ is equal to 50% (resp., -10%) of its value in the Cobb Douglas case in Case 1 (resp., Case 2). For both cases we choose the parameter s such that $\bar{\bar{\gamma}}(y^*, y^*) = -1$ for $y^* = \$250,000$, which implies that the cross-wage effects from a labor supply change of an individual who earns \$250,000 are the same on average as in the Cobb-Douglas case.[51]

Figure 5: $\bar{\gamma}(\theta, \theta^*)$ with $y(\theta^*) = \$250,000$ (left panel) and tax incidence (right panel)



[50]Here we show two such examples, and two more in Appendix C.5.

[51]In Appendix C.5, we consider cases where s is chosen such that $\bar{\bar{\gamma}}(y^*, y^*) = -1$ for $y^* = \$80,000$ as an alternative specification. In all cases, we set the parameter c to 0.5. Varying this parameter only slightly influences the results.

In the right panel of Figure 5 we evaluate the incidence of the tax reforms $\delta_{y^*}(\cdot)$ starting from a CRP tax schedule, as we did in Figure 2 for a CES technology. This graph not only confirms, but actually reinforces, the results obtained in Proposition 3 and Figure 2 regarding the benefits on government revenue of increasing the progressivity of the tax schedule. The ideal comparison here is the value at $y^* = \$250,000$ (represented by a vertical line), for which the average wage effects are the same as in the Cobb Douglas case,[52] which allows us to isolate the effects of the distance-dependence. We see that the revenue gains from raising the marginal tax rate at this income level are higher when the production function is Translog. Moreover, the stronger the distance-dependence of the cross-wage effects (i.e., going from the bold red to the dashed blue curves), the higher the revenue gains from raising the progressivity, relative to the partial equilibrium benchmark. For incomes $y^* > \$250,000$, the difference between the Translog and CES cases is even larger. In Appendix C.5, we confirm these insights for different specifications of the Translog production function. We moreover plot the optimal tax rates and show that the U-shape pattern is again more pronounced than in partial equilibrium, confirming our results of Section 4.3.

5 Conclusion

In this paper we jointly study the incidence and the optimal design of nonlinear income taxes in general equilibrium with a continuum of endogenous wages. Our methodological contribution is to characterize this problem using the variational approach and the theory of integral equations. The analysis of tax incidence delivers novel economic insights about the optimum, and reveals that the implications of general equilibrium can be reversed when considering reforms of suboptimal tax codes. The revenue gains from increasing the progressivity of the U.S. tax schedule are larger, and the U-shape of optimal marginal tax rates is more pronounced, than in partial equilibrium.

[52] That is, the own-wage effect is $\bar{\bar{\gamma}}(\theta^*, \theta^*) = -1$, and hence, by Euler's theorem, the average cross-wage effects $\int_\Theta \bar{\gamma}(\theta, \theta^*)\, y(\theta)\, f_\theta(\theta)\, d\theta$ is the same as in the CES case.

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Appendix

A A primer on tax incidence

In this section we summarize some of the classical results on tax incidence in a framework with one good, two factors of production (typically labor and capital, see e.g. (Kotlikoff and Summers, 1987; Salanie, 2011)), and linear taxation of these two factors. We consider instead the case where the two factors are high-skilled and low-skilled labor, as our primary goal is to study a model with a continuum of labor inputs. The proofs of this section are gathered in Section B.1.

A.1 Equilibrium

Individuals have preferences over consumption c and labor supply l given by $U(c,l) = u(c - v(l))$, where the functions u and v are twice continuously differentiable and strictly increasing, u is concave, and v is strictly convex. In particular, note that there are no income effects on labor supply. There are two skill levels $\theta_1 < \theta_2$, with respective masses F_1 and $F_2 = 1 - F_1$.

An individual of type θ_i earns a wage $w(\theta_i)$, which she takes as given. She chooses her labor supply $l(\theta_i)$ and earns taxable income $y(\theta_i) = w(\theta_i) l(\theta_i)$. The government levies linear income taxes τ_i on income of type $i \in \{1,2\}$. Individual θ_i therefore solves the following problem:

$$l(\theta_i) = \arg\max_{l \in \mathbb{R}_+} u\left[(1-\tau_i) w(\theta_i) l - v(l)\right].$$

The optimal labor supply $l(\theta_i)$ chosen by an individual θ_i is thus the solution to the first-order condition:

$$w(\theta_i) = \frac{v'(l(\theta_i))}{(1-\tau_i)}. \tag{55}$$

We denote by $\mathcal{U}(\theta_i)$ the indirect utility function attained by individual θ_i. Finally, the total amount of labor supplied by individuals of type θ_i is denoted by $L(\theta_i) \equiv l(\theta_i) F_i$. Equation (55) defines a decreasing "supply curve" in the plane (w,l).

There is a continuum of identical firms that produce output using both types of labor θ_1, θ_2. The resulting aggregate production function $\mathscr{F}$ is defined as:

$$\mathscr{Y} = \mathscr{F}(L(\theta_1), L(\theta_2)).$$

We assume that the production function has constant returns to scale. The representative firm chooses the (relative) demand of inputs (labor of each type), taking as given the wages $w(\theta_i)$, to maximize its profit

$$\max_{L_1, L_2} \left[\mathscr{F}(L_1, L_2) - \sum_{i=1}^{2} w(\theta_i) L_i\right].$$

As a result, in equilibrium the firm earns no profits and the wage $w(\theta)$ is equal to the marginal productivity of the type-θ labor, i.e.,

$$w(\theta_i) = \mathscr{F}_i'(L(\theta_1), L(\theta_2)), \tag{56}$$

for all $i \in \{1,2\}$, where $\mathscr{F}_i'$ denotes the partial derivative of the production function with respect to its i^{th} variable.

The equilibrium wages and quantities are derived by equating (56), which are infinitely elastic demand curves and (55) which are increasing supply curves.

A.2 Elasticity concepts

We first define the structural labor supply elasticity $\varepsilon_i = \varepsilon_{l,1-\tau}(\theta_i)$ as the change in the labor supply of individuals of type θ_i when the tax rate on their income, τ_i, is increased.[53] We let, for $i \in \{1,2\}$,

$$\varepsilon_i = \left.\frac{\partial \ln l(\theta_i)}{\partial \ln(1-\tau_i)}\right|_{w(\theta)} = \frac{v'(l(\theta_i))}{l(\theta_i)\, v''(l(\theta_i))}. \tag{57}$$

Note that this is a elasticity in partial equilibrium, since the change in labor supply is computed for a constant individual wage.

Next, we define the wage elasticity $\gamma_{ij} = \gamma(\theta_i, \theta_j)$ as the effect of a marginal increase in the labor supply of type θ_j, $L(\theta_j)$, on the wage of type θ_i, $w(\theta_i)$, keeping the labor supply of type $k \neq j$, $L(\theta_k)$, constant. That is, for $(i,j) \in \{1,2\}^2$,

$$\gamma_{ij} = \frac{\partial \ln w(\theta_i)}{\partial \ln L(\theta_j)} = \frac{L(\theta_j)\, \mathscr{F}_{i,j}''(L(\theta_1), L(\theta_2))}{\mathscr{F}_i'(L(\theta_1), L(\theta_2))}.$$

A.3 Tax incidence

To analyze the tax incidence problem in the simple two-type model laid out in this section, we start by deriving the general effects of arbitrary infinitesimal perturbations ("tax reforms") $(d\tau_1, d\tau_2)$ of the baseline tax system (τ_1, τ_2). Denote by $d\hat{l}_i = \frac{dl_i}{l_i}$ and $d\hat{w}_i = \frac{dw_i}{w_i}$ the implied percentage changes in labor supplies and wages induced by the tax reform. We show in the Appendix that we can express the labor supply effects as

$$\begin{pmatrix} d\hat{l}_1 \\ d\hat{l}_2 \end{pmatrix} = -\left[\begin{pmatrix} \frac{1}{\varepsilon_1} & 0 \\ 0 & \frac{1}{\varepsilon_2} \end{pmatrix} - \begin{pmatrix} \gamma_{11} & \gamma_{12} \\ \gamma_{21} & \gamma_{22} \end{pmatrix}\right]^{-1} \begin{pmatrix} \frac{d\tau_1}{1-\tau_1} \\ \frac{d\tau_2}{1-\tau_2} \end{pmatrix} \tag{58}$$

and the wage effects as

$$\begin{pmatrix} d\hat{w}_1 \\ d\hat{w}_2 \end{pmatrix} = \begin{pmatrix} \gamma_{11} & \gamma_{12} \\ \gamma_{21} & \gamma_{22} \end{pmatrix} \begin{pmatrix} d\hat{l}_1 \\ d\hat{l}_2 \end{pmatrix}. \tag{59}$$

[53]Note that with the utility function U that we consider, the Marshallian (uncompensated) and Hicksian (compensated) elasticities are identical.

The impact on individual welfare is given by

$$dU_i = (1 - \tau_i)\, y_i U_i' \left[-\frac{d\tau_i}{1-\tau_i} + d\hat{w}_i \right]. \tag{60}$$

Finally the impact on government revenue is given by

$$d\mathscr{R} = \sum_{i=1}^{2} (F_i y_i)\, d\tau_i + \sum_{i=1}^{2} (F_i \tau_i y_i)\, d\hat{l}_i + \sum_{i=1}^{2} (F_i \tau_i y_i)\, d\hat{w}_i. \tag{61}$$

Equation (58) shows that the changes in labor supplies $\left(d\hat{l}_1, d\hat{l}_2\right)$, induced by the changes in marginal tax rates $(d\tau_1, d\tau_2)$, are given by the sum of: (i) the partial equilibrium effects, captured by the diagonal matrix of labor supply elasticities ε_1 and ε_2 in equation (58); and (ii) the general equilibrium effects, coming from the fact that the initial labor supply changes trigger own- and cross-changes in wages, which in turn affect labor supplies, etc. This infinite sequence of feedback effects between equilibrium wages and labor supplies is captured by the inverse of the matrix of wage elasticities $(\gamma_{ij})_{i,j\in\{1,2\}}$ in equations (58) and (59).[54]

Equation (60) shows that individual utilities are affected in two ways. First, their income (and hence utility) is directly affected by the changes in taxes $d\tau_i$, holding wages and labor supply fixed. Second, their income is indirectly affected, holding labor supply fixed. Note that the endogenous change in labor supply has no first-order impact on utility by the envelope theorem.

The first term in (61) is the mechanical effect of the perturbation, i.e. the change in government revenue due to the change in tax rates, assuming that both labor supply behavior and wages remain constant. The second term is the behavioral effect, due to the change in labor supplies which induces a change in government revenue proportional to the marginal tax rate τ_i. The third term is the general equilibrium effect, coming from the fact that perturbing the marginal tax rates impacts individual wages and hence government revenue directly. The analysis so far thus shows that the size and the sign of the impacts of a particular tax reform $(d\tau_1, d\tau_2)$ depends generally on the baseline tax rates (τ_1, τ_2), individual preferences (through the elasticities $(\varepsilon_1, \varepsilon_2)$), and the production technology (through the elasticities $(\gamma_{11}, \gamma_{12}, \gamma_{21}, \gamma_{22})$).

We now focus on the particularly simple case where one of the factors, say the labor supply of type θ_2, is in fixed supply, i.e. $\varepsilon_2 = 0$. More precisely, we consider the limit of the previous expression as the elasticity of labor supply of type θ_2 is positive but small, so that l_2 is always paid its marginal productivity. Suppose in addition that we perturb only the tax on the elastic factor, $l(\theta_1)$. This amounts to reducing the model we have analyzed so far to its partial equilibrium limit, where the effects of the tax on θ_1 affect only the quantity of that factor (since θ_2 is in fixed supply). In this case, we show in the Appendix by directly inverting the matrix in (58) and letting $\varepsilon_2 \to 0$ that we obtain the following familiar result (see equation (2.6) in Kotlikoff and Summers (1987) and

[54] Using the matrix equality $(\mathbf{I} - \mathbf{A})^{-1} = \sum_{n=0}^{\infty} \mathbf{A}^n$ (assuming that the norm of the matrix is smaller than 1), we can solve equation (58) to express $\left(d\hat{l}_1, d\hat{l}_2\right)$ as an infinite sum, the economic interpretation of which is the sequence of feedback effects on labor supply coming from the general equilibrium wage effects. This expression parallels (22) in Section 2 for the case of a continuum of labor inputs and general nonlinear tax reforms.

Section 1.1.1 in Salanie (2011)): the impact on the wage of types θ_1 and θ_2 is given by

$$d\hat{w}_1 = \frac{\varepsilon_1}{\varepsilon_1 - \gamma_{11}^{-1}} \frac{d\tau_1}{1-\tau_1} \quad \text{and} \quad d\hat{w}_2 = -\frac{w_1}{w_2} d\hat{w}_1. \tag{62}$$

Moreover, the impact on the labor supply of type θ_1 is given by

$$d\hat{l}_1 = \frac{\varepsilon_1 \gamma_{11}^{-1}}{\varepsilon_1 - \gamma_{11}^{-1}} \frac{d\tau_1}{1-\tau_1} \quad . \tag{63}$$

Formula (62) shows that the effect on the own wage w_1 of an increase in the tax rate τ_1 on factor θ_1, depends on the elasticity of supply of factor 1, ε_1, and the elasticity of the demand of factor 1, $-1/\gamma_{11} > 0$.[55] This expression implies that the gross wage increases all the more that demand is less elastic relative to supply. In other words, when labor supply of a given type is much less elastic than labor demand ($\varepsilon_1 \ll \left|\gamma_{11}^{-1}\right|$), the cost of labor hardly changes and workers of type θ_1 bear almost the full burden.

B Proofs

B.1 Proofs of Section A

We start by evaluating the incidence effects of a general perturbation $(d\tau_1, d\tau_2)$ of the baseline tax system.

Proof of equations (58) to (61). The perturbed individual first-order condition of individual θ_i writes

$$(1-\tau_i - d\tau_i)\, \tilde{w}_i = v'\left(l_i + dl_i\right),$$

where the perturbed wage rate $\tilde{w}_i$ satisfies, to a first-order in $(d\tau_1, d\tau_2)$,

$$\begin{aligned}
&\tilde{w}_i\left(\left(l_1+dl_1\right)F_1, \left(l_2+dl_2\right)F_2\right) - w_i\left(l_1F_1, l_2F_2\right)\\
=&\mathscr{F}_i'\left(\left(l_1+dl_1\right)F_1, \left(l_2+dl_2\right)F_2\right) - \mathscr{F}_i'\left(l_1F_1, l_2F_2\right)\\
=&\sum_{n=1}^{2} \mathscr{F}_{i,n}''\left(l_1F_1, l_2F_2\right)F_n dl_n = \sum_{n=1}^{2} \frac{l_nF_n\mathscr{F}_{i,n}''}{\mathscr{F}_i'}\mathscr{F}_i' \frac{dl_n}{l_n} = \sum_{n=1}^{2} w_i \gamma_{i,n} d\hat{l}_n.
\end{aligned}$$

Denoting by $d\hat{w}_i \equiv \frac{\tilde{w}_i - w_i}{w_i}$, this is a linear system of two equations (indexed by $i \in \{1,2\}$) with two unknowns $(d\hat{w}_1, d\hat{w}_2)$, which can be rewritten as

$$d\hat{w}_i = \gamma_{i,1} d\hat{l}_1 + \gamma_{i,2} d\hat{l}_2, \ \forall i \in \{1,2\},$$

[55] $1/\gamma_{11}$ is the standard elasticity of demand when the supply of factor θ_2 is fixed, as it is constructed as the change in the labor supply $l(\theta_1)$ induced by a change in the wage $w(\theta_1)$, keeping $l(\theta_2)$ fixed.

which immediately leads to the matrix form (59).

We thus get, to a first order in $(d\tau_1, d\tau_2)$,

$$(1-\tau_i)\,w_i + (1-\tau_i)\,(\tilde{w}_i - w_i) - w_i d\tau_i = v'\,(l_i) + v''\,(l_i)\,dl_i,$$

i.e., using the first-order condition at the baseline tax system,

$$\begin{aligned} w_i d\tau_i &= (1-\tau_i)\,(\tilde{w}_i - w_i) - v''\,(l_i)\,dl_i = (1-\tau_i)\sum_{n=1}^{2} w_i\gamma_{i,n} d\hat{l}_n - l_i v''\,(l_i)\,d\hat{l}_i \\ &= \sum_{n=1}^{2}\left[(1-\tau_i)\,w_i\gamma_{i,n} - l_i v''\,(l_i)\,\mathbb{I}_{\{n=i\}}\right] d\hat{l}_n. \end{aligned}$$

Using the formula (57) for the labor supply elasticity, we can rewrite this equation as

$$\begin{aligned} \frac{d\tau_i}{1-\tau_i} &= \sum_{n=1}^{2}\left[\gamma_{i,n} - \frac{l_i v''\,(l_i)}{(1-\tau_i)\,w_i}\mathbb{I}_{\{n=i\}}\right] d\hat{l}_n = \sum_{n=1}^{2}\left[\gamma_{i,n} - \frac{l_i v''\,(l_i)}{v'\,(l_i)}\mathbb{I}_{\{n=i\}}\right] d\hat{l}_n \\ &= \sum_{n=1}^{2}\left[\gamma_{i,n} - \frac{1}{\varepsilon_i}\mathbb{I}_{\{n=i\}}\right] d\hat{l}_n. \end{aligned}$$

This is a linear system of two equations (indexed by $i \in \{1,2\}$) with two unknowns $(d\hat{l}_1, d\hat{l}_2)$, which can be rewritten as

$$\begin{aligned} \left(\gamma_{1,1} - \frac{1}{\varepsilon_1}\right) d\hat{l}_1 + \gamma_{1,2} d\hat{l}_2 &= \frac{d\tau_1}{1-\tau_1} \\ \gamma_{2,1} d\hat{l}_1 + \left(\gamma_{2,2} - \frac{1}{\varepsilon_2}\right) d\hat{l}_2 &= \frac{d\tau_2}{1-\tau_2}, \end{aligned}$$

and hence, in matrix form, as

$$\left[\begin{pmatrix} \gamma_{11} & \gamma_{12} \\ \gamma_{21} & \gamma_{22} \end{pmatrix} - \begin{pmatrix} \frac{1}{\varepsilon_1} & 0 \\ 0 & \frac{1}{\varepsilon_2} \end{pmatrix}\right]\begin{pmatrix} d\hat{l}_1 \\ d\hat{l}_2 \end{pmatrix} = \begin{pmatrix} \frac{d\tau_1}{1-\tau_1} \\ \frac{d\tau_2}{1-\tau_2} \end{pmatrix}.$$

Assuming that the matrix in square brackets on the left hand side is invertible, we immediately obtain (58).

The utility of individual θ_i changes, to a first-order in $(d\tau_1, d\tau_2)$, by

$$\begin{aligned} dU_i \equiv& U\left((1-\tau_i - d\tau_i)\,\tilde{w}_i\,(l_i + dl_i) - v\,(l_i + dl_i)\right) - U\left((1-\tau_i)\,w_i l_i - v\,(l_i)\right) \\ =& \left[-w_i l_i d\tau_i + (1-\tau_i)\,l_i\,(\tilde{w}_i - w_i) + (1-\tau_i)\,w_i dl_i - v'\,(l_i)\,dl_i\right] U'\left((1-\tau_i)\,w_i l_i - v\,(l_i)\right) \\ =& (1-\tau_i)\,w_i l_i\left[-\frac{d\tau_i}{1-\tau_i} + d\hat{w}_i + d\hat{l}_i - \frac{v'\,(l_i)}{(1-\tau_i)\,w_i} d\hat{l}_i\right] U'\left((1-\tau_i)\,w_i l_i - v\,(l_i)\right). \end{aligned}$$

But the individual's first order condition (55) implies $\frac{v'(l_i)}{(1-\tau_i)w_i} = 1$, so that we obtain (60).

Government revenue changes, to a first-order in $(d\tau_1, d\tau_2)$, by

$$\begin{aligned} d\mathscr{R} &\equiv \sum_{i=1}^{2} \left[(\tau_i + d\tau_i)\, \tilde{w}_i \,(l_i + dl_i) - \tau_i w_i l_i \right] F_i = \sum_{i=1}^{2} \left[w_i l_i d\tau_i + \tau_i l_i \left(\tilde{w}_i - w_i \right) + \tau_i w_i dl_i \right] F_i \\ &= \sum_{i=1}^{2} \tau_i w_i l_i \left(\frac{d\tau_i}{\tau_i} + d\hat{w}_i + d\hat{l}_i \right) F_i, \end{aligned}$$

which implies (61).

$\square$

Next we focus on the partial equilibrium case.

Proof of equations (62) and (63). With two types, we can easily invert explicitly the matrix in equation (58) to obtain

$$\begin{pmatrix} d\hat{l}_1 \\ d\hat{l}_2 \end{pmatrix} = \begin{pmatrix} \gamma_{11} - \frac{1}{\varepsilon_1} & \gamma_{12} \\ \gamma_{21} & \gamma_{22} - \frac{1}{\varepsilon_2} \end{pmatrix}^{-1} \begin{pmatrix} \frac{d\tau_1}{1-\tau_1} \\ \frac{d\tau_2}{1-\tau_2} \end{pmatrix} = \frac{\begin{pmatrix} \gamma_{22} - \frac{1}{\varepsilon_2} & -\gamma_{12} \\ -\gamma_{21} & \gamma_{11} - \frac{1}{\varepsilon_1} \end{pmatrix} \begin{pmatrix} \frac{d\tau_1}{1-\tau_1} \\ \frac{d\tau_2}{1-\tau_2} \end{pmatrix}}{\left(\gamma_{11} - \frac{1}{\varepsilon_1} \right) \left(\gamma_{22} - \frac{1}{\varepsilon_2} \right) - \gamma_{12}\gamma_{21}}.$$

Consider a perturbation of τ_2 only, i.e., $d\tau_2 = 0$. Then the previous expression implies that the changes in labor supplies are given by

$$\begin{pmatrix} d\hat{l}_1 \\ d\hat{l}_2 \end{pmatrix} = \frac{\begin{pmatrix} \gamma_{22} - \frac{1}{\varepsilon_2} \\ -\gamma_{21} \end{pmatrix} \frac{d\tau_1}{1-\tau_1}}{\left(\gamma_{11} - \frac{1}{\varepsilon_1} \right) \left(\gamma_{22} - \frac{1}{\varepsilon_2} \right) - \gamma_{12}\gamma_{21}} = \frac{\begin{pmatrix} \varepsilon_2 \gamma_{22} - 1 \\ -\varepsilon_2 \gamma_{21} \end{pmatrix} \frac{d\tau_1}{1-\tau_1}}{\left(\gamma_{11} - \frac{1}{\varepsilon_1} \right) \left(\varepsilon_2 \gamma_{22} - 1 \right) - \varepsilon_2 \gamma_{12}\gamma_{21}}.$$

Now take the limit as type-θ_2 labor supply becomes inelastic, i.e., $\varepsilon_2 \to 0$. We get

$$\begin{pmatrix} d\hat{l}_1 \\ d\hat{l}_2 \end{pmatrix} \to \frac{\frac{d\tau_1}{1-\tau_1}}{\frac{1}{\varepsilon_1} - \gamma_{11}} \begin{pmatrix} -1 \\ 0 \end{pmatrix}, \text{ i.e., } \begin{cases} d\hat{l}_1 = & -\frac{1}{\frac{1}{\varepsilon_1} - \gamma_{11}} \frac{d\tau_1}{1-\tau_1}, \\ d\hat{l}_2 = & 0, \end{cases}$$

which shows (63). Finally, (59) implies that

$$d\hat{w}_1 = \gamma_{1,1} d\hat{l}_1 = -\frac{\gamma_{1,1}}{\frac{1}{\varepsilon_1} - \gamma_{11}} \frac{d\tau_1}{1-\tau_1}$$

and $d\hat{w}_2 = \gamma_{2,1} d\hat{l}_1 = \frac{\gamma_{2,1}}{\gamma_{1,1}} d\hat{w}_1$. The Euler's homogeneous equation theorem states that

$$w_1 L_1 + w_2 L_2 = F\left(L_1, L_2\right)$$

so that

$$w_1 \gamma_{1,1} + w_2 \gamma_{2,1} = L_1 \frac{\partial w_1}{\partial L_1} + L_2 \frac{\partial w_2}{\partial L_1} = 0,$$

and hence $d\hat{w}_2 = -\frac{w_1}{w_2} d\hat{w}_1$.

$\square$

B.2 Proofs of Section 1

B.2.1 Proof of equations (6), (7), and (8)

We first derive the expressions for the labor supply elasticities.

Proof. We first derive the labor supply elasticity along the linear budget constraint (6). Rewrite the first-order condition (1) as

$$v'(l(\theta)) = (1-\tau(\theta))\, w(\theta),$$

where $\tau(\theta) = T'(w(\theta)\, l(\theta))$ is the marginal tax rate of agent θ. The first-order effect of perturbing the marginal tax rate $\tau(\theta)$ by $d\tau_\theta$ on the labor supply $l(\theta)$, in partial equilibrium (i.e., keeping $w(\theta)$ constant), is obtained by a Taylor approximation of the first-order condition characterizing the perturbed equilibrium,

$$v'(l(\theta) + dl_\theta) = (1-\tau(\theta) - d\tau_\theta)\, w(\theta),$$

around the baseline equilibrium. We obtain

$$v'(l(\theta)) + v''(l(\theta))\, dl_\theta = (1-\tau(\theta))\, w(\theta) - w(\theta)\, d\tau_\theta = v'(l(\theta)) - \frac{v'(l(\theta))}{1-\tau(\theta)} d\tau_\theta,$$

and thus

$$\frac{dl_\theta}{l(\theta)} = -\frac{v'(l(\theta))}{l(\theta)\, v''(l(\theta))} \frac{d\tau_\theta}{1-\tau(\theta)},$$

which immediately leads equation (6).

Next we derive the labor supply elasticity along the non-linear budget constraint (6), keeping the wage constant. The perturbed individual first-order condition writes

$$v'(l(\theta) + dl_\theta) = [1 - T'(w(\theta)(l(\theta) + dl_\theta)) - d\tau_\theta]\, w(\theta).$$

A first-order Taylor expansion implies

$$v'(l(\theta)) + v''(l(\theta))\, dl_\theta = (1 - T'(w(\theta)\, l(\theta)))\, w(\theta) - T''(w(\theta)\, l(\theta))\, (w(\theta))^2\, dl_\theta - w(\theta)\, d\tau_\theta,$$

i.e.,

$$\begin{aligned}
\frac{dl_\theta}{l(\theta)} &= -\frac{1}{l(\theta)} \frac{w(\theta)}{v''(l(\theta)) + T''(w(\theta)\, l(\theta))\, w_\theta^2} d\tau_\theta \\
&= -\frac{\frac{v'(l(\theta))}{l(\theta)}}{(1 - T'(w(\theta)\, l(\theta)))\, v''(l(\theta)) + v'(l(\theta))\, w(\theta)\, T''(w(\theta)\, l(\theta))} d\tau_\theta \\
&= \frac{\varepsilon_{l,1-\tau}(\theta)\,(1 - T'(w(\theta)\, l(\theta)))}{1 - T'(w(\theta)\, l(\theta)) + \varepsilon_{l,1-\tau}(\theta)\, w(\theta)\, l(\theta)\, T''(w(\theta)\, l(\theta))} \frac{d(1-\tau(\theta))}{1 - T'(w(\theta)\, l(\theta))},
\end{aligned}$$

which yields equation (7).

Finally, we derive the partial equilibrium labor supply elasticity with respect to the wage along the non-linear budget constraint (8). The perturbed individual first-order condition writes

$$v'\left(l\left(\theta\right)+dl_{\theta}\right)=\left[1-T'\left(\left(w\left(\theta\right)+dw_{\theta}\right)\left(l\left(\theta\right)+dl_{\theta}\right)\right)\right]\left(w\left(\theta\right)+dw_{\theta}\right).$$

A first-order Taylor expansion then implies

$$\begin{aligned}v'\left(l\left(\theta\right)\right)+v''\left(l\left(\theta\right)\right)dl_{\theta}=&\left(1-T'\left(w\left(\theta\right)l\left(\theta\right)\right)\right)w\left(\theta\right)-T''\left(w\left(\theta\right)l\left(\theta\right)\right)\left(w\left(\theta\right)\right)^{2}dl_{\theta}\\&-T''\left(w\left(\theta\right)l\left(\theta\right)\right)w\left(\theta\right)l\left(\theta\right)dw_{\theta}+\left(1-T'\left(w\left(\theta\right)l\left(\theta\right)\right)\right)dw_{\theta},\end{aligned}$$

i.e.,

$$\begin{aligned}\frac{dl_{\theta}}{l\left(\theta\right)}=&\frac{\left(1-T'\left(w\left(\theta\right)l\left(\theta\right)\right)-w\left(\theta\right)l\left(\theta\right)T''\left(w\left(\theta\right)l\left(\theta\right)\right)\right)w\left(\theta\right)}{v''\left(l\left(\theta\right)\right)+\left(w\left(\theta\right)\right)^{2}T''\left(w\left(\theta\right)l\left(\theta\right)\right)}\frac{dw_{\theta}}{w\left(\theta\right)l\left(\theta\right)}\\=&\frac{\left(1-T'\left(w\left(\theta\right)l\left(\theta\right)\right)-w\left(\theta\right)l\left(\theta\right)T''\left(w\left(\theta\right)l\left(\theta\right)\right)\right)\frac{v'\left(l\left(\theta\right)\right)}{l\left(\theta\right)v''\left(l\left(\theta\right)\right)}}{\left(1-T'\left(w\left(\theta\right)l\left(\theta\right)\right)\right)+w\left(\theta\right)l\left(\theta\right)\frac{v'\left(l\left(\theta\right)\right)}{l\left(\theta\right)v''\left(l\left(\theta\right)\right)}T''\left(w\left(\theta\right)l\left(\theta\right)\right)}\frac{dw_{\theta}}{w\left(\theta\right)},\end{aligned}$$

which implies (8).

□

B.2.2 Proofs for Examples 1 and 2

We first provide algebra details for the CES technology. It is straightforward to show that the cross-wage elasticities are given by (15). We have

$$\bar{\gamma}\left(\theta,\theta^{*}\right)=\left(1-\rho\right)\frac{a\left(\theta^{*}\right)L\left(\theta^{*}\right)^{\rho}}{\int_{\Theta}a\left(x\right)L\left(x\right)^{\rho}dx},$$

which implies, for all $\theta\in\Theta$,

$$\int_{\Theta}\bar{\gamma}\left(\theta,\theta^{*}\right)d\theta^{*}=1-\rho=\frac{1}{\sigma}.$$

Moreover, denoting interchangeably by $\bar{\gamma}\left(y,y^{*}\right)\equiv\bar{\gamma}\left(\theta,\theta^{*}\right)$ where $y=y\left(\theta\right)$ and $y^{*}=y\left(\theta^{*}\right)$, Euler's homogeneous function theorem writes

$$\int_{\Theta}\gamma\left(\theta,\theta^{*}\right)y\left(\theta\right)f_{\theta}\left(\theta\right)d\theta=0,$$

which can be rewritten as

$$\int_{\Theta}\bar{\gamma}\left(\theta,\theta^{*}\right)y\left(\theta\right)f_{\theta}\left(\theta\right)d\theta+\bar{\bar{\gamma}}\left(\theta^{*},\theta^{*}\right)y\left(\theta^{*}\right)f_{\theta}\left(\theta^{*}\right)=0,$$

or equivalently,

$$\int_{\mathbb{R}_{+}}\bar{\gamma}\left(y,y^{*}\right)yf_{y}\left(y\right)dy-\left(1-\rho\right)y^{*}f_{y}\left(y^{*}\right)\frac{dy\left(\theta^{*}\right)}{d\theta}=0.$$

Since $\bar{\gamma}(y, y^*) = \bar{\gamma}(\theta, \theta^*)$ does not depend on y (or θ), this implies

$$(1-\rho)\frac{a(\theta^*) L(\theta^*)^\rho}{\int_\Theta a(x) L(x)^\rho dx} \int_{\mathbb{R}_+} y f_y(y)\, dy - (1-\rho) y^* f_y(y^*) \frac{dy(\theta^*)}{d\theta} = 0,$$

i.e.,

$$\frac{a(\theta^*) L(\theta^*)^\rho}{\int_\Theta a(x) L(x)^\rho dx} = \frac{y^* f_y(y^*)}{\int_{\mathbb{R}_+} y f_y(y)\, dy} \frac{dy(\theta^*)}{d\theta}.$$

Substituting in expression (15), we hence get

$$\bar{\gamma}(y, y(\theta^*)) = (1-\rho) \frac{y(\theta^*) f_y(y(\theta^*))}{\int_{\mathbb{R}_+} y f_y(y)\, dy} \frac{dy(\theta^*)}{d\theta}. \tag{64}$$

We now provide the algebra details for the Translog production function.

Proof. First, we show that if the conditions θ, θ', $\int_\Theta a(\theta')\, d\theta' = 1$, $\bar{\beta}(\theta, \theta') = \bar{\beta}(\theta', \theta)$, and $\bar{\bar{\beta}}(\theta, \theta) = -\int_\Theta \bar{\beta}(\theta, \theta')\, d\theta'$ hold, then the production function (16) has constant returns to scale. Indeed, we can then write

$$\begin{aligned}\ln \mathscr{F}(\lambda \mathscr{L}) = & \ln \mathscr{F}(\mathscr{L}) + \left(\int_\Theta \alpha_\theta d\theta\right) \ln \lambda + \frac{1}{2} (\ln \lambda)^2 \left(\int_\Theta \bar{\bar{\beta}}_{\theta,\theta} d\theta + \int_{\Theta \times \Theta} \bar{\beta}_{\theta,\theta'} d\theta d\theta'\right) \\ & + \ln \lambda \left(\int_\Theta \alpha_\theta d\theta + \int_\Theta \bar{\bar{\beta}}_{\theta,\theta} \ln L(\theta)\, d\theta + \int_{\Theta\times\Theta} \bar{\beta}_{\theta,\theta'} \ln L(\theta)\, d\theta d\theta'\right)\end{aligned}$$

which is equal to $\ln \mathscr{F}(\mathscr{L}) + \ln \lambda$.

Second, we derive expression (17) for the wage. We have

$$\begin{aligned}& \ln \mathscr{F}(\mathscr{L} + \mu \delta_{\theta^*}) - \ln \mathscr{F}(\mathscr{L}) \\ = & \int_\Theta \alpha_\theta \left[\ln(L(\theta) + \mu \delta_{\theta^*}(\theta)) - \ln L(\theta)\right] d\theta + \frac{1}{2} \int_\Theta \bar{\bar{\beta}}_{\theta,\theta} \left[\ln^2(L(\theta) + \mu \delta_{\theta^*}(\theta)) - \ln^2 L(\theta)\right] d\theta \\ & + \frac{1}{2} \int_{\Theta\times\Theta} \bar{\beta}_{\theta,\theta'} \left[\ln(L(\theta) + \mu \delta_{\theta^*}(\theta)) \ln(L(\theta') + \mu \delta_{\theta^*}(\theta')) - \ln(L(\theta)) \ln(L(\theta'))\right] d\theta d\theta'.\end{aligned}$$

A first-order Taylor approximation of the right-hand side as $\mu \to 0$ yields

$$\begin{aligned}& \ln \mathscr{F}(\mathscr{L} + \mu \delta_{\theta^*}) - \ln \mathscr{F}(\mathscr{L}) \\ \underset{\mu\to 0}{=} & \mu \int_\Theta \alpha_\theta \frac{1}{L(\theta)} \delta_{\theta^*}(\theta)\, d\theta + \mu \int_\Theta \bar{\bar{\beta}}_{\theta,\theta} \frac{\ln L(\theta)}{L(\theta)} \delta_{\theta^*}(\theta)\, d\theta \\ & + \frac{1}{2}\mu \int_{\Theta\times\Theta} \bar{\beta}_{\theta,\theta'} \left[\delta_{\theta^*}(\theta) \frac{\ln(L(\theta'))}{L(\theta)} + \delta_{\theta^*}(\theta') \frac{\ln(L(\theta))}{L(\theta')}\right] d\theta d\theta' + o(\mu) \\ = & \mu \left\{\frac{\alpha_{\theta^*}}{L(\theta^*)} + \bar{\bar{\beta}}_{\theta^*,\theta^*} \frac{\ln L(\theta^*)}{L(\theta^*)} + \frac{1}{2} \int_\Theta \bar{\beta}_{\theta^*,\theta'} \frac{\ln(L(\theta'))}{L(\theta^*)} d\theta' + \frac{1}{2} \int_\Theta \bar{\beta}_{\theta,\theta^*} \frac{\ln(L(\theta))}{L(\theta^*)} d\theta\right\} + o(\mu),\end{aligned}$$

Thus the wage of type θ^* is equal to

$$
\begin{aligned}
w(\theta^*) &= \lim_{\mu\to 0}\frac{1}{\mu}\left[\mathscr{F}(\mathscr{L}+\mu\delta_{\theta^*})-\mathscr{F}(\mathscr{L})\right]\\
&=\frac{\mathscr{F}(\mathscr{L})}{L(\theta^*)}\left\{\alpha_{\theta^*}+\bar{\bar{\beta}}_{\theta^*,\theta^*}\ln L(\theta^*)+\int_\Theta \bar{\beta}_{\theta^*,\theta''}\ln\left(L(\theta'')\right)d\theta''\right\},
\end{aligned}
$$

where the second equality follows from the symmetry of $\left\{\bar{\beta}_{\theta,\theta'}\right\}_{(\theta,\theta')\in\Theta^2}$. Note that the same expression can be obtained heuristically by computing the derivative of $\mathscr{F}\left(\{L(\theta'')\}_{\theta''\in\Theta}\right)$ with respect to $L(\theta^*)$:

$$
w(\theta^*) = \frac{\mathscr{F}(\mathscr{L})}{L(\theta^*)}\frac{\partial \ln \mathscr{F}\left(\{L(\theta'')\}_{\theta''\in\Theta}\right)}{\partial \ln L(\theta^*)}.
$$

Third, we derive the cross-wage elasticities and own-wage elasticities (18). For simplicity we derive these formulas heuristically by directly evaluating the derivatives; they can be easily obtained rigorously following the same steps as above for the wages. For the cross-wage elasticities, suppose that $\theta\neq\theta'$. We have

$$
\begin{aligned}
\bar{\gamma}(\theta,\theta') &= \frac{\partial \ln\left(\mathscr{F}(\mathscr{L})/L(\theta)\right)}{\partial \ln L(\theta')}+\frac{\partial \ln\left\{\alpha_\theta+\bar{\bar{\beta}}_{\theta,\theta}\ln L(\theta)+\int_\Theta \bar{\beta}_{\theta,\theta''}\ln L(\theta'')\,d\theta''\right\}}{\partial \ln L(\theta')}\\
&=\left\{\alpha_{\theta'}+\bar{\bar{\beta}}_{\theta',\theta'}\ln L(\theta')+\int_\Theta \bar{\beta}_{\theta',\theta''}\ln L(\theta'')\,d\theta''\right\}+\frac{\bar{\beta}_{\theta,\theta'}}{\alpha_\theta+\bar{\bar{\beta}}_{\theta,\theta}\ln L(\theta)+\int_\Theta \bar{\beta}_{\theta,\theta''}\ln L(\theta'')\,d\theta''}\\
&=\left(\frac{w(\theta')L(\theta')}{\mathscr{F}(\mathscr{L})}\right)+\left(\frac{w(\theta)L(\theta)}{\mathscr{F}(\mathscr{L})}\right)^{-1}\bar{\beta}_{\theta,\theta'}.
\end{aligned}
$$

Similarly, the own wage elasticities are given by:

$$
\begin{aligned}
\bar{\bar{\gamma}}(\theta,\theta) &= \frac{\partial \ln\left(Y(\mathscr{L})/L(\theta)\right)}{\partial \ln L(\theta)}+\frac{\partial \ln\left\{\alpha_\theta+\bar{\bar{\beta}}_{\theta,\theta}\ln L(\theta)+\int_\Theta \bar{\beta}_{\theta,\theta''}\ln L(\theta'')\,d\theta''\right\}}{\partial \ln L(\theta)}-\bar{\gamma}(\theta,\theta)\\
&=-1+\frac{\bar{\bar{\beta}}_{\theta,\theta}}{\alpha_\theta+\bar{\bar{\beta}}_{\theta,\theta}\ln L(\theta)+\int_\Theta \bar{\beta}_{\theta,\theta''}\ln L(\theta'')\,d\theta''}=-1+\left(\frac{w(\theta)L(\theta)}{\mathscr{F}(\mathscr{L})}\right)^{-1}\bar{\bar{\beta}}_{\theta,\theta}.
\end{aligned}
$$

Finally, note that

$$
\begin{aligned}
\ln\left(\frac{w(\theta)}{w(\theta')}\right) &= \ln\left(\frac{L(\theta')}{L(\theta)}\right)+\ln\frac{\alpha_\theta+\bar{\bar{\beta}}_{\theta,\theta}\ln L(\theta)+\int_\Theta \bar{\beta}_{\theta,\theta''}\ln L(\theta'')\,d\theta''}{\alpha_{\theta^*}+\bar{\bar{\beta}}_{\theta',\theta'}\ln L(\theta')+\int_\Theta \bar{\beta}_{\theta',\theta''}\ln L(\theta'')\,d\theta''}\\
&=\ln\left(\frac{L(\theta')}{L(\theta)}\right)+\ln\frac{\alpha_\theta+\ln L(\theta)\left\{\bar{\bar{\beta}}_{\theta,\theta}+\int_\Theta \bar{\beta}_{\theta,\theta''}d\theta''\right\}+\int_\Theta \bar{\beta}_{\theta,\theta''}\left\{\ln L(\theta'')-\ln L(\theta)\right\}d\theta''}{\alpha_{\theta^*}+\ln L(\theta')\left\{\bar{\bar{\beta}}_{\theta',\theta'}+\int_\Theta \bar{\beta}_{\theta',\theta''}d\theta''\right\}+\int_\Theta \bar{\beta}_{\theta',\theta''}\left\{\ln L(\theta'')-\ln L(\theta')\right\}d\theta''}\\
&=\ln\left(\frac{L(\theta')}{L(\theta)}\right)+\ln\frac{\alpha_\theta+\int_\Theta \bar{\beta}_{\theta,\theta''}\ln\left(L(\theta'')/L(\theta)\right)d\theta''}{\alpha_{\theta'}+\int_\Theta \bar{\beta}_{\theta',\theta''}\ln\left(L(\theta'')/L(\theta')\right)d\theta''},
\end{aligned}
$$

so that the elasticities of substitution are given by:

$$
\begin{aligned}
&-\frac{1}{\sigma(\theta,\theta')}=\frac{\partial\ln(w(\theta)/w(\theta'))}{\partial\ln(L(\theta)/L(\theta'))}\\
&=-1-\frac{\bar{\beta}_{\theta,\theta'}}{\alpha_{\theta}+\int_{\Theta}\bar{\beta}_{\theta,\theta''}\ln\left(\frac{L(\theta'')}{L(\theta)}\right)d\theta''}-\frac{\bar{\beta}_{\theta',\theta}}{\alpha_{\theta'}+\int_{\Theta}\bar{\beta}_{\theta',\theta''}\ln\left(\frac{L(\theta'')}{L(\theta')}\right)d\theta''}\\
&=-1-\frac{\bar{\beta}_{\theta,\theta'}}{\alpha_{\theta}+\bar{\bar{\beta}}_{\theta,\theta}\ln L(\theta)+\int_{\Theta}\bar{\beta}_{\theta,\theta''}\ln L(\theta'')\,d\theta''}-\frac{\bar{\beta}_{\theta',\theta}}{\alpha_{\theta'}+\bar{\bar{\beta}}_{\theta',\theta'}\ln L(\theta')+\int_{\Theta}\bar{\beta}_{\theta',\theta''}\ln L(\theta'')\,d\theta''}\\
&=-1-\left[\left(\frac{w(\theta)L(\theta)}{\mathscr{F}(\mathscr{L})}\right)^{-1}+\left(\frac{w(\theta')L(\theta')}{\mathscr{F}(\mathscr{L})}\right)^{-1}\right]\bar{\beta}_{\theta,\theta'}.
\end{aligned}
$$

This concludes the derivations of the wage elasticities for a general Translog production function.

□

B.3 Proofs of Section 2

B.3.1 Proof of Lemma 1

Proof. Denote the perturbed tax function by $\tilde{T}(y)=T(y)+\mu h(y)$ (later we let $\mu\to 0$). Denote by $\mathrm{d}l(\theta,h)$ the Gateaux derivative of the labor supply of type θ in response to this perturbation, and let $\mathrm{d}L(\theta,h)\equiv\mathrm{d}l(\theta,h)f_{\theta}(\theta)$. The labor supply response $\mathrm{d}l(\theta,h)$ of type θ is given by the solution to the perturbed first-order condition:

$$
\begin{aligned}
0=&v'(l(\theta)+\mu\mathrm{d}l(\theta,h))\\
&-\{1-T'[\tilde{w}(\theta)\times(l(\theta)+\mu\mathrm{d}l(\theta,h))]-\mu h'[\tilde{w}(\theta)\times(l(\theta)+\mu\mathrm{d}l(\theta,h))]\}\,\tilde{w}(\theta),
\end{aligned}
\tag{65}
$$

where $\tilde{w}(\theta)$ is the perturbed wage schedule. Heuristically, $\tilde{w}(\theta)$ satisfies

$$
\begin{aligned}
&\tilde{w}(\theta)-w(\theta)=\mathscr{F}'_{L(\theta)}\left(\{(l(\theta')+\mu\mathrm{d}l(\theta',h))f_{\theta}(\theta')\}_{\theta'\in\Theta}\right)-\mathscr{F}'_{L(\theta)}\left(\{l(\theta')f_{\theta}(\theta')\}_{\theta'\in\Theta}\right)\\
&=\int_{\Theta}\mathscr{F}''_{L(\theta),L(\theta')}\mu\mathrm{d}l(\theta',h)f_{\theta}(\theta')\,d\theta'+o(\mu)=\mu\mathscr{F}'_{L(\theta)}\int_{\Theta}\frac{L(\theta')\mathscr{F}''_{L(\theta),L(\theta')}}{\mathscr{F}'_{L(\theta)}}\mathrm{d}\hat{l}(\theta',h)\,d\theta'+o(\mu),
\end{aligned}
$$

so that, to a first order as $\mu\to 0$,

$$
\tilde{w}(\theta)-w(\theta)=\mu w(\theta)\int_{\Theta}\gamma(\theta,\theta')\,\mathrm{d}\hat{l}(\theta',h)\,d\theta'. \tag{66}
$$

To derive this equation formally, denote by $\mathrm{d}\mathscr{L}(h)\in\mathcal{M}$ the measure on Θ defined by the Gateaux derivative of the measure $\mathscr{L}$ in the direction h. We have $\mathrm{d}\mathscr{L}(h)=\int_{\Theta}\mathrm{d}L(\theta',h)\,\delta_{\theta'}d\theta'$. We then have

$$
\tilde{w}(\theta)-w(\theta)=\omega\{\theta,L(\theta)+\mu\mathrm{d}L(\theta,h),\mathscr{L}+\mu\mathrm{d}\mathscr{L}(h)\}-\omega(\theta,L(\theta),\mathscr{L}).
$$

The right hand side of this equation is equal to

$$\begin{aligned}
&\omega_2\left(\theta, L\left(\theta\right), \mathscr{L}\right) \mu \mathrm{d}L\left(\theta, h\right) + \int_\Theta \left\{\omega\left(\theta, L\left(\theta\right), \mathscr{L} + \mu \mathrm{d}L\left(\theta', h\right) \delta_{\theta'}\right) - \omega\left(\theta, L\left(\theta\right), \mathscr{L}\right)\right\} d\theta' \\
=&\bar{\bar{\gamma}}\left(\theta, \theta\right) \frac{w\left(\theta\right)}{L\left(\theta\right)} \mu \mathrm{d}L\left(\theta, h\right) + \int_\Theta \bar{\gamma}\left(\theta, \theta'\right) \frac{w\left(\theta\right)}{L\left(\theta'\right)} \mu \mathrm{d}L\left(\theta', h\right) d\theta' + o\left(\mu\right) \\
=&\mu w\left(\theta\right) \left\{\bar{\bar{\gamma}}\left(\theta, \theta\right) \mathrm{d}\hat{l}\left(\theta, h\right) + \int_\Theta \bar{\gamma}\left(\theta, \theta'\right) \mathrm{d}\hat{l}\left(\theta', h\right) d\theta'\right\} + o\left(\mu\right),
\end{aligned}$$

which leads to expression (66).

Next, for any function g, we have, to a first order as $\mu \to 0$,

$$\begin{aligned}
&g\left[\tilde{w}\left(\theta\right)\left(l\left(\theta\right) + \mu \mathrm{d}l\left(\theta, h\right)\right)\right] - g\left[w\left(\theta\right) l\left(\theta\right)\right] \\
=&\left\{\left[\tilde{w}\left(\theta\right) - w\left(\theta\right)\right] l\left(\theta\right) + w\left(\theta\right) \mu \mathrm{d}l\left(\theta, h\right)\right\} g'\left(w\left(\theta\right) l\left(\theta\right)\right) \\
=&\mu \left\{\int_\Theta \gamma\left(\theta, \theta'\right) \mathrm{d}\hat{l}\left(\theta', h\right) d\theta' + \mathrm{d}\hat{l}\left(\theta, h\right)\right\} w\left(\theta\right) l\left(\theta\right) g'\left(w\left(\theta\right) l\left(\theta\right)\right).
\end{aligned}$$

A first-order Taylor expansion of the perturbed first-order conditions (65) around the baseline allocation then yields:

$$\begin{aligned}
0 =& v''\left(l\left(\theta\right)\right) \mu \mathrm{d}l\left(\theta, h\right) + \mu h'\left(w\left(\theta\right) l\left(\theta\right)\right) w\left(\theta\right) \\
&- \left\{\left[\tilde{w}\left(\theta\right) - w\left(\theta\right)\right]\left(1 - T'\left(w\left(\theta\right) l\left(\theta\right)\right)\right) - \left[T'\left(\tilde{w}\left(\theta\right)\left(l\left(\theta\right) + \mu \mathrm{d}l\left(\theta, h\right)\right)\right) - T'\left(w\left(\theta\right) l\left(\theta\right)\right)\right] w\left(\theta\right)\right\}.
\end{aligned}$$

Using (66), we obtain

$$\begin{aligned}
0 =& \frac{l\left(\theta\right) v''\left(l\left(\theta\right)\right)}{w\left(\theta\right)} \mu \mathrm{d}\hat{l}\left(\theta, h\right) + \mu h'\left(w\left(\theta\right) l\left(\theta\right)\right) - \mu \left[\int_\Theta \gamma\left(\theta, \theta'\right) \mathrm{d}\hat{l}\left(\theta', h\right) d\theta'\right]\left(1 - T'\left(w\left(\theta\right) l\left(\theta\right)\right)\right) \\
&+ \mu \left[\int_\Theta \gamma\left(\theta, \theta'\right) \mathrm{d}\hat{l}\left(\theta', h\right) d\theta' + \mathrm{d}\hat{l}\left(\theta, h\right)\right] w\left(\theta\right) l\left(\theta\right) T''\left(w\left(\theta\right) l\left(\theta\right)\right).
\end{aligned}$$

Solving for $\mathrm{d}\hat{l}\left(\theta, h\right)$,

$$\begin{aligned}
\mathrm{d}\hat{l}\left(\theta, h\right) =& \frac{\left[1 - T'\left(y\left(\theta\right)\right) - y\left(\theta\right) T''\left(y\left(\theta\right)\right)\right] \int_\Theta \bar{\gamma}\left(\theta, \theta'\right) \mathrm{d}\hat{l}\left(\theta', h\right) d\theta' - h'\left(y\left(\theta\right)\right)}{\left[\left(1 - T'\left(y\left(\theta\right)\right)\right) \frac{l(\theta) v''(l(\theta))}{v'(l(\theta))} + y\left(\theta\right) T''\left(y\left(\theta\right)\right)\right] - \left[1 - T'\left(y\left(\theta\right)\right) - y\left(\theta\right) T''\left(y\left(\theta\right)\right)\right] \bar{\bar{\gamma}}\left(\theta, \theta\right)} \\
=& \frac{\frac{1 - T'(y(\theta)) - y(\theta) T''(y(\theta))}{1 - T'(y(\theta)) + \varepsilon_{l,1-\tau}(\theta) y(\theta) T''(y(\theta))} \varepsilon_{l,1-\tau}\left(\theta\right)}{1 - \frac{1 - T'(y(\theta)) - y(\theta) T''(y(\theta))}{1 - T'(y(\theta)) + \varepsilon_{l,1-\tau}(\theta) y(\theta) T''(y(\theta))} \varepsilon_{l,1-\tau}\left(\theta\right) \bar{\bar{\gamma}}\left(\theta, \theta\right)} \int_\Theta \bar{\gamma}\left(\theta, \theta'\right) \mathrm{d}\hat{l}\left(\theta', h\right) d\theta' \\
&- \frac{\frac{\varepsilon_{l,1-\tau}(\theta)}{1 - T'(y(\theta)) + \varepsilon_{l,1-\tau}(\theta) y(\theta) T''(y(\theta))}}{1 - \frac{1 - T'(y(\theta)) - y(\theta) T''(y(\theta))}{1 - T'(y(\theta)) + \varepsilon_{l,1-\tau}(\theta) y(\theta) T''(y(\theta))} \varepsilon_{l,1-\tau}\left(\theta\right) \bar{\bar{\gamma}}\left(\theta, \theta\right)} h'\left(y\left(\theta\right)\right),
\end{aligned}$$

which leads to equation (21).

□

B.3.2 Proof of Proposition 1

We now derive the general solution (22) of the integral equation (21). Assume that the condition $\int_{\Theta^2} |K_1(\theta,\theta')|^2 \, d\theta d\theta' < 1$ holds.

Proof. For ease of exposition, denote by $\hat{h}'(\theta) = \frac{h'(y(\theta))}{1-T'(y(\theta))}$, and $g(\theta) \equiv \mathrm{d}\hat{l}(\theta,h)$. The integral equation writes

$$g(\theta) = -\tilde{E}_{l,1-\tau}(\theta)\,\hat{h}(\theta) + \int_\Theta K_1(\theta,\theta')\,g(\theta')\,d\theta',$$

where $K_1(\theta,\theta') = \tilde{E}_{l,w}(\theta)\,\bar{\gamma}(\theta,\theta')$. Substituting for $g(\theta')$ in the integral using the r.h.s. of the integral equation yields:

$$\begin{aligned}
g(\theta) &= -\tilde{E}_{l,1-\tau}(\theta)\,\hat{h}'(\theta) + \int_\Theta K_1(\theta,\theta')\left\{-\tilde{E}_{l,1-\tau}(\theta')\,\hat{h}'(\theta') + \int_\Theta K_1(\theta',\theta'')\,g(\theta'')\,d\theta''\right\}d\theta' \\
&= -\left\{\tilde{E}_{l,1-\tau}(\theta)\,\hat{h}'(\theta) + \int_\Theta K_1(\theta,\theta')\,\tilde{E}_{l,1-\tau}(\theta')\,\hat{h}'(\theta')\,d\theta'\right\} \\
&\quad + \int_\Theta\left\{\int_\Theta K_1(\theta,\theta')\,K_1(\theta',\theta'')\,g(\theta'')\,d\theta''\right\}d\theta'.
\end{aligned}$$

Applying Fubini's theorem yields

$$\begin{aligned}
g(\theta) &= -\left\{\tilde{E}_{l,1-\tau}(\theta)\,\hat{h}'(\theta) + \int_\Theta K_1(\theta,\theta')\,\tilde{E}_{l,1-\tau}(\theta')\,\hat{h}'(\theta')\,d\theta'\right\} \\
&\quad + \int_\Theta\left\{\int_\Theta K_1(\theta,\theta')\,K_1(\theta',\theta'')\,d\theta'\right\}g(\theta'')\,d\theta'' \\
&\equiv -\left\{\tilde{E}_{l,1-\tau}(\theta)\,\hat{h}'(\theta) + \int_\Theta K_1(\theta,\theta')\,\tilde{E}_{l,1-\tau}(\theta')\,\hat{h}'(\theta')\,d\theta'\right\} + \int_\Theta K_2(\theta,\theta')\,g(\theta')\,d\theta',
\end{aligned}$$

where $K_2(\theta,\theta') = \int_\Theta K(\theta,\theta'')\,K(\theta'',\theta')\,d\theta''$. A second substitution yields:

$$\begin{aligned}
g(\theta) &= -\left\{\tilde{E}_{l,1-\tau}(\theta)\,\hat{h}'(\theta) + \int_\Theta K_1(\theta,\theta')\,\tilde{E}_{l,1-\tau}(\theta')\,\hat{h}'(\theta')\,d\theta'\right\} \\
&\quad + \int_\Theta K_2(\theta,\theta')\left\{-\tilde{E}_{l,1-\tau}(\theta')\,\hat{h}'(\theta') + \int_\Theta K_1(\theta',\theta'')\,g(\theta'')\,d\theta''\right\}d\theta',
\end{aligned}$$

which can be rewritten, following the same steps as above, as

$$\begin{aligned}
g(\theta) &= -\left\{\tilde{E}_{l,1-\tau}(\theta)\,\hat{h}'(\theta) + \int_\Theta K(\theta,\theta')\,\tilde{E}_{l,1-\tau}(\theta')\,\hat{h}'(\theta')\,d\theta' + \int_\Theta K_2(\theta,\theta')\,\tilde{E}_{l,1-\tau}(\theta')\,\hat{h}'(\theta')\,d\theta'\right\} \\
&\quad + \int_\Theta K_3(\theta,\theta')\,g(\theta')\,d\theta',
\end{aligned}$$

where $K_3(\theta,\theta') = \int_\Theta K_2(\theta,\theta'')\,K(\theta'',\theta')\,d\theta''$. By repeated substitution, we thus obtain: for all

$n \geq 1$,

$$g(\theta) = -\left\{ \tilde{E}_{l,1-\tau}(\theta) \hat{h}'(\theta) + \sum_{i=1}^{n} \int_{\Theta} K_i(\theta, \theta') \tilde{E}_{l,1-\tau}(\theta') \hat{h}'(\theta') d\theta' \right\}$$
$$+ \int_{\Theta} K_{n+1}(\theta, \theta') g(\theta') d\theta',$$

where for all n, $K_{n+1}(\theta, \theta') = \int_{\Theta} K_n(\theta, \theta'') K_1(\theta'', \theta') d\theta''$. Now the last step is to show that that $\int_{\Theta} K_{n+1}(\theta, \theta') g(\theta') d\theta'$ converges to zero as $n \to \infty$.

Now, applying the Cauchy-Schwartz inequality to the interated kernel yields

$$|K_{n+1}(\theta, \theta')|^2 \leq \left(\int_{\Theta} |K_n(\theta, \theta'')|^2 d\theta'' \right) \left(\int_{\Theta} |K_1(\theta'', \theta')|^2 d\theta'' \right).$$

Integrating this inequality with respect to θ' implies

$$\int_{\Theta} |K_{n+1}(\theta, \theta')|^2 d\theta' \leq \left(\int_{\Theta} |K_n(\theta, \theta'')|^2 d\theta'' \right) \left(\int_{\Theta} \int_{\Theta} |K_1(\theta'', \theta')|^2 d\theta'' d\theta' \right)$$
$$= \|K_1\|_2^2 \times \int_{\Theta} |K_n(\theta, \theta'')|^2 d\theta''.$$

By induction, we obtain

$$\int_{\Theta} |K_{n+1}(\theta, \theta')|^2 d\theta' \leq \|K_1\|_2^{2n} \times \int_{\Theta} |K_1(\theta, \theta'')|^2 d\theta''.$$

We thus have, using the Cauchy-Schwartz inequality again,

$$\left| \int_{\Theta} K_{n+1}(\theta, \theta') \tilde{E}_{l,1-\tau}(\theta') \hat{h}'(\theta') d\theta' \right|^2$$
$$\leq \left(\int_{\Theta} |K_{n+1}(\theta, \theta'')|^2 d\theta'' \right) \left(\int_{\Theta} \left| \tilde{E}_{l,1-\tau}(\theta') \hat{h}'(\theta') \right|^2 d\theta'' \right)$$
$$\leq \|K_1\|_2^{2n} \times \left(\int_{\Theta} |K_1(\theta, \theta'')|^2 d\theta'' \right) \times \left\| \tilde{E}_{l,1-\tau} \hat{h}' \right\|_2^2$$

Thus, for all $\theta \in \Theta$,

$$\left| \int_{\Theta} K_i(\theta, \theta') \tilde{E}_{l,1-\tau}(\theta') \hat{h}'(\theta') d\theta' \right| \leq \left[\left\| \tilde{E}_{l,1-\tau} \hat{h}' \right\|_2 \sqrt{\int_{\Theta} |K_1(\theta, \theta'')|^2 d\theta''} \right] \times \|K_1\|_2^i.$$

Denote by

$$\kappa_n(\theta) \equiv \sum_{i=1}^{n} \int_{\Theta} K_i(\theta, \theta') \tilde{E}_{l,1-\tau}(\theta') \hat{h}'(\theta') d\theta'.$$

Since $\|K_1\|_2 < 1$, the previous arguments imply that the sequence $\{\kappa_n(\theta)\}_{n \geq 1}$ is dominated by a convergent geometric series of positive terms, and therefore it converges absolutely and uniformly to a unique limit $\kappa(\theta)$ on Θ. Similarly, we have

$$\lim_{n\to\infty}\left|\int_{\Theta} K_{n+1}(\theta,\theta') g(\theta') d\theta'\right| = 0.$$

Therefore, we can write

$$g(\theta) = -\tilde{E}_{l,1-\tau}(\theta)\hat{h}'(\theta) - \sum_{i=1}^{\infty}\int_{\Theta} K_i(\theta,\theta')\tilde{E}_{l,1-\tau}(\theta')\hat{h}'(\theta') d\theta',$$

which proves equation (22).

To show the uniqueness of the solution, suppose that $g_1(\theta)$ and $g_2(\theta)$ are two solutions to (22). Then $\Delta(\theta) \equiv g_2(\theta) - g_1(\theta)$ satisfies the homogeneous integral equation

$$\Delta(\theta) = \int_{\Theta} K_1(\theta,\theta')\Delta(\theta') d\theta'.$$

The Cauchy-Schwartz inequality reads

$$|\Delta(\theta)|^2 \leq \left(\int_{\Theta} |K_1(\theta,\theta')|^2 d\theta'\right)\left(\int_{\Theta} |\Delta(\theta')|^2 d\theta'\right).$$

Integrating with respect to θ yields

$$\int_{\Theta} |\Delta(\theta)|^2 d\theta \leq \|K_1\|^2 \int_{\Theta} |\Delta(\theta')|^2 d\theta',$$

Assumption $\|K_1\|^2 < 1$ then implies $\int_{\Theta} |\Delta(\theta)|^2 d\theta$, i.e., $\Delta(\theta) = 0$ for all $\theta \in \Theta$.

□

In case the condition $\|K_1\|_2^2 < 1$ does not hold, there exist methods to express the solution to the integral equation (21). Schmidt's method consists of showing that the kernel of the integral equation (21) can be written as the sum of two kernels, $K_1(\theta,\theta') = \bar{K}(\theta,\theta') + K_\varepsilon(\theta,\theta')$, where $\bar{K}(\theta,\theta')$ is separable and $K_\varepsilon(\theta,\theta')$ satisfies the condition $\|K_\varepsilon\|_2^2 < 1$. This decomposition is allowed by the Weierstrass theorem: by appropriately choosing a separable polynomial in θ and θ' for $\bar{K}(\theta,\theta')$, we can make the norm of $K_\varepsilon(\theta,\theta')$ arbitrarily small. It is then easy to see that the solution to (21) satisfies an integral equation with kernel $K_\varepsilon(\theta,\theta')$, where the exogenous function on the right hand side (outside of the integral) is itself the solution to an integral equation with separable kernel $\bar{K}(\theta,\theta')$. The former integral equation can be analyzed using the same arguments as in the proof of Proposition 1. The latter integral equation can be analyzed using the arguments of the proof of Proposition 3. We can then derive a solution to (21) of the same form as (22), with a more general resolvent. This technique is detailed in Section 2.4 of Zemyan (2012).

B.3.3 Proof of Corollaries 1 and 2

We now derive the incidence of tax reforms on wages, utilities, government revenue, and social welfare.

Proof. First note that we can also write this equation as

$$\begin{aligned}
\mathrm{d}\hat{l}(\theta,h) =& \frac{\left[1-T'(w(\theta)l(\theta))-y(\theta)T''(w(\theta)l(\theta))\right]\int_\Theta \gamma(\theta,\theta')\,\mathrm{d}\hat{l}(\theta',h)\,d\theta' - h'(y(\theta))}{(1-T'(y(\theta)))\frac{l(\theta)v''(l(\theta))}{v'(l(\theta))}+y(\theta)T''(w(\theta)l(\theta))} \\
=& \frac{\left[1-T'(y(\theta))-y(\theta)T''(y(\theta))\right]\varepsilon_{l,1-\tau}(\theta)}{1-T'(y(\theta))+\varepsilon_{l,1-\tau}(\theta)y(\theta)T''(y(\theta))}\int_\Theta \gamma(\theta,\theta')\,\mathrm{d}\hat{l}(\theta',h)\,d\theta' \\
&-\frac{\varepsilon_{l,1-\tau}(\theta)}{1-T'(y(\theta))+\varepsilon_{l,1-\tau}(\theta)y(\theta)T''(y(\theta))}h'(y(\theta)) \\
=& \tilde{\varepsilon}_{l,w}(\theta)\int_\Theta \gamma(\theta,\theta')\,\mathrm{d}\hat{l}(\theta',h)\,d\theta' - \tilde{\varepsilon}_{l,1-\tau}(\theta)\frac{h'(y(\theta))}{1-T'(y(\theta))},
\end{aligned}$$

which, along with (66), proves equation (26).

Next, the first-order effects of the tax reform h on individual θ's tax liability are given by:

$$\begin{aligned}
& \mathrm{d}_h T(w(\theta)l(\theta)) \\
\equiv & \lim_{\mu\to 0}\left\{\frac{1}{\mu}\left[T(\tilde{w}(\theta)(l(\theta)+\mu\mathrm{d}l(\theta,h)))-T(w(\theta)l(\theta))\right]+h(\tilde{w}(\theta)(l(\theta)+\mu\mathrm{d}l(\theta,h)))\right\} \\
=& T'(y(\theta))y(\theta)\left[\mathrm{d}\hat{l}(\theta,h)+\int_\Theta \gamma(\theta,\theta')\,\mathrm{d}\hat{l}(\theta',h)\,d\theta'\right]+h(y(\theta)) \\
=& T'(y(\theta))y(\theta)\left[\left(1+\frac{1}{\tilde{\varepsilon}_{l,w}(\theta)}\right)\mathrm{d}\hat{l}(\theta,h)+\frac{\tilde{\varepsilon}_{l,1-\tau}(\theta)}{\tilde{\varepsilon}_{l,w}(\theta)}\frac{h'(y(\theta))}{1-T'(y(\theta))}\right]+h(y(\theta)).
\end{aligned}$$

The first-order effects of the tax reform h on individual welfare are given by

$$\begin{aligned}
& \mathrm{d}_h u(y(\theta)-T(y(\theta))-v(l(\theta))) = \left[\mathrm{d}_h y(\theta)-\mathrm{d}_h T(y(\theta))-h(y(\theta))-\mathrm{d}_h v(l(\theta))\right]u'(\theta) \\
=& \left[(1-T'(y(\theta)))y(\theta)\,\mathrm{d}\hat{y}(\theta,h)-l(\theta)v'(l(\theta))\,\mathrm{d}\hat{l}(\theta,h)-h(y(\theta))\right]u'(\theta) \\
=& (1-T'(y(\theta)))y(\theta)\left[\left(1+\frac{1}{\tilde{\varepsilon}_{l,w}(\theta)}\right)\mathrm{d}\hat{l}(\theta,h)+\frac{\tilde{\varepsilon}_{l,1-\tau}(\theta)}{\tilde{\varepsilon}_{l,w}(\theta)}\frac{h'(y(\theta))}{1-T'(y(\theta))}\right]u'(\theta) \\
&-l(\theta)v'(l(\theta))\,\mathrm{d}\hat{l}(\theta,h)u'(\theta)-h(y(\theta))u'(\theta) \\
=& (1-T'(y(\theta)))y(\theta)\left[\frac{1}{\tilde{\varepsilon}_{l,w}(\theta)}\mathrm{d}\hat{l}(\theta,h)+\frac{\tilde{\varepsilon}_{l,1-\tau}(\theta)}{\tilde{\varepsilon}_{l,w}(\theta)}\frac{h'(y(\theta))}{1-T'(y(\theta))}\right]u'(\theta)-h(y(\theta))u'(\theta).
\end{aligned}$$

The first-order effects of the tax reform h on government revenue are given by

$$\begin{aligned}
& \mathrm{d}\mathscr{R}(T,h) = \mathrm{d}_h\left[\int_\Theta T(y(\theta))f_\theta(\theta)\,d\theta\right] \\
=& \int_\Theta h(y(\theta))f_\theta(\theta)\,d\theta \\
&+\int_\Theta T'(y(\theta))\left[\frac{\tilde{\varepsilon}_{l,1-\tau}(y(\theta))}{\tilde{\varepsilon}_{l,w}(y(\theta))}\frac{h'(y(\theta))}{1-T'(y(\theta))}+\left(1+\frac{1}{\tilde{\varepsilon}_{l,w}(y(\theta))}\right)\mathrm{d}\hat{l}(\theta,h)\right]y(\theta)f_\theta(\theta)\,d\theta \\
=& \int_{\mathbb{R}_+} h(y)f_y(y)\,dy+\int_{\mathbb{R}_+} T'(y)\left[\frac{\tilde{\varepsilon}_{l,1-\tau}(y)}{\tilde{\varepsilon}_{l,w}(y)}\frac{h'(y)}{1-T'(y)}+\left(1+\frac{1}{\tilde{\varepsilon}_{l,w}(y)}\right)\mathrm{d}\hat{l}(y,h)\right]yf_y(y)\,dy.
\end{aligned}$$

The first-order effects of the tax reform h on the social objective are given by

$$\lambda^{-1}\mathrm{d}\mathscr{G}(T,h) = \lambda^{-1}\mathrm{d}_h \int_\Theta u\left(y(\theta) - T(y(\theta)) - v(l(\theta))\right)\tilde{f}_\theta(\theta)\,d\theta$$
$$= \int_\Theta \left[\frac{1-T'(y(\theta))}{\tilde{\varepsilon}_{l,w}(\theta)} y(\theta)\,\mathrm{d}\hat{l}(\theta,h) + \frac{\tilde{\varepsilon}_{l,1-\tau}(\theta)}{\tilde{\varepsilon}_{l,w}(\theta)} y(\theta)\,h'(y(\theta)) - h(y(\theta))\right]\lambda^{-1}u'(\theta)\,\tilde{f}_\theta(\theta)\,d\theta$$
$$= \int_\Theta \left[\frac{1-T'(y(\theta))}{\tilde{\varepsilon}_{l,w}(\theta)} y(\theta)\,\mathrm{d}\hat{l}(\theta,h) + \frac{\tilde{\varepsilon}_{l,1-\tau}(\theta)}{\tilde{\varepsilon}_{l,w}(\theta)} y(\theta)\,h'(y(\theta)) - h(y(\theta))\right] g_\theta(\theta)\,f_\theta(\theta)\,d\theta$$
$$= \int_{\mathbb{R}_+} \left[\frac{1-T'(y)}{\tilde{\varepsilon}_{l,w}(y)}\mathrm{d}\hat{l}(y,h) + \frac{\tilde{\varepsilon}_{l,1-\tau}(y)}{\tilde{\varepsilon}_{l,w}(y)} h'(y)\right] y g_y(y)\,f_y(y)\,dy - \int_{\mathbb{R}_+} h(y)\,g_y(y)\,f_y(y)\,dy.$$

The first-order effects of the tax reform h on social welfare are finally given by

$$\mathrm{d}\mathscr{W}(T,h) = \mathrm{d}\mathscr{R}(T,h) + \lambda^{-1}\mathrm{d}\mathscr{G}(T,h)$$
$$= \int_{\mathbb{R}_+} (1 - g_y(y))\,h(y)\,f_y(y)\,dy$$
$$+ \int_{\mathbb{R}_+} T'(y)\left[\frac{\tilde{\varepsilon}_{l,1-\tau}(y)}{\tilde{\varepsilon}_{l,w}(y)}\frac{h'(y)}{1-T'(y)} + \left(1 + \frac{1}{\tilde{\varepsilon}_{l,w}(y)}\right)\mathrm{d}\hat{l}(y,h)\right] y f_y(y)\,dy$$
$$+ \int_{\mathbb{R}_+} \left[\frac{1-T'(y)}{\tilde{\varepsilon}_{l,w}(y)}\mathrm{d}\hat{l}(y,h) + \frac{\tilde{\varepsilon}_{l,1-\tau}(y)}{\tilde{\varepsilon}_{l,w}(y)} h'(y)\right] y g_y(y)\,f_y(y)\,dy.$$

This concludes the proof of equations (28) and (29).

□

B.3.4 Proof of Proposition 2

We now specialize the production function to be CES. In this case the integral equation (21) has a simple solution.

Proof. If the production function is CES, the cross wage elasticities $\bar{\gamma}(\theta,\theta')$ do not depend on θ (see equation (15)). Hence the kernel of the integral equation, $K_1(\theta,\theta') = \tilde{E}_{l,w}(\theta)\,\bar{\gamma}(\theta,\theta')$, is multiplicatively separable, i.e., the product of a function of θ only and a function of θ' only:

$$K_1(\theta,\theta') = \frac{\tilde{\varepsilon}_{l,w}(\theta)}{1 - \bar{\bar{\gamma}}(\theta,\theta)\,\tilde{\varepsilon}_{l,w}(\theta)}\bar{\gamma}(\theta,\theta')$$
$$= \left[\frac{\tilde{\varepsilon}_{l,w}(\theta)}{1 - \bar{\bar{\gamma}}(\theta,\theta)\,\tilde{\varepsilon}_{l,w}(\theta)}\right] \times \left[\frac{(1-\rho)\,a(\theta')\,L(\theta')^\rho}{\int_\Theta a(x)\,L(x)^\rho\,dx}\right] \equiv \kappa_1(\theta) \times \kappa_2(\theta').$$

The integral equation (21) then writes, letting $\hat{h}'(y) = \frac{h'(y)}{1-T'(y)}$,

$$\mathrm{d}\hat{l}(\theta,h) = -\,\tilde{E}_{l,1-\tau}(\theta)\,\hat{h}'(y(\theta)) + \kappa_1(\theta)\int_\Theta \kappa_2(\theta')\,\mathrm{d}\hat{l}(\theta',h)\,d\theta'$$

and can be solved as follows. Multiplying by $\kappa_2(\theta')$ both sides of the integral equation evaluated at

θ' yields

$$\kappa_2(\theta')\,\mathrm{d}\hat{l}(\theta',h) = -\,\kappa_2(\theta')\,\tilde{E}_{l,1-\tau}(\theta')\,\hat{h}'(y(\theta')) + \kappa_1(\theta')\,\kappa_2(\theta')\int_\Theta \kappa_2(\theta'')\,\mathrm{d}\hat{l}(\theta'',h)\,d\theta''.$$

Integrating with respect to θ' gives

$$\int_\Theta \kappa_2(\theta')\,\mathrm{d}\hat{l}(\theta',h)\,d\theta' = -\int_\Theta \kappa_2(\theta')\,\tilde{E}_{l,1-\tau}(\theta')\,\hat{h}'(y(\theta'))\,d\theta' + \left(\int_\Theta \kappa_1(\theta')\,\kappa_2(\theta')\,d\theta'\right)\left(\int_\Theta \kappa_2(\theta'')\,\mathrm{d}\hat{l}(\theta'',h)\,d\theta''\right),$$

i.e.,

$$\int_\Theta \kappa_2(\theta')\,\mathrm{d}\hat{l}(\theta',h)\,d\theta' = -\frac{\int_\Theta \kappa_2(\theta')\,\tilde{E}_{l,1-\tau}(\theta')\,\hat{h}'(y(\theta'))\,d\theta'}{1-\int_\Theta \kappa_1(\theta')\,\kappa_2(\theta')\,d\theta'}.$$

Substituting into the right hand side of the integral equation (21) leads to

$$\mathrm{d}\hat{l}(\theta,h) = -\,\tilde{E}_{l,1-\tau}(\theta)\,\hat{h}'(y(\theta)) - \kappa_1(\theta)\,\frac{\int_\Theta \kappa_2(\theta')\,\tilde{E}_{l,1-\tau}(\theta')\,\hat{h}'(y(\theta'))\,d\theta'}{1-\int_\Theta \kappa_1(\theta')\,\kappa_2(\theta')\,d\theta'},$$

which implies equation (30).

Suppose in particular, as in Saez (2001), that the tax reform h is the step function $h(y) = \mathbb{I}_{\{y\geq y^*\}}$, so that $h'(y) = \delta_{y^*}(y)$ is the Dirac delta function (i.e., marginal tax rates are perturbed at income y^* only). To apply this equality to this non-differentiable perturbation, construct a sequence of smooth funtions $\varphi_{y^*,\varepsilon}(y)$ such that

$$\delta_{y^*}(y) = \lim_{\varepsilon\to 0}\varphi_{y^*,\varepsilon}(y),$$

in the sense that for all continuous functions ψ with compact support,

$$\lim_{\varepsilon\to 0}\int_{\mathbb{R}}\varphi_{y^*,\varepsilon}(y)\,\psi(y)\,dy = \psi(y^*),$$

i.e., changing variables in the integral,

$$\lim_{\varepsilon\to 0}\int_\Theta \varphi_{y^*,\varepsilon}(y(\theta'))\left\{\psi(y(\theta'))\,\frac{dy(\theta')}{d\theta}\right\}d\theta' = \psi(y^*).$$

This can be obtained by defining an absolutely integrable and smooth function $\varphi_{y^*}(y)$ with compact support and $\int_{\mathbb{R}}\varphi_{y^*}(y)\,dy = 1$, and letting $\varphi_{y^*,\varepsilon}(y) = \varepsilon^{-1}\varphi_{y^*}\left(\frac{y}{\varepsilon}\right)$. We then have, for all $\varepsilon > 0$,

$$\mathrm{d}\hat{l}(\theta,\Phi_{y^*,\varepsilon}) = -\,\tilde{E}_{l,1-\tau}(\theta)\,\frac{\varphi_{y^*,\varepsilon}(y(\theta))}{1-T'(y(\theta^*))} - \kappa_1(\theta)\,\frac{\int_\Theta \kappa_2(\theta')\,\tilde{E}_{l,1-\tau}(\theta')\,\frac{\varphi_{y^*,\varepsilon}(y(\theta'))}{1-T'(y(\theta'))}d\theta'}{1-\int_\Theta \kappa_1(\theta')\,\kappa_2(\theta')\,d\theta'}.$$

Letting $\varepsilon \to 0$, we get

$$\begin{aligned}
\mathrm{d}\hat{l}(\theta,h) &= -\tilde{E}_{l,1-\tau}(\theta)\frac{\delta_{y^*}(y(\theta))}{1-T'(y(\theta^*))} - \kappa_1(\theta)\frac{\left(\frac{dy(\theta^*)}{d\theta}\right)^{-1}\kappa_2(\theta^*)\tilde{E}_{l,1-\tau}(\theta^*)\frac{1}{1-T'(y(\theta^*))}}{1-\int_\Theta \kappa_1(\theta')\kappa_2(\theta')\,d\theta'}, \\
&= -\frac{\tilde{E}_{l,1-\tau}(y(\theta^*))}{1-T'(y(\theta^*))}\left[\delta_{y^*}(y(\theta)) + \frac{1}{y'(\theta^*)}\frac{\tilde{E}_{l,w}(y(\theta))\bar{\gamma}(\theta,\theta^*)}{1-\int_\Theta \tilde{E}_{l,w}(y(\theta'))\bar{\gamma}(\theta',\theta')\,d\theta'}\right].
\end{aligned} \tag{67}$$

This formula will be useful for the proof of Corollary 3.

□

B.3.5 Proof of Corollary 3

Next, assume that the production function is CES and the baseline tax schedule is CRP, given by (31) for $p \in (-\infty, 1)$, so that in particular, $1-T'(y) = (1-\tau)y^{-p}$ and $T''(y) = p(1-\tau)y^{-p-1}$. We start by deriving prelimilary properties of the labor supply and wage elasticities.

The labor supply elasticities (7), (7), and (12) are given by

$$\begin{aligned}
\tilde{\varepsilon}_{l,1-\tau}(y) =& \frac{1-T'(y)}{1-T'(y)+\varepsilon y T''(y)}\varepsilon = \frac{(1-\tau)y^{-p}}{(1-\tau)y^{-p}+\varepsilon y p(1-\tau)y^{-p-1}}\varepsilon = \frac{\varepsilon}{1+p\varepsilon}, \\
\tilde{\varepsilon}_{l,w}(y) =& \frac{1-T'(y)-yT''(y)}{1-T'(y)+\varepsilon y T''(y)}\varepsilon = \frac{(1-\tau)y^{-p}-yp(1-\tau)y^{-p-1}}{(1-\tau)y^{-p}+\varepsilon y p(1-\tau)y^{-p-1}}\varepsilon = \frac{(1-p)\varepsilon}{1+p\varepsilon},
\end{aligned}$$

and

$$\begin{aligned}
\tilde{E}_{l,1-\tau}(y) =& \frac{\tilde{\varepsilon}_{l,1-\tau}(\theta)}{1-\bar{\bar{\gamma}}(\theta,\theta)\tilde{\varepsilon}_{l,w}(\theta)} = \frac{\frac{\varepsilon}{1+p\varepsilon}}{1+\frac{(1-p)\varepsilon}{(1+p\varepsilon)\sigma}} = \frac{\varepsilon}{1+p\varepsilon+(1-p)\frac{\varepsilon}{\sigma}}, \\
\tilde{E}_{l,w}(y) =& \frac{\tilde{\varepsilon}_{l,w}(\theta)}{1-\bar{\bar{\gamma}}(\theta,\theta)\tilde{\varepsilon}_{l,w}(\theta)} = \frac{\frac{(1-p)\varepsilon}{1+p\varepsilon}}{1+\frac{(1-p)\varepsilon}{(1+p\varepsilon)\sigma}} = \frac{(1-p)\varepsilon}{1+p\varepsilon+(1-p)\frac{\varepsilon}{\sigma}}.
\end{aligned}$$

Note that all of these elasticities are constant. This is because the curvature of the CRP tax schedule (captured by the parameter p) is constant. This feature allows us to simplify further equation (29) to obtain (32), respectively.

We can now prove Corollary 3.

Proof. Suppose first that the baseline tax schedule is linear, i.e., $p = 0$ in (31). In this case we have

$$\int_\Theta \tilde{E}_{l,w}(y(\theta'))\bar{\gamma}(\theta',\theta')\,d\theta' = \frac{\varepsilon}{1+\frac{\varepsilon}{\sigma}}\int_\Theta \bar{\gamma}(\theta',\theta')\,d\theta' = \frac{\varepsilon}{1+\frac{\varepsilon}{\sigma}} \times \frac{1}{\sigma},$$

where the last equality follows from expression (15) for the cross-wage elasticities $\bar{\gamma}(\theta',\theta')$. Thus the integral equation (30) becomes

$$\mathrm{d}\hat{l}(\theta,h) = -\frac{1}{1-\tau}\frac{\varepsilon}{1+\frac{\varepsilon}{\sigma}}h'(y(\theta)) - \frac{1}{1-\tau}\left(\frac{\varepsilon}{1+\frac{\varepsilon}{\sigma}}\right)^2\left(\frac{\sigma+\varepsilon}{\sigma}\right)\int_\Theta \bar{\gamma}(\theta,\theta')h'(y(\theta'))\,d\theta'.$$

Note that the first term in this expression for individual θ is proportional to the exogenous marginal tax rate perturbation at income $y(\theta)$, and the second term is a constant independent of θ.

Now integrate this equation to get the effect of the perturbation on government revenue. Formula (28) writes

$$
\begin{aligned}
&\mathrm{d}\mathscr{R}(T,h)\\
&=\int_{\mathbb{R}_+} h(y) f_y(y)\, dy + \int_{\mathbb{R}_+} T'(y)\left[\frac{\tilde{\varepsilon}_{l,1-\tau}(y)}{\tilde{\varepsilon}_{l,w}(y)}\frac{h'(y)}{1-T'(y)} + \left(1+\frac{1}{\tilde{\varepsilon}_{l,w}(y)}\right)\mathrm{d}\hat{l}(y,h)\right] y f_y(y)\, dy\\
&=\int_{\mathbb{R}_+} h(y) f_y(y)\, dy + \frac{\tau}{1-\tau}\int_{\mathbb{R}_+} h'(y)\, y f_y(y)\, dy + \left(1+\frac{1}{\varepsilon}\right)\tau\int_{\mathbb{R}_+} \mathrm{d}\hat{l}(y,h)\, y f_y(y)\, dy\\
&=\int_{\mathbb{R}_+} h(y) f_y(y)\, dy - \frac{(\sigma-1)\varepsilon}{\sigma+\varepsilon}\frac{\tau}{1-\tau}\int_{\mathbb{R}_+} h'(y)\, y f_y(y)\, dy\\
&\quad -\frac{\sigma(1+\varepsilon)\varepsilon}{\sigma+\varepsilon}\frac{\tau}{1-\tau}\int_{\mathbb{R}_+}\left[\int_{\mathbb{R}_+}\bar{\gamma}(y,y')\, h'(y')\left(\frac{dy(\theta')}{d\theta}\right)^{-1} dy'\right] y f_y(y)\, dy,
\end{aligned}
$$

where the last equality uses a change of variables to rewrite the integral $\int_\Theta \bar{\gamma}(\theta,\theta')\, h'(y(\theta'))\, d\theta'$. But using expression (64) to substitute for $\bar{\gamma}(y,y')$, we obtain

$$
\begin{aligned}
&\int_{\mathbb{R}_+}\left[\int_{\mathbb{R}_+}\bar{\gamma}(y,y')\, h'(y')\left(\frac{dy(\theta')}{d\theta}\right)^{-1} dy'\right] y f_y(y)\, dy\\
&=\left[\int_{\mathbb{R}_+}(1-\rho)\frac{y' f_y(y')}{\int_{\mathbb{R}_+} y f_y(y)\, dy} h'(y')\, dy'\right]\mathbb{E}[y] = (1-\rho)\int_{\mathbb{R}_+} h'(y)\, y f_y(y)\, dy.
\end{aligned}
$$

Hence we obtain

$$
\begin{aligned}
&\mathrm{d}\mathscr{R}(T,h)\\
&=\int_{\mathbb{R}} h(y) f_y(y)\, dy - \frac{(\sigma-1)\varepsilon}{\sigma+\varepsilon}\frac{\tau}{1-\tau}\int_{\mathbb{R}} h'(y)\, y f_y(y)\, dy - \frac{(1+\varepsilon)\varepsilon}{\sigma+\varepsilon}\frac{\tau}{1-\tau}\int_{\mathbb{R}} h'(y)\, y f_y(y)\, dy\\
&=\int_{\mathbb{R}} h(y) f_y(y)\, dy - \frac{\tau}{1-\tau}\varepsilon\int_{\mathbb{R}} h'(y)\, y f_y(y)\, dy,
\end{aligned}
$$

which is exactly the partial equilibrium formula.

Now suppose more generally that the baseline tax schedule is CRP with $p\in(-\infty,1)$. Consider the Saez perturbation at income y^*, i.e. $h(y)=\mathbb{I}_{\{y\geq y^*\}}$ and $h'(y)=\delta_{y^*}(y)$. Substituting for the labor supply elasticities and using the fact that the elasticities $\tilde{E}_{l,1-\tau}(\theta), \tilde{E}_{l,w}(\theta)$ are constant, equation (67) can be rewritten as

$$
\begin{aligned}
\mathrm{d}\hat{l}(y(\theta),h) &= -\frac{\tilde{E}_{l,1-\tau}(y(\theta^*))}{1-T'(y(\theta^*))}\left[\delta_{y^*}(y) + \left(\frac{dy(\theta^*)}{d\theta}\right)^{-1}\frac{\tilde{E}_{l,w}(y(\theta^*))\,\bar{\gamma}(\theta,\theta^*)}{1-\int_\Theta \tilde{E}_{l,w}(y(\theta'))\,\bar{\gamma}(\theta',\theta')\, d\theta'}\right]\\
&= -\frac{\tilde{E}_{l,1-\tau}}{1-T'(y^*)}\left[\delta_{y^*}(y) + \left(\frac{dy(\theta^*)}{d\theta}\right)^{-1}\frac{\tilde{E}_{l,w}}{1-\tilde{E}_{l,w}/\sigma}\bar{\gamma}(y,y^*)\right].
\end{aligned}
$$

Substituting in the expression (28) that gives the revenue effects of the tax reform, we obtain:

$$\begin{aligned}
&\mathrm{d}\mathscr{R}\left(T,h\right) \\
=&\int_{\mathbb{R}_+}\mathbb{I}_{\{y\geq y^*\}}f_y\left(y\right)dy+\int_{\mathbb{R}_+}T'\left(y\right)\left[\frac{\tilde{\varepsilon}_{l,1-\tau}}{\tilde{\varepsilon}_{l,w}}\frac{\delta_{y^*}\left(y\right)}{1-T'\left(y\right)}+\left(1+\frac{1}{\tilde{\varepsilon}_{l,w}}\right)\mathrm{d}\hat{l}\left(y,h\right)\right]yf_y\left(y\right)dy \\
=&1-F_y\left(y^*\right)+\int_{\mathbb{R}_+}\frac{T'\left(y\right)}{1-T'\left(y^*\right)}\left[\frac{\tilde{\varepsilon}_{l,1-\tau}}{\tilde{\varepsilon}_{l,w}}-\left(1+\frac{1}{\tilde{\varepsilon}_{l,w}}\right)\tilde{E}_{l,1-\tau}\right]yf_y\left(y\right)\delta_{y^*}\left(y\right)dy \\
&-\int_{\mathbb{R}_+}\frac{T'\left(y\right)}{1-T'\left(y^*\right)}\left(1+\frac{1}{\tilde{\varepsilon}_{l,w}}\right)\frac{\tilde{E}_{l,1-\tau}\tilde{E}_{l,w}}{1-\tilde{E}_{l,w}/\sigma}\bar{\gamma}\left(y,y^*\right)\left(\frac{dy\left(\theta^*\right)}{d\theta}\right)^{-1}yf_y\left(y\right)dy,
\end{aligned}$$

i.e.,

$$\begin{aligned}
\mathrm{d}\mathscr{R}\left(T,h\right)=&1-F_y\left(y^*\right)-\tilde{\varepsilon}_{l,1-\tau}\frac{T'\left(y^*\right)}{1-T'\left(y^*\right)}y^*f_y\left(y^*\right) \\
&+\left(1+\frac{1}{\tilde{\varepsilon}_{l,w}}\right)\left(\tilde{\varepsilon}_{l,1-\tau}-\tilde{E}_{l,1-\tau}\right)\frac{T'\left(y^*\right)}{1-T'\left(y^*\right)}y^*f_y\left(y^*\right) \\
&-\left(1+\frac{1}{\tilde{\varepsilon}_{l,w}}\right)\tilde{E}_{l,1-\tau}\frac{\frac{\tilde{\varepsilon}_{l,w}}{1+\frac{\tilde{\varepsilon}_{l,w}}{\sigma}}}{1-\frac{\tilde{\varepsilon}_{l,w}}{\sigma+\tilde{\varepsilon}_{l,w}}}\int_{\mathbb{R}_+}\frac{T'\left(y\right)}{1-T'\left(y^*\right)}\bar{\gamma}\left(y,y^*\right)\left(\frac{dy\left(\theta^*\right)}{d\theta}\right)^{-1}yf_y\left(y\right)dy \\
=&1-F_y\left(y^*\right)-\tilde{\varepsilon}_{l,1-\tau}\frac{T'\left(y^*\right)}{1-T'\left(y^*\right)}y^*f_y\left(y^*\right)-\tilde{E}_{l,1-\tau}\left(1+\tilde{\varepsilon}_{l,w}\right)\times\ldots \\
&\left\{-\frac{1}{\sigma}\frac{T'\left(y^*\right)}{1-T'\left(y^*\right)}y^*f_y\left(y^*\right)+\left(\frac{dy\left(\theta^*\right)}{d\theta}\right)^{-1}\int_{\mathbb{R}_+}\frac{T'\left(y\right)}{1-T'\left(y^*\right)}\bar{\gamma}\left(y,y^*\right)yf_y\left(y\right)dy\right\}.
\end{aligned}$$

The terms in curly brackets are equal to

$$\begin{aligned}
&\left(\frac{dy\left(\theta^*\right)}{d\theta}\right)^{-1}\left\{\int_{\mathbb{R}_+}\frac{T'\left(y\right)}{1-T'\left(y^*\right)}\bar{\gamma}\left(y,y^*\right)yf_y\left(y\right)dy+\frac{T'\left(y^*\right)}{1-T'\left(y^*\right)}\bar{\bar{\gamma}}\left(y^*,y^*\right)y^*f_y\left(y^*\right)\frac{dy\left(\theta^*\right)}{d\theta}\right\} \\
=&\left(\frac{dy\left(\theta^*\right)}{d\theta}\right)^{-1}\left\{\int_{\Theta}\frac{T'\left(y\left(\theta\right)\right)}{1-T'\left(y\left(\theta^*\right)\right)}\bar{\gamma}\left(y\left(\theta\right),y\left(\theta^*\right)\right)y\left(\theta\right)f_\theta\left(\theta\right)d\theta\right. \\
&\left.+\frac{T'\left(y\left(\theta^*\right)\right)}{1-T'\left(y\left(\theta^*\right)\right)}\bar{\bar{\gamma}}\left(y\left(\theta^*\right),y\left(\theta^*\right)\right)y\left(\theta^*\right)f_\theta\left(\theta^*\right)\right\} \\
=&\left(\frac{dy\left(\theta^*\right)}{d\theta}\right)^{-1}\left\{\int_{\Theta}\frac{T'\left(y\left(\theta\right)\right)}{1-T'\left(y\left(\theta^*\right)\right)}\gamma\left(y\left(\theta\right),y\left(\theta^*\right)\right)y\left(\theta\right)f_\theta\left(\theta\right)d\theta\right\} \\
=&\left(\frac{dy\left(\theta^*\right)}{d\theta}\right)^{-1}\left\{\int_{\mathbb{R}_+}\frac{T'\left(y\right)}{1-T'\left(y^*\right)}\gamma\left(y,y^*\right)yf_y\left(y\right)dy\right\}.
\end{aligned}$$

Thus we obtain

$$\begin{aligned}
\mathrm{d}\mathscr{R}\left(T,h\right)=&1-F_y\left(y^*\right)-\tilde{\varepsilon}_{l,1-\tau}\frac{T'\left(y^*\right)}{1-T'\left(y^*\right)}y^*f_y\left(y^*\right) \\
&-\tilde{E}_{l,1-\tau}\left(1+\tilde{\varepsilon}_{l,w}\right)\left(\frac{dy\left(\theta^*\right)}{d\theta}\right)^{-1}\int_{\mathbb{R}_+}\frac{T'\left(y\right)}{1-T'\left(y^*\right)}\gamma\left(y,y^*\right)yf_y\left(y\right)dy.
\end{aligned}$$

Another way to write this formula is as follows: using the expression (64) derived above for the cross wage elasticities and the fact that $\bar{\gamma}(y, y^*)$ depends only on y^* (so that it can be taken out of the integral), we get

$$\begin{aligned}
&\left(\frac{dy(\theta^*)}{d\theta}\right)^{-1} \bar{\gamma}(y, y^*) \int_{\mathbb{R}_+} \frac{T'(y)}{1-T'(y^*)} y f_y(y)\, dy + \frac{T'(y^*)}{1-T'(y^*)} \bar{\bar{\gamma}}(y^*, y^*) y^* f_y(y^*) \\
=& \frac{1}{\sigma} \frac{y^* f_y(y^*)}{\int_{\mathbb{R}_+} y f_y(y)\, dy} \int_{\mathbb{R}_+} \frac{T'(y)}{1-T'(y^*)} y f_y(y)\, dy - \frac{1}{\sigma} \frac{T'(y^*)}{1-T'(y^*)} y^* f_y(y^*) \\
=& \frac{1}{\sigma} \frac{y^* f_y(y^*)}{\int_{\mathbb{R}_+} y f_y(y)\, dy} \left\{ \int_{\mathbb{R}_+} \frac{T'(y) - T'(y^*)}{1-T'(y^*)} y f_y(y)\, dy \right\},
\end{aligned}$$

so that we can write

$$\begin{aligned}
d\mathscr{R}(T, h) =& 1 - F_y(y^*) - \tilde{\varepsilon}_{l,1-\tau} \frac{T'(y^*)}{1-T'(y^*)} y^* f_y(y^*) \\
& - \frac{1}{\sigma} \tilde{E}_{l,1-\tau} (1 + \tilde{\varepsilon}_{l,w}) \frac{y^* f_y(y^*)}{\int_{\mathbb{R}_+} y f_y(y)\, dy} \int_{\mathbb{R}_+} \frac{T'(y) - T'(y^*)}{1-T'(y^*)} y f_y(y)\, dy.
\end{aligned}$$

We can rewrite this more concisely as

$$\begin{aligned}
d\mathscr{R}(T, h) =& 1 - F_y(y^*) - \frac{1}{1-T'(y^*)} y^* f_y(y^*) \ldots \\
& \times \left[T'(y^*) \tilde{\varepsilon}_{l,1-\tau} + \frac{1}{\sigma} \tilde{E}_{l,1-\tau} (1 + \tilde{\varepsilon}_{l,w}) \int_{\mathbb{R}_+} (T'(y) - T'(y^*)) \frac{y f_y(y)}{\int_{\mathbb{R}_+} y' f_y(y')\, dy'} dy \right].
\end{aligned}$$

Finally, the integral in the last expression can be easily calculated since the tax schedule is CRP. We have

$$\begin{aligned}
&\frac{1}{\int_{\mathbb{R}_+} y f_y(y)\, dy} \int_{\mathbb{R}_+} \frac{T'(y) - T'(y^*)}{1-T'(y^*)} y f_y(y)\, dy = \frac{1}{\int_{\mathbb{R}_+} y f_y(y)\, dy} \int_{\mathbb{R}_+} \frac{(y^*)^{-p} - y^{-p}}{(y^*)^{-p}} y f_y(y)\, dy \\
=& \frac{1}{\int_{\mathbb{R}_+} y f_y(y)\, dy} \left[\int_{\mathbb{R}_+} y f_y(y)\, dy - (y^*)^p \int_{\mathbb{R}_+} y^{1-p} f_y(y)\, dy \right] = 1 - \frac{\int_{\mathbb{R}_+} \left(\frac{y}{y^*}\right)^{1-p} dF_y(y)}{\int_{\mathbb{R}_+} \left(\frac{y}{y^*}\right) dF_y(y)} \\
=& 1 - \frac{\int_{\mathbb{R}_+} \frac{\frac{1-\tau}{1-p} y^{1-p}}{\frac{1-\tau}{1-p} (y^*)^{1-p}} f_y(y)\, dy}{\int_{\mathbb{R}_+} \frac{y}{y^*} f_y(y)\, dy} = 1 - \frac{\int_{\mathbb{R}_+} (c(y)/c(y^*)) f_y(y)\, dy}{\int_{\mathbb{R}_+} (y/y^*) f_y(y)\, dy},
\end{aligned}$$

where $c \equiv c(y)$ and $c^* \equiv c(y^*)$ are the consumptions (disposable incomes) of types θ and θ^* respectively. Therefore

$$\begin{aligned}
d\mathscr{R}(T, h) =& 1 - F_y(y^*) - \frac{T'(y^*)}{1-T'(y^*)} \tilde{\varepsilon}_{l,1-\tau} y^* f_y(y^*) \\
& - \frac{1}{\sigma} \tilde{E}_{l,1-\tau} (1 + \tilde{\varepsilon}_{l,w}) y^* f_y(y^*) \left(1 - \frac{\mathbb{E}[c/c^*]}{\mathbb{E}[y/y^*]}\right).
\end{aligned}$$

Finally, we derive the effects of the perturbation on social welfare. We have

$$
\begin{aligned}
&\lambda^{-1}\mathrm{d}\mathscr{G}(T,h) \\
=&-\int_{\mathbb{R}_+} g_y(y)\,\mathbb{I}_{\{y\geq y^*\}} f_y(y)\,dy+\int_{\mathbb{R}_+} g_y(y)\left[\frac{\tilde{\varepsilon}_{l,1-\tau}}{\tilde{\varepsilon}_{l,w}}\delta_{y^*}(y)+\frac{1-T'(y)}{\tilde{\varepsilon}_{l,w}}\mathrm{d}\hat{l}(y,h)\right] y f_y(y)\,dy \\
=&-\int_{y^*}^{\infty} g_y(y) f_y(y)\,dy+\int_{\mathbb{R}_+} g_y(y)\left[\frac{\tilde{\varepsilon}_{l,1-\tau}}{\tilde{\varepsilon}_{l,w}}-\frac{1-T'(y)}{1-T'(y^*)}\frac{\tilde{E}_{l,1-\tau}}{\tilde{\varepsilon}_{l,w}}\right] y f_y(y)\,\delta_{y^*}(y)\,dy \\
&-\int_{\mathbb{R}_+} g_y(y)\frac{1-T'(y)}{1-T'(y^*)}\frac{\tilde{E}_{l,1-\tau}}{1-\tilde{E}_{l,w}/\sigma}\frac{\tilde{E}_{l,w}}{\tilde{\varepsilon}_{l,w}}\bar{\gamma}(y,y^*)\left(\frac{dy(\theta^*)}{d\theta}\right)^{-1} y f_y(y)\,dy.
\end{aligned}
$$

The second and third integrals of this expression are equal to

$$
\begin{aligned}
&\left(\frac{dy(\theta^*)}{d\theta}\right)^{-1}\left\{\frac{\tilde{\varepsilon}_{l,1-\tau}-\tilde{E}_{l,1-\tau}}{\tilde{\varepsilon}_{l,w}} g_y(y^*)\, y^* f_\theta(\theta^*)\right. \\
&\qquad\qquad\left.-\frac{\tilde{E}_{l,1-\tau}}{1-\tilde{E}_{l,w}/\sigma}\frac{\tilde{E}_{l,w}}{\tilde{\varepsilon}_{l,w}}\int_{\mathbb{R}_+} g_y(y)\frac{1-T'(y)}{1-T'(y^*)}\bar{\gamma}(y,y^*)\, y f_y(y)\,dy\right\} \\
=&\left(\frac{dy(\theta^*)}{d\theta}\right)^{-1}\left\{\tilde{E}_{l,1-\tau} g_y(y^*)\frac{1}{\sigma} y^* f_\theta(\theta^*)-\tilde{E}_{l,1-\tau}\int_{\mathbb{R}_+} g_y(y)\frac{1-T'(y)}{1-T'(y^*)}\bar{\gamma}(y,y^*)\, y f_y(y)\,dy\right\} \\
=&-\tilde{E}_{l,1-\tau}\left(\frac{dy(\theta^*)}{d\theta}\right)^{-1}\int_{\mathbb{R}_+} g_y(y)\frac{1-T'(y)}{1-T'(y^*)}\gamma(y,y^*)\, y f_y(y)\,dy.
\end{aligned}
$$

Using the fact that $\bar{\gamma}(y,y^*)$ depends only on y^* and equation (64), we obtain another way of writing these terms:

$$
\begin{aligned}
&\tilde{E}_{l,1-\tau}\frac{1}{\sigma} g_y(y^*)\, y^* f_y(y^*)-\tilde{E}_{l,1-\tau}\left(\frac{dy(\theta^*)}{d\theta}\right)^{-1}\bar{\gamma}(y,y^*)\int_{\mathbb{R}_+} g_y(y)\frac{1-T'(y)}{1-T'(y^*)} y f_y(y)\,dy \\
=&\tilde{E}_{l,1-\tau}\frac{1}{\sigma} g_y(y^*)\, y^* f_y(y^*)-\tilde{E}_{l,1-\tau}\frac{1}{\sigma}\frac{y^* f_y(y^*)}{\int_{\mathbb{R}_+} y f_y(y)\,dy}\int_{\mathbb{R}_+} g_y(y)\frac{1-T'(y)}{1-T'(y^*)} y f_y(y)\,dy \\
=&\frac{\tilde{E}_{l,1-\tau}}{\sigma}\frac{y^* f_y(y^*)}{\int_{\mathbb{R}_+} y f_y(y)\,dy} g_y(y^*)\int_{\mathbb{R}_+}\left(1-\frac{(1-T'(y))\, g_y(y)}{(1-T'(y^*))\, g_y(y^*)}\right) y f_y(y)\,dy.
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
&\lambda^{-1}\mathrm{d}\mathscr{G}(T,h) \\
=&-\int_{y^*}^{\infty} g_y(y) f_y(y)\,dy-\tilde{E}_{l,1-\tau}\left(\frac{dy(\theta^*)}{d\theta}\right)^{-1}\int_{\mathbb{R}_+} g_y(y)\frac{1-T'(y)}{1-T'(y^*)}\gamma(y,y^*)\, y f_y(y)\,dy \\
=&-\int_{y^*}^{\infty} g_y(y) f_y(y)\,dy+\frac{\tilde{E}_{l,1-\tau}}{\sigma}\frac{y^* f_y(y^*)}{\int_{\mathbb{R}_+} y f_y(y)\,dy} g_y(y^*)\int_{\mathbb{R}_+}\left(1-\frac{(1-T'(y))\, g_y(y)}{(1-T'(y^*))\, g_y(y^*)}\right) y f_y(y)\,dy.
\end{aligned}
$$

Finally, the effect of the perturbation on social welfare is given by

$$
\mathrm{d}\mathscr{W}(T,h)=\mathrm{d}\mathscr{R}(T,h)+\lambda^{-1}\mathrm{d}\mathscr{G}(T,h).
$$

We thus obtain

$$
\begin{aligned}
d\mathscr{W}(T,h) =& 1-F_y(y^*) - \int_{y^*}^{\infty} g(y) f_y(y)\, dy - \frac{T'(y^*)}{1-T'(y^*)}\tilde{\varepsilon}_{l,1-\tau} y^* f_y(y^*) - \frac{1}{1-T'(y^*)} y^* f_y(y^*) \ldots \\
&\times \left[\frac{1}{\sigma}\tilde{E}_{l,1-\tau}(1+\tilde{\varepsilon}_{l,w}) \int_{\mathbb{R}_+} (T'(y)-T'(y^*)) \frac{y f_y(y)}{\int_{\mathbb{R}_+} y' f_y(y')\, dy'} dy \ldots \right. \\
&\left. + \frac{1}{\sigma}\tilde{E}_{l,1-\tau} \int_{\mathbb{R}_+} ((1-T'(y)) g_y(y) - (1-T'(y^*)) g_y(y^*)) \frac{y f_y(y)}{\int_{\mathbb{R}_+} y' f_y(y')\, dy'} dy \right].
\end{aligned}
$$

Reorganizing the terms easily leads to formula (32).

□

B.3.6 Proof of Proposition 3

Suppose that the production function is Translog, as defined in Example 2, with the functional form specification (19).

Proof. A Taylor expansion of the right hand side of (19) as $\theta \to \theta'$ writes

$$
\bar{\beta}(\theta,\theta') = \alpha\left[1-\exp\left(-\frac{1}{2s^2}(\theta-\theta')^2\right)\right] = -\alpha \sum_{n=1}^{N} \frac{1}{n!}\left(-\frac{1}{2s^2}(\theta-\theta')^2\right)^n + o(\theta-\theta')^{2N+1}.
$$

Ignoring for now the error term $o(\theta-\theta')^{2N+1}$ and denoting by $\bar{\beta}_N(\theta,\theta')$ the first N terms of the Taylor expansion, we get

$$
\begin{aligned}
\bar{\gamma}_N(\theta,\theta') =& \chi(\theta') + \frac{1}{\chi(\theta)}\bar{\beta}_N(\theta,\theta') = \chi(\theta') + \frac{\alpha}{\chi(\theta)} \sum_{n=1}^{N} \frac{(-1)^{n+1}}{2^n s^{2n} n!} ((\theta-\mu_\theta)-(\theta'-\mu_\theta))^{2n} \\
=& \chi(\theta') + \frac{\alpha}{\chi(\theta)} \sum_{n=1}^{N} \frac{(-1)^{n+1}}{2^n s^{2n} n!} \left\{ \sum_{k=0}^{2n} \begin{pmatrix} 2n \\ k \end{pmatrix} (\theta-\mu_\theta)^k (\theta'-\mu_\theta)^{2n-k} \right\} \\
=& \chi(\theta') + \frac{\alpha}{\chi(\theta)} \sum_{k=0}^{2N} \sum_{n=\max\left\{1,\lceil \frac{k}{2} \rceil\right\}}^{N} \frac{(-1)^{n+1}}{2^n s^{2n} n!} \begin{pmatrix} 2n \\ k \end{pmatrix} (\theta-\mu_\theta)^k (\theta'-\mu_\theta)^{2n-k} \\
=& \chi(\theta') + \frac{\alpha}{\chi(\theta)} \sum_{k=0}^{2N} A_k(\theta')(\theta-\mu_\theta)^k,
\end{aligned}
$$

where $\mu_\theta = \int_0^\infty \theta f_\theta(\theta)\, d\theta$ and

$$
A_k(\theta') \equiv \sum_{n=\max\left\{1,\lceil \frac{k}{2} \rceil\right\}}^{N} \frac{(-1)^{n+1}}{2^n s^{2n} n!} \begin{pmatrix} 2n \\ k \end{pmatrix} (\theta'-\mu_\theta)^{2n-k}.
$$

We therefore obtain that $\bar{\gamma}_N(\theta,\theta')$, and hence the approximate kernel $K_1(\theta,\theta') = \tilde{E}_{l,w}(\theta)\bar{\gamma}_N(\theta,\theta')$ of the integral equation (21), can be written as the sum of multiplicatively separable functions, which allows us to derive a simple closed-form solution for $\mathrm{d}\hat{l}\,(\theta, h)$ at an arbitrary degree of precision. Letting $\hat{h}'\,(y) \equiv \frac{h'(y)}{1-T'(y)}$, we can write the approximate solution to the integral equation (21) as

$$\mathrm{d}\hat{l}_N\,(\theta, h) = -\,\tilde{E}_{l,1-\tau}\,(\theta)\,\hat{h}'\,(y\,(\theta)) + \int_\Theta \left\{ \sum_{i=0}^{2N+1} \kappa_{i,1}\,(\theta)\,\kappa_{i,2}\,(\theta') \right\} \mathrm{d}\hat{l}_N\,(\theta', h)\,d\theta', \tag{68}$$

where

$$\kappa_{0,1}\,(\theta) = \tilde{E}_{l,w}\,(\theta)\,,\ \kappa_{0,2}\,(\theta') = \chi\,(\theta')\,,$$

and for all $i \in \{1, \dots, 2N+1\}$,

$$\kappa_{i,1}\,(\theta) = \frac{\tilde{E}_{l,w}\,(\theta)}{\chi\,(\theta)}\,(\theta - \mu_\theta)^{i-1}\,,\ \kappa_{i,2}\,(\theta') = A_{i-1}\,(\theta')\,.$$

This equation can thus be rewritten as

$$\mathrm{d}\hat{l}_N\,(\theta, h) = -\,\tilde{E}_{l,1-\tau}\,(\theta)\,\hat{h}'\,(y\,(\theta)) + \sum_{i=0}^{2N+1} a_i \kappa_{i,1}\,(\theta)\,,$$

where the constants $(a_i)_{i\geq 0}$ are given by

$$a_i = \int_\Theta \kappa_{i,2}\,(\theta')\,\mathrm{d}\hat{l}\,(\theta', h)\,d\theta'.$$

We thus obtain immediately that the solution to (21) is of the form

$$\mathrm{d}\hat{l}_N\,(\theta, h) = -\,\tilde{E}_{l,1-\tau}\,(\theta)\,\hat{h}'\,(y\,(\theta)) + \tilde{E}_{l,w}\,(\theta) \left[a_0 + \frac{1}{\chi\,(\theta)} \sum_{n=0}^{N} a_{n+1}\,(\theta - \mu_\theta)^n \right].$$

To characterize the constants $(a_i)_{i\geq 0}$ in closed form, integrate over Θ both sides of (68) evaluated at θ' and multiplied by $\kappa_{i,2}\,(\theta')$:

$$\begin{aligned} a_i = \int_\Theta \kappa_{i,2}\,(\theta')\,\mathrm{d}\hat{l}_N\,(\theta', h)\,d\theta' = &-\int_\Theta \kappa_{i,2}\,(\theta')\,\tilde{E}_{l,1-\tau}\,(\theta')\,\hat{h}'\,(y\,(\theta'))\,d\theta' \\ &+ \sum_{j=0}^{2N+1} a_j \int_\Theta \kappa_{j,1}\,(\theta')\,\kappa_{i,2}\,(\theta')\,d\theta', \end{aligned}$$

so that the vector $\mathbf{a} = (a_i)_{0\leq i\leq 2N+1}$ is the solution to the linear system

$$[\mathbf{I}_{2N+2} - \mathbf{A}]\,\mathbf{a} = \mathbf{h},$$

where $\mathbf{I}_{2N+2}$ is the $(2N+2)\times(2N+2)$-identity matrix, and the matrix $\mathbf{A} = (\mathbf{A}_{i,j})_{0\leq i,j\leq 2N+1}$ and the vector $\mathbf{h} = (\mathbf{h}_i)_{0\leq i\leq 2N+1}$ are given by:

$$\mathbf{A}_{i,j} = \int_\Theta \kappa_{j,1}(\theta)\,\kappa_{i,2}(\theta)\,d\theta,$$
$$\mathbf{h}_i = \int_\Theta \kappa_{i,2}(\theta)\,\tilde{E}_{l,1-\tau}(\theta)\,\hat{h}'(y(\theta))\,d\theta.$$

We assume that the determinant $\det(\mathbf{I}_{2N+2} - \mathbf{A}) \neq 0$, so that this system can be inverted. The inverse matrix $[\mathbf{I}_{2N+2} - \mathbf{A}]^{-1}$ can be expressed as the transpose of the matrix of cofactors from $[\mathbf{I}_{2N+2} - \mathbf{A}]$, which we denote by $\mathbf{C}$, normalized by the Fredholm determinant $\det(\mathbf{I}_{2N+2} - \mathbf{A})$. Thus we have

$$a_i = \frac{1}{\det(\mathbf{I}_{2N+2} - \mathbf{A})} \sum_{j=0}^{2N+1} \mathbf{C}_{j,i}\mathbf{h}_j,$$

so that the solution to the integral equation writes

$$\begin{aligned}
\mathrm{d}\hat{l}_N(\theta,h) =& -\tilde{E}_{l,1-\tau}(\theta)\,\hat{h}'(y(\theta)) + \sum_{i=0}^{2N+1}\left[\frac{1}{\det(\mathbf{I}_{2N+2} - \mathbf{A})}\sum_{j=0}^{2N+1}\mathbf{C}_{j,i}\mathbf{h}_j\right]\kappa_{i,1}(\theta) \\
=& -\tilde{E}_{l,1-\tau}(\theta)\,\hat{h}'(y(\theta)) \\
&+ \int_\Theta \left[\frac{1}{\det(\mathbf{I}_{2N+2} - \mathbf{A})}\sum_{i=0}^{2N+1}\sum_{j=0}^{2N+1}\mathbf{C}_{j,i}\kappa_{i,1}(\theta)\,\kappa_{j,2}(\theta')\right]\tilde{E}_{l,1-\tau}(\theta')\,\hat{h}'(y(\theta'))\,d\theta',
\end{aligned}$$

or, denoting by $\mathcal{R}_N(\theta,\theta')$ the term in square brackets in the previous equation, or the resolvent kernel of the integral equation,

$$\mathrm{d}\hat{l}_N(\theta,h) = -\tilde{E}_{l,1-\tau}(\theta)\,\hat{h}'(y(\theta)) + \int_\Theta \mathcal{R}_N(\theta,\theta')\,\tilde{E}_{l,1-\tau}(\theta')\,\hat{h}'(y(\theta'))\,d\theta'.$$

Finally, we can show that

$$\sum_{i=0}^{2N+1}\sum_{j=0}^{2N+1}\mathbf{C}_{j,i}\kappa_{i,1}(\theta)\,\kappa_{j,2}(\theta') = -\det(\mathbf{D}(\theta,\theta'))$$

with $\mathbf{D}_{1,1}(\theta,\theta') = 0$, $\mathbf{D}_{i,1}(\theta,\theta') = \kappa_{i-2,2}(\theta')$ for $i \geq 2$, $\mathbf{D}_{1,j}(\theta,\theta') = \kappa_{j-2,1}(\theta')$ for $j \geq 2$, and $(\mathbf{D}_{i,j}(\theta,\theta'))_{2\leq i,j\leq 2N+3} = \mathbf{I}_{2N+2} - \mathbf{A}$, so that

$$\mathcal{R}_N(\theta,\theta') = -\frac{\det(\mathbf{D}(\theta,\theta'))}{\det(\mathbf{I}_2 - \mathbf{A})}.$$

This proves the first Fredholm theorem with a separable kernel.

We now compute a bound on the error in the approximation of the true solution $\mathrm{d}\hat{l}(\theta,h)$ by $\mathrm{d}\hat{l}_N(\theta,h)$. We have

$$|\bar{\gamma}(\theta,\theta') - \bar{\gamma}_N(\theta,\theta')| \leq \frac{\alpha\,|\theta-\theta'|^{2N+1}}{2^{N+1}s^{2(N+1)}\,(N+1)!},$$

which implies, denoting $K_{1,N}(\theta,\theta') = \tilde{E}_{l,w}(\theta)\,\bar{\gamma}_N(\theta,\theta')$,

$$\begin{aligned}
\int_\Theta \left|K_1(\theta,\theta') - K_{1,N}(\theta,\theta')\right| d\theta' \leq& \int_\Theta \left|\tilde{E}_{l,w}(\theta)\right| \frac{\alpha\left|\theta-\theta'\right|^{2N+1}}{2^{N+1}s^{2(N+1)}(N+1)!}d\theta' \\
\leq& \frac{\alpha\left(\bar{\theta}-\underline{\theta}\right)^{2(N+1)}\sup_\Theta\left|\tilde{E}_{l,w}(\theta)\right|}{2^{N+1}s^{2(N+1)}(N+1)!} \equiv \varepsilon_N,
\end{aligned}$$

which converges to zero as $N\to\infty$. Assume that $\varepsilon_N(1+M_N) < 1$, where

$$M_N = \sup_{\theta\in\Theta}\int_\Theta \left|\mathcal{R}_N(\theta,\theta')\right| d\theta'.$$

We then have (see Theorem 2.6.1 in Zemyan (2012) and Section 13.14 in Polyanin and Manzhirov (2008)):

$$\left|\mathrm{d}\hat{l}(\theta,h) - \mathrm{d}\hat{l}_N(\theta,h)\right| \leq \varepsilon_N \frac{(1+M_N)^2\sup_\Theta\left|\tilde{E}_{l,1-\tau}(\theta)\,\hat{h}'(y(\theta))\right|}{1-\varepsilon_N(1+M_N)}.$$

Now consider the approximation of $\mathrm{d}\hat{l}(\theta,h)$ obtained from the third-order Taylor expansion of $\bar{\beta}(\theta,\theta')$:

$$\begin{aligned}
\mathrm{d}\hat{l}(\theta,h) \approx& -\tilde{E}_{l,1-\tau}(\theta)\,\hat{h}'(y(\theta)) + \tilde{E}_{l,w}(\theta)\left[a_0 + \frac{1}{\chi(\theta)}\sum_{n=0}^{2} a_{n+1}(\theta-\mu_\theta)^n\right] \\
=& -\tilde{E}_{l,1-\tau}(\theta)\,\hat{h}'(y(\theta)) + \tilde{E}_{l,w}(\theta)\left[a_0 + \frac{a_1}{\chi(\theta)} + \frac{a_3}{\chi(\theta)}\left((\theta-\mu_\theta)^2 + \frac{a_2}{a_3}(\theta-\mu_\theta)\right)\right] \\
=& -\tilde{E}_{l,1-\tau}(\theta)\,\hat{h}'(y(\theta)) + \tilde{E}_{l,w}(\theta)\left[a_0 + \frac{1}{\chi(\theta)}\left(a_1 - \frac{a_2^2}{4a_3}\right) + \frac{a_3}{\chi(\theta)}\left(\theta-\mu_\theta+\frac{a_2}{2a_3}\right)^2\right].
\end{aligned}$$

Letting $\tilde{\theta} = \mu_\theta - \frac{a_2}{2a_3}$ and recalling the approximation $\bar{\gamma}(\theta,\theta') = \chi(\theta') + \frac{\alpha}{2s^2}\frac{(\theta-\theta')^2}{\chi(\theta)}$, we get

$$\begin{aligned}
\mathrm{d}\hat{l}(\theta,h) \approx& -\tilde{E}_{l,1-\tau}(\theta)\,\hat{h}'(y(\theta)) + \tilde{E}_{l,w}(\theta)\times\ldots \\
&\left[a_0 - \frac{2s^2a_3}{\alpha}\chi\left(\tilde{\theta}\right) + \frac{1}{\chi(\theta)}\left(a_1 - \frac{a_2^2}{4a_3}\right) + \frac{2s^2}{\alpha}a_3\left(\chi\left(\tilde{\theta}\right) + \frac{\alpha}{2s^2}\frac{1}{\chi(\theta)}\left(\theta-\tilde{\theta}\right)^2\right)\right] \\
=& -\tilde{E}_{l,1-\tau}(\theta)\,\hat{h}'(y(\theta)) + \tilde{E}_{l,w}(\theta)\left[c_1 + \frac{c_2}{\chi(\theta)} + c_3\bar{\gamma}\left(\theta,\tilde{\theta}\right)\right],
\end{aligned}$$

which proves equation (33).

We can obtain (33) by computing the second-order approximation directly. We find

$$\begin{aligned}
\bar{\beta}(\theta,\theta') \approx& \frac{1}{4s^2}\alpha(\theta)(\theta-\mu_\theta)^2\alpha(\theta') + \frac{1}{4s^2}\alpha(\theta)\alpha(\theta')(\theta'-\mu_\theta)^2 \\
&-\frac{1}{2s^2}\alpha(\theta)(\theta-\mu_\theta)\alpha(\theta')(\theta'-\mu_\theta),
\end{aligned}$$

so that

$$\mathrm{d}\hat{l}\left(\theta,h\right)\approx-\tilde{E}_{l,1-\tau}\left(\theta\right)\hat{h}'\left(y\left(\theta\right)\right)+\int_{\Theta}\left\{\sum_{i=1}^{4}\kappa_{i,1}\left(\theta\right)\kappa_{i,2}\left(\theta'\right)\right\}\mathrm{d}\hat{l}\left(\theta',h\right)d\theta',$$

with

$$\begin{aligned}\kappa_{1,1}\left(\theta\right)=&\tilde{E}_{l,w}\left(\theta\right)\\ \kappa_{2,1}\left(\theta\right)=&\tilde{E}_{l,w}\left(\theta\right)\frac{\alpha\left(\theta\right)}{\chi\left(\theta\right)}\\ \kappa_{3,1}\left(\theta\right)=&\tilde{E}_{l,w}\left(\theta\right)\frac{\alpha\left(\theta\right)}{\chi\left(\theta\right)}\left(\theta-\mu_{\theta}\right)\\ \kappa_{4,1}\left(\theta\right)=&\tilde{E}_{l,w}\left(\theta\right)\frac{\alpha\left(\theta\right)}{\chi\left(\theta\right)}\left(\theta-\mu_{\theta}\right)^{2},\end{aligned}$$

and

$$\begin{aligned}\kappa_{1,2}\left(\theta'\right)=&\chi\left(\theta'\right)\\ \kappa_{2,2}\left(\theta'\right)=&\frac{\alpha\left(\theta'\right)}{4s^{2}}\left(\theta'-\mu_{\theta}\right)^{2}\\ \kappa_{3,2}\left(\theta'\right)=&-\frac{\alpha\left(\theta'\right)}{2s^{2}}\left(\theta'-\mu_{\theta}\right)\\ \kappa_{4,2}\left(\theta'\right)=&\frac{\alpha\left(\theta'\right)}{4s^{2}}.\end{aligned}$$

Thus

$$\mathrm{d}\hat{l}\left(\theta,h\right)=-\tilde{E}_{l,1-\tau}\left(\theta\right)\hat{h}'\left(y\left(\theta\right)\right)+\sum_{i=1}^{4}a_{i}\kappa_{i,1}\left(\theta\right),$$

with

$$\begin{pmatrix}a_{1}\\ a_{2}\\ a_{3}\\ a_{4}\end{pmatrix}=\left[\mathbf{I}_{4}-\mathbf{A}\right]^{-1}\begin{pmatrix}\int_{\Theta}\kappa_{1,2}\left(\theta\right)\tilde{E}_{l,1-\tau}\left(\theta\right)\hat{h}'\left(y\left(\theta\right)\right)d\theta\\ \int_{\Theta}\kappa_{2,2}\left(\theta\right)\tilde{E}_{l,1-\tau}\left(\theta\right)\hat{h}'\left(y\left(\theta\right)\right)d\theta\\ \int_{\Theta}\kappa_{3,2}\left(\theta\right)\tilde{E}_{l,1-\tau}\left(\theta\right)\hat{h}'\left(y\left(\theta\right)\right)d\theta\\ \int_{\Theta}\kappa_{4,2}\left(\theta\right)\tilde{E}_{l,1-\tau}\left(\theta\right)\hat{h}'\left(y\left(\theta\right)\right)d\theta\end{pmatrix},$$

where the 4×4 matrix $\mathbf{A}$ is defined by

$$\mathbf{A}_{i,j}=\int_{\Theta}\kappa_{j,1}\left(\theta\right)\kappa_{i,2}\left(\theta\right)d\theta.$$

We can finally write

$$\begin{aligned}\mathrm{d}\hat{l}\left(\theta,h\right)\approx&-\tilde{E}_{l,1-\tau}\left(\theta\right)\hat{h}'\left(y\left(\theta\right)\right)+\tilde{E}_{l,w}\left(\theta\right)\left[a_{1}+\frac{\alpha\left(\theta\right)}{\chi\left(\theta\right)}\left(a_{2}+a_{3}\left(\theta-\mu_{\theta}\right)+a_{4}\left(\theta-\mu_{\theta}\right)^{2}\right)\right]\\ =&-\tilde{E}_{l,1-\tau}\left(\theta\right)\hat{h}'\left(y\left(\theta\right)\right)+\tilde{E}_{l,w}\left(\theta\right)\left[a_{1}+\frac{\alpha\left(\theta\right)}{\chi\left(\theta\right)}a_{2}+a_{4}\frac{\alpha\left(\theta\right)}{\chi\left(\theta\right)}\left(\left(\theta-\mu_{\theta}\right)^{2}+\frac{a_{3}}{a_{4}}\left(\theta-\mu_{\theta}\right)\right)\right].\end{aligned}$$

The term in square brackets can be rewritten as

$$
\begin{aligned}
& a_1 + \frac{\alpha(\theta)}{\chi(\theta)} a_2 + a_4 \frac{\alpha(\theta)}{\chi(\theta)} \left(\left(\theta - \mu_\theta + \frac{a_3}{2a_4} \right)^2 - \left(\frac{a_3}{2a_4} \right)^2 \right) \\
= & a_1 + \left(a_2 - \frac{a_3^2}{4a_4} \right) \frac{\alpha(\theta)}{\chi(\theta)} + a_4 \frac{\alpha(\theta)}{\chi(\theta)} \left(\theta - \tilde{\theta} \right)^2 \\
= & \left(a_1 - 2s^2 a_4 \chi\left(\tilde{\theta}\right) \right) + \left(a_2 - \frac{a_3^2}{4a_4} \right) \frac{\alpha(\theta)}{\chi(\theta)} + 2s^2 a_4 \left(\chi\left(\tilde{\theta}\right) + \frac{\alpha(\theta)}{2s^2 \chi(\theta)} \left(\theta - \tilde{\theta} \right)^2 \right),
\end{aligned}
$$

so that

$$
\mathrm{d}\hat{l}(\theta, h) \approx - \tilde{E}_{l,1-\tau}(\theta) \hat{h}'(y(\theta)) + \tilde{E}_{l,w}(\theta) \left[c_1 + c_2 \frac{\alpha(\theta)}{\chi(\theta)} + c_3 \bar{\gamma}\left(\theta, \tilde{\theta}\right) \right],
$$

where

$$
\begin{aligned}
\tilde{\theta} = & \mu_\theta - \frac{a_3}{2a_4}, \\
c_1 = & a_1 - 2s^2 a_4 \chi\left(\tilde{\theta}\right), \\
c_2 = & a_2 - \frac{a_3^2}{4a_4}, \\
c_3 = & 2s^2 a_4.
\end{aligned}
$$

This concludes the proof.

□

B.4 Proofs of Sections 3 and 4

B.4.1 Proof of Proposition 4

We start by deriving the formula (42) for optimal taxes using mechanism design tools, i.e., by solving the optimal control problem (37, 38, 40, 41).

Proof. The Lagrangian writes:

$$
\begin{aligned}
\mathcal{L} = & \int_\Theta u(V(\theta)) \tilde{f}_\theta(\theta) d\theta + \lambda \left\{ \mathscr{F}(\mathscr{L}) - \int_\Theta [V(\theta) + v(l(\theta))] f_\theta(\theta) d\theta \right\} \\
& - \int_\Theta \mu(\theta) v'(l(\theta)) l(\theta) \frac{\omega_1[\theta, l(\theta) f_\theta(\theta), \mathscr{L}] + [l(\theta) f_\theta'(\theta) + b(\theta) f_\theta(\theta)] \omega_2[\theta, l(\theta) f_\theta(\theta), \mathscr{L}]}{\omega[\theta, l(\theta) f_\theta(\theta), \mathscr{L}]} d\theta \\
& - \int_\Theta \mu'(\theta) V(\theta) d\theta - \int_\Theta \eta(\theta) b(\theta) d\theta - \int_\Theta \eta'(\theta) l(\theta) d\theta.
\end{aligned}
$$

For simplicity of notation we denote

$$
\begin{aligned}
\hat{w}(\theta) \equiv & \hat{\omega}[\theta, l(\theta) f_\theta(\theta), \mathscr{L}] \\
\equiv & \frac{\omega_1[\theta, l(\theta) f_\theta(\theta), \mathscr{L}] + [l(\theta) f_\theta'(\theta) + b(\theta) f_\theta(\theta)] \omega_2[\theta, l(\theta) f_\theta(\theta), \mathscr{L}]}{\omega[\theta, l(\theta) f_\theta(\theta), \mathscr{L}]}.
\end{aligned} \tag{69}
$$

The first-order condition for $V(\theta)$ writes:

$$u'\left(V(\theta)\right)\tilde{f}_\theta(\theta)-\lambda f_\theta(\theta)-\mu'(\theta)=0. \tag{70}$$

The first-order conditions for $b(\theta)$ writes:

$$-\mu(\theta)v'\left(l(\theta)\right)l(\theta)\frac{\partial\hat{w}(\theta)}{\partial b(\theta)}-\eta(\theta)=0. \tag{71}$$

The first-order conditions for $l(\theta)$ is obtained by perturbing $\mathscr{L}$ in the Dirac direction δ_θ and evaluating the Gateaux derivative of $\mathcal{L}$ (i.e., heuristically, "$\frac{\partial\mathcal{L}}{\partial l(\theta)}$"):

$$\begin{aligned}\mathrm{d}\mathcal{L}\left(\mathscr{L},\delta_\theta\right)=&\lambda w(\theta)f_\theta(\theta)-\lambda v'(l(\theta))f_\theta(\theta)-\mu(\theta)v''\left(l(\theta)\right)l(\theta)\hat{w}(\theta)-\mu(\theta)v'(l(\theta))\hat{w}(\theta)\\&-\int_\Theta\mu(\theta')v'(l(\theta'))l(\theta')\mathrm{d}\hat{\omega}\left(\theta',\delta_\theta\right)d\theta'-\eta'(\theta)=0,\end{aligned} \tag{72}$$

where $\mathrm{d}\hat{\omega}\left(\theta',\delta_\theta\right)$ (or, heuristically, "$\frac{\partial\hat{w}(\theta')}{\partial l(\theta)}$") is defined as:

$$\mathrm{d}\hat{\omega}\left(\theta',\delta_\theta\right)\equiv\lim_{\mu\to0}\frac{1}{\mu}\left\{\hat{\omega}\left[\theta',\left(l\left(\theta'\right)+\mu\delta_\theta\left(\theta'\right)\right)f_\theta\left(\theta'\right),\mathscr{L}+\mu\delta_\theta\right]-\hat{\omega}\left[\theta',l\left(\theta'\right)f_\theta\left(\theta'\right),\mathscr{L}\right]\right\}. \tag{73}$$

Now, note that
$$\frac{\partial\hat{w}(\theta)}{\partial b(\theta)}=\frac{\omega_2\left[\theta,l(\theta)f_\theta(\theta),\mathscr{L}\right]f_\theta(\theta)}{\omega(\theta,l(\theta)f_\theta(\theta),\mathscr{L})}=\frac{\bar{\bar{\gamma}}(\theta,\theta)}{l(\theta)},$$
where the second equality comes from the fact that, by definition of the own-wage elasticity,

$$\bar{\bar{\gamma}}\left(\theta,\theta\right)=\frac{\omega_2\left[\theta,l(\theta)f_\theta(\theta),\mathscr{L}\right]f_\theta(\theta)}{w\left(\theta\right)}l\left(\theta\right) \tag{74}$$

(intuitively, $\bar{\bar{\gamma}}=\frac{l}{w}\frac{\partial w}{\partial l}$, keeping $\mathscr{L}$ constant). Thus, we can rewrite (71) as:

$$\eta(\theta)=-\mu(\theta)v'\left(l(\theta)\right)l(\theta)\frac{\bar{\bar{\gamma}}\left(\theta,\theta\right)}{l(\theta)}=-\mu(\theta)v'\left(l(\theta)\right)\bar{\bar{\gamma}}\left(\theta,\theta\right), \tag{75}$$

which implies

$$\eta'(\theta)=-\mu'(\theta)v'\left(l(\theta)\right)\bar{\bar{\gamma}}\left(\theta,\theta\right)-\mu(\theta)v''\left(l(\theta)\right)l'(\theta)\bar{\bar{\gamma}}\left(\theta,\theta\right)-\mu(\theta)v'\left(l(\theta)\right)\bar{\bar{\gamma}}'\left(\theta,\theta\right).$$

Using this expression to substitute for $\eta'(\theta)$ into (72) yields

$$\begin{aligned}0=&\lambda w(\theta)f_\theta(\theta)-\lambda v'(l(\theta))f_\theta(\theta)-\mu(\theta)v''\left(l(\theta)\right)l(\theta)\hat{w}(\theta)-\mu(\theta)v'(l(\theta))\hat{w}(\theta)\\&+\mu'(\theta)v'\left(l(\theta)\right)\bar{\bar{\gamma}}\left(\theta,\theta\right)+\mu(\theta)v''\left(l(\theta)\right)b\left(\theta\right)\bar{\bar{\gamma}}\left(\theta,\theta\right)+\mu(\theta)v'\left(l(\theta)\right)\bar{\bar{\gamma}}'\left(\theta,\theta\right)\\&-\int_\Theta\mu(\theta')v'(l(\theta'))l(\theta')\frac{\partial\hat{w}(\theta')}{\partial l(\theta)}d\theta'.\end{aligned} \tag{76}$$

We now analyze the last line of this equation. From (73), we have

$$\begin{aligned}
d\hat{\omega}\left(\theta', \delta_\theta\right) =& \hat{\omega}_2\left(\theta', l\left(\theta'\right) f_\theta\left(\theta'\right), \mathscr{L}\right) f_\theta\left(\theta'\right) \delta_\theta\left(\theta'\right) \\
&+ \lim_{\mu \to 0} \frac{1}{\mu}\left\{\hat{\omega}\left[\theta', l\left(\theta'\right) f_\theta\left(\theta'\right), \mathscr{L} + \mu\delta_\theta\right] - \hat{\omega}\left[\theta', l\left(\theta'\right) f_\theta\left(\theta'\right), \mathscr{L}\right]\right\} \\
\equiv& \hat{\omega}_2\left(\theta', l\left(\theta'\right) f_\theta\left(\theta'\right), \mathscr{L}\right) f_\theta\left(\theta'\right) \delta_\theta\left(\theta'\right) + \hat{\omega}_{3,\theta}\left[\theta', l\left(\theta'\right) f_\theta\left(\theta'\right), \mathscr{L}\right],
\end{aligned}$$

where we introduce the short-hand notation $\hat{\omega}_{3,\theta}$ in the last line for simplicity of exposition. Denote by $\hat{\omega}_{13,\theta}$ and $\hat{\omega}_{23,\theta}$ the derivatives of $\hat{\omega}_{3,\theta}$ with respect to its first and second variables, respectively.

Now recall the notation (69) and note that

$$\begin{aligned}
\hat{\omega}_2(\theta, l(\theta) f_\theta(\theta), \mathscr{L}) f_\theta(\theta) =& \frac{\left[f_\theta(\theta)\omega_{12} + f_\theta'(\theta)\omega_2 + \left(l(\theta) f_\theta'(\theta) + b(\theta) f_\theta(\theta)\right) f_\theta(\theta)\omega_{22}\right] w(\theta)}{w^2(\theta)} \\
&- \frac{\left[\omega_1 + \left(l(\theta) f_\theta'(\theta) + b(\theta) f_\theta(\theta)\right)\omega_2\right] f_\theta(\theta)\omega_2}{w^2(\theta)},
\end{aligned}$$

so that, using the definition of $\hat{w}(\theta)$ and equation (74),

$$\hat{\omega}_2\left[\theta, l(\theta) f_\theta(\theta), \mathscr{L}\right] f_\theta(\theta) = \frac{\omega_{12} + \frac{f_\theta'(\theta)}{f_\theta(\theta)}\omega_2 + \left[l(\theta) f_\theta'(\theta) + b(\theta) f_\theta(\theta)\right]\omega_{22}}{w(\theta)} f_\theta(\theta) - \frac{\hat{w}(\theta)}{l(\theta)}\bar{\bar{\gamma}}(\theta, \theta).$$

Now, we have

$$\begin{aligned}
\left(\frac{\bar{\bar{\gamma}}(\theta, \theta)}{l(\theta)}\right)' =& \frac{\left\{\left[\omega_{21} + \left(l(\theta) f_\theta'(\theta) + b(\theta) f_\theta(\theta)\right)\omega_{22}\right] f_\theta(\theta) + \omega_2 f_\theta'(\theta)\right\} w(\theta)}{w^2(\theta)} \\
&- \frac{\omega_2 f_\theta(\theta)\left[\omega_1 + \left(l(\theta) f_\theta'(\theta) + b(\theta) f_\theta(\theta)\right)\omega_2\right]}{w^2(\theta)} \\
=& \frac{\omega_{12} + \frac{f_\theta'(\theta)}{f_\theta(\theta)}\omega_2 + \left[l(\theta) f_\theta'(\theta) + b(\theta) f_\theta(\theta)\right]\omega_{22}}{w(\theta)} f_\theta(\theta) - \frac{\hat{w}(\theta)}{l(\theta)}\bar{\bar{\gamma}}(\theta, \theta).
\end{aligned}$$

Therefore, the previous two equalities imply

$$\hat{\omega}_2\left[\theta, l(\theta) f_\theta(\theta), \mathscr{L}\right] f_\theta(\theta) = \left(\frac{\bar{\bar{\gamma}}(\theta, \theta)}{l(\theta)}\right)' = \frac{\bar{\bar{\gamma}}'(\theta, \theta) l(\theta) - \bar{\bar{\gamma}}(\theta, \theta) b(\theta)}{l^2(\theta)}. \tag{77}$$

Next, from definition (69) we have (omitting the arguments $(\theta', L(\theta'), \mathscr{L})$ on the right hand side)

$$\begin{aligned}
&\hat{\omega}_{3,\theta}\left[\theta', l\left(\theta'\right) f_\theta\left(\theta'\right), \mathscr{L}\right] \\
=& \frac{\omega_{13,\theta} + \left(l(\theta') f_\theta'(\theta') + b(\theta') f_\theta(\theta')\right)\omega_{23,\theta}}{w\left(\theta'\right)} - \frac{\left[\omega_1 + \left(l(\theta') f_\theta'(\theta') + b(\theta') f_\theta(\theta')\right)\omega_2\right]\omega_{3,\theta}}{w^2\left(\theta'\right)} \\
=& \frac{\omega_{13,\theta} + \left(l(\theta') f_\theta'(\theta') + b(\theta') f_\theta(\theta')\right)\omega_{23,\theta}}{w\left(\theta'\right)} - \frac{\hat{w}\left(\theta'\right)}{l(\theta)}\bar{\gamma}\left(\theta', \theta\right),
\end{aligned} \tag{78}$$

where the second equality follows from the definition of $\hat{w}(\theta')$ and of the cross-wage elasticities

$$\begin{aligned}\bar{\gamma}(\theta',\theta) =& \frac{l(\theta)}{w(\theta')} \times \lim_{\mu\to 0} \frac{1}{\mu}\left\{\omega\left[\theta', l(\theta') f_\theta(\theta'), \mathscr{L}+\mu\delta_\theta\right] - \omega\left[\theta', l(\theta') f_\theta(\theta'), \mathscr{L}\right]\right\} \\ =& \frac{l(\theta)}{w(\theta')}\omega_{3,\theta}\left(\theta', l(\theta') f_\theta(\theta'), \mathscr{L}\right).\end{aligned}$$

Note moreover that this equality implies

$$\begin{aligned}\frac{\partial\bar{\gamma}(\theta',\theta)}{\partial\theta'} =& l(\theta)\frac{\omega_{13,\theta}\left(\theta', l(\theta') f_\theta(\theta'), \mathscr{L}\right) + (l(\theta')f_\theta'(\theta') + b(\theta')f_\theta(\theta'))\,\omega_{23,\theta}\left(\theta', l(\theta') f_\theta(\theta'), \mathscr{L}\right)}{w(\theta')} \\ & - l(\theta)\frac{\omega_{3,\theta}\left(\theta', l(\theta') f_\theta(\theta'), \mathscr{L}\right)}{w^2(\theta')} \times \ldots \\ & \left[\omega_1\left(\theta', l(\theta') f_\theta(\theta'), \mathscr{L}\right) + (l(\theta')f_\theta'(\theta') + b(\theta')f_\theta(\theta'))\,\omega_2\left(\theta', l(\theta') f_\theta(\theta'), \mathscr{L}\right)\right] \\ =& l(\theta)\frac{\omega_{13,\theta} + (l(\theta')f_\theta'(\theta') + b(\theta')f_\theta(\theta'))\,\omega_{23,\theta}}{w(\theta')} - \hat{w}(\theta')\,\bar{\gamma}(\theta',\theta),\end{aligned}$$

and thus, from (78)

$$\hat{\omega}_{3,\theta}\left[\theta', l(\theta') f_\theta(\theta'), \mathscr{L}\right] = \frac{1}{l(\theta)}\frac{\partial\bar{\gamma}(\theta',\theta)}{\partial\theta'}. \tag{79}$$

Substitute equations (77) and (79) in (76) to get:

$$\begin{aligned}0 =& \lambda w(\theta)f_\theta(\theta) - \lambda v'(l(\theta))f_\theta(\theta) - \mu(\theta)v''(l(\theta))\,l(\theta)\hat{w}(\theta) - \mu(\theta)v'(l(\theta))\hat{w}(\theta) \\ & + \mu'(\theta)v'(l(\theta))\,\bar{\bar{\gamma}}(\theta,\theta) + \mu(\theta)v''(l(\theta))\,l'(\theta)\bar{\bar{\gamma}}(\theta,\theta) + \mu(\theta)v'(l(\theta))\,\bar{\bar{\gamma}}'(\theta,\theta) \\ & - \int_\Theta \mu(\theta')v'(l(\theta'))l(\theta') \times \ldots \\ & \left\{\hat{\omega}_2\left(\theta', l(\theta') f_\theta(\theta'), \mathscr{L}\right) f_\theta(\theta')\,\delta_\theta(\theta') + \hat{\omega}_{3,\theta}\left[\theta', l(\theta') f_\theta(\theta'), \mathscr{L}\right]\right\} d\theta' \\ =& \lambda w(\theta)f_\theta(\theta) - \lambda v'(l(\theta))f_\theta(\theta) - \mu(\theta)v''(l(\theta))\,l(\theta)\hat{w}(\theta) - \mu(\theta)v'(l(\theta))\hat{w}(\theta) \\ & + \mu'(\theta)v'(l(\theta))\,\bar{\bar{\gamma}}(\theta,\theta) + \mu(\theta)v''(l(\theta))\,b(\theta)\,\bar{\bar{\gamma}}(\theta,\theta) \\ & + \left\{\mu(\theta)v'(l(\theta))\,\frac{b(\theta)}{l(\theta)}\bar{\bar{\gamma}}(\theta,\theta) + \mu(\theta)v'(l(\theta))\,l(\theta)\,\hat{\omega}_2 f_\theta(\theta)\right\} \\ & - \left\{\mu(\theta)v'(l(\theta))l(\theta)\hat{\omega}_2\left(\theta, l(\theta) f_\theta(\theta), \mathscr{L}\right) f_\theta(\theta) + \int_\Theta \mu(\theta')v'(l(\theta'))l(\theta')\frac{1}{l(\theta)}\frac{\partial\bar{\gamma}(\theta',\theta)}{\partial\theta'}d\theta'\right\},\end{aligned}$$

and hence,

$$\begin{aligned}0 =& \lambda w(\theta)f_\theta(\theta) - \lambda v'(l(\theta))f_\theta(\theta) - \mu(\theta)v''(l(\theta))\,l(\theta)\hat{w}(\theta) - \mu(\theta)v'(l(\theta))\hat{w}(\theta) \\ & + \mu'(\theta)v'(l(\theta))\,\bar{\bar{\gamma}}(\theta,\theta) + \mu(\theta)v''(l(\theta))\,b(\theta)\,\bar{\bar{\gamma}}(\theta,\theta) + \mu(\theta)v'(l(\theta))\,\frac{b(\theta)}{l(\theta)}\bar{\bar{\gamma}}(\theta,\theta) \\ & - \int_\Theta \mu(\theta')v'(l(\theta'))\frac{l(\theta')}{l(\theta)}\frac{\partial\bar{\gamma}(\theta',\theta)}{\partial\theta'}d\theta'.\end{aligned} \tag{80}$$

Using the definition of the labor supply elasticity (6), we finally obtain

$$\begin{aligned}0 =&\lambda w(\theta)f_\theta(\theta) - \lambda v'(l(\theta))f_\theta(\theta) - \mu(\theta)v''\left(l(\theta)\right)l(\theta)\hat{w}(\theta) - \mu(\theta)v'(l(\theta))\hat{w}(\theta)\\ &+ \mu'(\theta)v'\left(l(\theta)\right)\bar{\bar{\gamma}}\left(\theta,\theta\right) + \mu(\theta)\left(1+\frac{1}{\varepsilon_{l,1-\tau}(\theta)}\right)v'\left(l(\theta)\right)\frac{b\left(\theta\right)}{l\left(\theta\right)}\bar{\bar{\gamma}}\left(\theta,\theta\right)\\ &- \int_\Theta \mu(\theta')v'(l(\theta'))\frac{l(\theta')}{l\left(\theta\right)}\frac{\partial\bar{\gamma}\left(\theta',\theta\right)}{\partial\theta'}d\theta'. \end{aligned} \tag{81}$$

Moreover, defining the wedge $(1-\tau(\theta))w(\theta) = v'(l(\theta))$ and noting that

$$v'\left(l(\theta)\right) + v''\left(l(\theta)\right)l(\theta) = (1-\tau(\theta))w(\theta)\left(1+\frac{1}{\varepsilon_{l,1-\tau}(\theta)}\right),$$

we can rewrite (81) as

$$\begin{aligned}0 =&\lambda w(\theta)f_\theta(\theta) - \lambda(1-\tau(\theta))w(\theta)f_\theta(\theta) - \mu(\theta)(1-\tau(\theta))w(\theta)\left(1+\frac{1}{\varepsilon_{l,1-\tau}(\theta)}\right)\hat{w}(\theta)\\ &+ \mu'(\theta)(1-\tau(\theta))w(\theta)\bar{\bar{\gamma}}\left(\theta,\theta\right) + \mu(\theta)\left(1+\frac{1}{\varepsilon_{l,1-\tau}(\theta)}\right)(1-\tau(\theta))w(\theta)\frac{b\left(\theta\right)}{l\left(\theta\right)}\bar{\bar{\gamma}}\left(\theta,\theta\right)\\ &- \frac{1}{l\left(\theta\right)}\int_\Theta \mu(\theta')(1-\tau(\theta'))y(\theta')\frac{\partial\bar{\gamma}\left(\theta',\theta\right)}{\partial\theta'}d\theta'\\ =&\lambda\tau(\theta)w(\theta)f_\theta(\theta) + \mu(\theta)(1-\tau(\theta))w(\theta)\left(1+\frac{1}{\varepsilon_{l,1-\tau}(\theta)}\right)\left\{\frac{l'\left(\theta\right)}{l\left(\theta\right)}\bar{\bar{\gamma}}\left(\theta,\theta\right) - \hat{w}(\theta)\right\}\\ &+ \mu'(\theta)(1-\tau(\theta))w(\theta)\bar{\bar{\gamma}}\left(\theta,\theta\right) - \frac{1}{l\left(\theta\right)}\int_\Theta \mu(\theta')(1-\tau(\theta'))y(\theta')\frac{\partial\bar{\gamma}\left(\theta',\theta\right)}{\partial\theta'}d\theta'.\end{aligned}$$

Using the fact that $\hat{w}(\theta) = \frac{w'(\theta)}{w(\theta)}$ and dividing through by $\lambda(1-\tau(\theta))w(\theta)f_\theta(\theta)$ yields

$$\begin{aligned}0 =&\frac{\tau(\theta)}{1-\tau(\theta)} + \frac{\mu(\theta)}{\lambda f_\theta(\theta)}\left(1+\frac{1}{\varepsilon_{l,1-\tau}(\theta)}\right)\left\{\frac{l'\left(\theta\right)}{l\left(\theta\right)}\bar{\bar{\gamma}}\left(\theta,\theta\right) - \frac{w'(\theta)}{w(\theta)}\right\}\\ &+ \frac{\mu'(\theta)}{\lambda f_\theta(\theta)}\bar{\bar{\gamma}}\left(\theta,\theta\right) - \frac{1}{\lambda(1-\tau(\theta))y(\theta)f_\theta(\theta)}\int_\Theta \mu(\theta')(1-\tau(\theta'))y(\theta')\frac{\partial\bar{\gamma}\left(\theta',\theta\right)}{\partial\theta'}d\theta',\end{aligned}$$

and hence, using the relationship between the densities of productivities and wages:

$$\begin{aligned}\frac{\tau(\theta)}{1-\tau(\theta)} =& \left(1+\frac{1}{\varepsilon_{l,1-\tau}(\theta)}\right)\frac{\mu(\theta)}{\lambda w(\theta)f_w(w(\theta))}\left(1-\bar{\bar{\gamma}}\left(\theta,\theta\right)\frac{\frac{l'(\theta)}{l(\theta)}}{\frac{w'(\theta)}{w(\theta)}}\right) - \frac{\mu'(\theta)}{\lambda f_\theta(\theta)}\bar{\bar{\gamma}}\left(\theta,\theta\right)\\ &+ \frac{1}{\lambda(1-\tau(\theta))y(\theta)f_\theta(\theta)}\int_\Theta \mu(\theta')(1-\tau(\theta'))y(\theta')\frac{\partial\bar{\gamma}\left(\theta',\theta\right)}{\partial\theta'}d\theta'.\end{aligned} \tag{82}$$

Note that for a CES production function, we have $\frac{\partial\bar{\gamma}\left(\theta',\theta\right)}{\partial\theta'} = 0$.

Finally, an alternative optimal tax formula is given by integrating the previous equation by parts (with the appropriate boundary conditions on $\mu\left(\theta'\right)$):

$$\int_\Theta \mu(\theta')(1-\tau(\theta'))y(\theta')\frac{\partial\bar{\gamma}\left(\theta',\theta\right)}{\partial\theta'}d\theta' = -\int_\Theta \bar{\gamma}\left(\theta',\theta\right)\frac{d}{d\theta'}\left[\mu(\theta')(1-\tau(\theta'))y(\theta')\right]d\theta'.$$

We therefore obtain

$$\begin{aligned}
\frac{\tau(\theta)}{1-\tau(\theta)} =& \left(1+\frac{1}{\varepsilon_{l,1-\tau}(\theta)}\right)\frac{\mu(\theta)}{\lambda w(\theta)f_w(w(\theta))}\left(1-\bar{\bar{\gamma}}\left(\theta,\theta\right)\frac{\frac{l'(\theta)}{l(\theta)}}{\frac{w'(\theta)}{w(\theta)}}\right) \\
&-\frac{\mu'(\theta)}{\lambda f_\theta(\theta)}\bar{\bar{\gamma}}\left(\theta,\theta\right)-\frac{\int_\Theta\left[\mu(x)v'(l(x))l(x)\right]'\bar{\gamma}\left(x,\theta\right)dx}{\lambda(1-\tau(\theta))y(\theta)f_\theta(\theta)} \\
=& \left(1+\frac{1}{\varepsilon_{l,1-\tau}(\theta)}\right)\frac{\mu(\theta)}{\lambda w(\theta)f_w(w(\theta))}\left(1-\bar{\bar{\gamma}}\left(\theta,\theta\right)\frac{\frac{l'(\theta)}{l(\theta)}}{\frac{w'(\theta)}{w(\theta)}}\right) \\
&-\frac{\int_\Theta\left[\mu(x)v'(l(x))l(x)\right]'\gamma\left(x,\theta\right)dx}{\lambda(1-\tau(\theta))y(\theta)f_\theta(\theta)}-\frac{\mu'(\theta)}{\lambda f_\theta(\theta)}\bar{\bar{\gamma}}\left(\theta,\theta\right) \\
&+\frac{\left[\mu'(\theta)v'(l(\theta))l(\theta)+\mu(\theta)v''(l(\theta))l(\theta)+\mu(\theta)v'(l(\theta))l'(\theta)\right]\bar{\bar{\gamma}}\left(\theta,\theta\right)}{\lambda(1-\tau(\theta))y(\theta)f_\theta(\theta)},
\end{aligned}$$

where the second equality uses the definition (11) of $\gamma\left(x,\theta\right)$. This implies

$$\begin{aligned}
\frac{\tau(\theta)}{1-\tau(\theta)} =& \left(1+\frac{1}{\varepsilon_{l,1-\tau}(\theta)}\right)\frac{\mu(\theta)}{\lambda w(\theta)f_w(w(\theta))}-\frac{\int_\Theta\left[\mu(x)v'(l(x))l(x)\right]'\gamma\left(x,\theta\right)dx}{\lambda(1-\tau(\theta))y(\theta)f_\theta(\theta)} \\
&-\bar{\bar{\gamma}}\left(\theta,\theta\right)\left\{\left(1+\frac{v''(l(\theta))l(\theta)}{v'(l(\theta))}\right)\frac{\mu(\theta)}{\lambda f_\theta(\theta)}\frac{l'\left(\theta\right)}{l\left(\theta\right)}+\frac{\mu'(\theta)}{\lambda f_\theta(\theta)}-\dots\right. \\
&\left.\frac{\mu'(\theta)v'(l(\theta))l(\theta)+\mu(\theta)v''(l(\theta))l'(\theta)l(\theta)+\mu(\theta)v'(l(\theta))l'(\theta)}{\lambda v'(l(\theta))l(\theta)f_\theta(\theta)}\right\}.
\end{aligned}$$

The terms in the curly brackets in the second and third lines cancel each other out, therefore

$$\frac{\tau(\theta)}{1-\tau(\theta)}=\left(1+\frac{1}{\varepsilon_{l,1-\tau}(\theta)}\right)\frac{\mu(\theta)}{\lambda w(\theta)f_w(w(\theta))}-\frac{\int_\Theta\left[\mu(x)v'(l(x))l(x)\right]'\gamma\left(x,\theta\right)dx}{\lambda(1-\tau(\theta))y(\theta)f_\theta(\theta)}.$$

Note finally that the first-order condition (70) implies

$$\mu'(\theta)=u'\left(V(\theta)\right)\tilde{f}_\theta(\theta)-\lambda f_\theta(\theta),$$

so that, using $\mu(\bar{\theta})=0$,

$$\begin{aligned}
\mu(\theta) =& -\int_\theta^{\bar{\theta}}\left[u'\left(V(x)\right)\tilde{f}_\theta(x)-\lambda f_\theta(x)\right]dx \\
=&\lambda\int_\theta^{\bar{\theta}}\left[1-\frac{u'\left(V(x)\right)\tilde{f}_\theta(x)}{\lambda f_\theta(x)}\right]f_\theta(x)dx=\lambda\int_\theta^{\bar{\theta}}\left(1-g_\theta\left(x\right)\right)f_\theta(x)dx.
\end{aligned}$$

This concludes the proof.

□

We now derive optimal taxes using the variational tools introduced in Section 2, i.e., we tackle problem (34, 35, 36) directly. We start by deriving the expression (48) for the counteracting perturbation h_2.

B.4.2 Proof of equation (48)

Proof. Consider the perturbation h_1 defined by $h_1(y) = \mathbb{I}_{\{y \geq y^*\}}$ and $h_1'(y) = \delta_{y^*}(y)$. As usual, denote by θ^* the type such that $y(\theta^*) = y^*$. We impose that $h_1 + h_2$ has the same effects on labor supply as those that h_1 induces in the partial equilibrium framework. The general equilibrium response to $h_1 + h_2$ is given by the solution to the following integral equation: for all $\theta \in \Theta$,

$$\begin{aligned} \mathrm{d}\hat{l}(\theta, h_1 + h_2) = &- \frac{\tilde{\varepsilon}_{l,1-\tau}(\theta)}{1 - \bar{\bar{\gamma}}(\theta,\theta)\,\tilde{\varepsilon}_{l,w}(\theta)} \left\{ \frac{h_1'(y(\theta)) + h_2'(y(\theta))}{1 - T'(y(\theta))} \right\} \\ &+ \frac{\tilde{\varepsilon}_{l,w}(\theta)}{1 - \bar{\bar{\gamma}}(\theta,\theta)\,\tilde{\varepsilon}_{l,w}(\theta)} \int_{\Theta} \bar{\gamma}(\theta,\theta')\,\mathrm{d}\hat{l}(\theta', h_1 + h_2)\,d\theta'. \end{aligned} \tag{83}$$

The partial equilibrium effect of h_1, on the other hand, is given by: for all $\theta \in \Theta$,

$$\begin{aligned} \mathrm{d}\hat{l}_{PE}(\theta, h_1) &= - \tilde{\varepsilon}_{l,1-\tau}(\theta) \left\{ \frac{h_1'(y(\theta))}{1 - T'(y(\theta))} \right\} \\ &= - \frac{\varepsilon_{l,1-\tau}(\theta^*)}{1 - T'(y(\theta^*)) + \varepsilon_{l,1-\tau}(\theta^*)\,y(\theta^*)\,T''(y(\theta^*))} \delta_{y^*}(y(\theta)). \end{aligned} \tag{84}$$

In particular, note that in partial equilibrium, we have $\mathrm{d}\hat{l}_{PE}(\theta, h_1) = 0$ for all $\theta \neq \theta^*$, i.e., the only individuals who respond to a change in the marginal tax rate at income y^* are those whose type is θ^* (and hence whose income is y^*). Substituting for (84) in the left hand side and under the integral sign of (83) yields:

$$\begin{aligned} - \frac{\tilde{\varepsilon}_{l,1-\tau}(\theta)}{1 - T'(y(\theta))} \delta_{y^*}(y(\theta)) = &- \frac{1}{1 - T'(y(\theta))} \frac{\tilde{\varepsilon}_{l,1-\tau}(\theta)}{1 - \bar{\bar{\gamma}}(\theta,\theta)\,\tilde{\varepsilon}_{l,w}(\theta)} \left\{ \delta_{y^*}(y(\theta)) + h_2'(y(\theta)) \right\} \\ &- \frac{\tilde{\varepsilon}_{l,w}(\theta)}{1 - \bar{\bar{\gamma}}(\theta,\theta)\,\tilde{\varepsilon}_{l,w}(\theta)} \int_{\Theta} \bar{\gamma}(\theta,\theta') \frac{\tilde{\varepsilon}_{l,1-\tau}(\theta')}{1 - T'(y(\theta'))} \delta_{y^*}(y(\theta'))\,d\theta', \end{aligned}$$

i.e., after changing variables in the integral in the second line of the previous expression,

$$\begin{aligned} \frac{\tilde{\varepsilon}_{l,1-\tau}(\theta)}{1 - \bar{\bar{\gamma}}(\theta,\theta)\,\tilde{\varepsilon}_{l,w}(\theta)} \frac{h_2'(y(\theta))}{1 - T'(y(\theta))} = &\left(\tilde{\varepsilon}_{l,1-\tau}(\theta) - \frac{\tilde{\varepsilon}_{l,1-\tau}(\theta)}{1 - \bar{\bar{\gamma}}(\theta,\theta)\,\tilde{\varepsilon}_{l,w}(\theta)} \right) \frac{\delta_{y^*}(y(\theta))}{1 - T'(y(\theta))} \\ &- \frac{\tilde{\varepsilon}_{l,w}(\theta)}{1 - \bar{\bar{\gamma}}(\theta,\theta)\,\tilde{\varepsilon}_{l,w}(\theta)} \left(\frac{dy(\theta^*)}{d\theta} \right)^{-1} \bar{\gamma}(\theta,\theta^*) \frac{\tilde{\varepsilon}_{l,1-\tau}(\theta^*)}{1 - T'(y(\theta^*))}. \end{aligned}$$

This implies

$$\begin{aligned} \frac{h_2'(y(\theta))}{1 - T'(y(\theta))} = &- \bar{\bar{\gamma}}(\theta,\theta)\,\tilde{\varepsilon}_{l,w}(\theta) \frac{\delta_{y^*}(y(\theta))}{1 - T'(y(\theta))} \\ &- \frac{1}{1 - T'(y(\theta^*))} \frac{\tilde{\varepsilon}_{l,w}(\theta)\,\tilde{\varepsilon}_{l,1-\tau}(\theta^*)}{\tilde{\varepsilon}_{l,1-\tau}(\theta)} \left(\frac{dy(\theta^*)}{d\theta} \right)^{-1} \bar{\gamma}(\theta,\theta^*), \end{aligned}$$

which can be rewritten as

$$\begin{aligned}
\frac{h_2'\left(y\left(\theta\right)\right)}{1-T'\left(y\left(\theta\right)\right)} &= -\frac{1}{1-T'\left(y\left(\theta^*\right)\right)}\frac{\tilde{\varepsilon}_{l,w}\left(\theta\right)\tilde{\varepsilon}_{l,1-\tau}\left(\theta^*\right)}{\tilde{\varepsilon}_{l,1-\tau}\left(\theta\right)}\left[\bar{\bar{\gamma}}\left(\theta^*,\theta^*\right)\delta_{y^*}\left(y\left(\theta\right)\right)+\left(\frac{dy\left(\theta^*\right)}{d\theta}\right)^{-1}\bar{\gamma}\left(\theta,\theta^*\right)\right] \\
&= -\frac{1}{1-T'\left(y\left(\theta^*\right)\right)}\frac{\tilde{\varepsilon}_{l,w}\left(\theta\right)\tilde{\varepsilon}_{l,1-\tau}\left(\theta^*\right)}{\tilde{\varepsilon}_{l,1-\tau}\left(\theta\right)}\left(\frac{dy\left(\theta^*\right)}{d\theta}\right)^{-1}\left[\bar{\bar{\gamma}}\left(\theta^*,\theta^*\right)\delta_{\theta^*}\left(\theta\right)+\bar{\gamma}\left(\theta,\theta^*\right)\right] \\
&= -\frac{1}{1-T'\left(y\left(\theta^*\right)\right)}\frac{\tilde{\varepsilon}_{l,w}\left(\theta\right)}{\tilde{\varepsilon}_{l,1-\tau}\left(\theta\right)}\tilde{\varepsilon}_{l,1-\tau}\left(\theta^*\right)\left(\frac{dy\left(\theta^*\right)}{d\theta}\right)^{-1}\gamma\left(\theta,\theta^*\right),
\end{aligned}$$

or

$$\begin{aligned}
h_2'\left(y\left(\theta\right)\right) = &-\left(1-T'\left(y\left(\theta\right)\right)\right)\frac{\left(1-T'\left(y\left(\theta\right)\right)-y\left(\theta\right)T''\left(y\left(\theta\right)\right)\right)\varepsilon_{l,1-\tau}\left(\theta\right)}{1-T'\left(y\left(\theta\right)\right)+\varepsilon_{l,1-\tau}\left(\theta\right)y\left(\theta\right)T''\left(y\left(\theta\right)\right)} \\
&\times\frac{1-T'\left(y\left(\theta\right)\right)+\varepsilon_{l,1-\tau}\left(\theta\right)y\left(\theta\right)T''\left(y\left(\theta\right)\right)}{\left(1-T'\left(y\left(\theta\right)\right)\right)\varepsilon_{l,1-\tau}\left(\theta\right)}\frac{\tilde{\varepsilon}_{l,1-\tau}\left(\theta^*\right)}{1-T'\left(y\left(\theta^*\right)\right)}\left(\frac{dy\left(\theta^*\right)}{d\theta}\right)^{-1}\gamma\left(\theta,\theta^*\right) \\
= &-\frac{\tilde{\varepsilon}_{l,1-\tau}\left(\theta^*\right)}{1-T'\left(y\left(\theta^*\right)\right)}\left(\frac{dy\left(\theta^*\right)}{d\theta}\right)^{-1}\times\left(1-T'\left(y\left(\theta\right)\right)-y\left(\theta\right)T''\left(y\left(\theta\right)\right)\right)\gamma\left(\theta,\theta^*\right).
\end{aligned}$$

This proves equation (48). Note that $h_2'\left(y\left(\theta\right)\right)$ is a smooth function, except for a jump (formally, a Dirac term) at $\theta=\theta^*$, which adds to the jump in marginal tax rates defined by the tax reform h_1 at θ^* so that the total response of labor supply of individuals with income y^* is equal to their response to h_1 in the partial equilibrium environment.

Now, integrate this expression from 0 to y (letting $h_2\left(0\right)=0$) to get $h_2\left(y\right)$:

$$h_2\left(y\left(\theta\right)\right) = -\frac{\tilde{\varepsilon}_{l,1-\tau}\left(\theta^*\right)}{1-T'\left(y\left(\theta^*\right)\right)}\left(\frac{dy\left(\theta^*\right)}{d\theta}\right)^{-1}\int_0^y\left(1-T'\left(y'\right)-y'T''\left(y'\right)\right)\gamma\left(y',y^*\right)dy'.$$

The integral in this expression can be rewritten as

$$\begin{aligned}
&\int_0^y\left(1-T'\left(y'\right)-y'T''\left(y'\right)\right)\bar{\bar{\gamma}}\left(y',y^*\right)\left(\frac{dy\left(\theta^*\right)}{d\theta}\right)\delta_{y^*}\left(y\right)dy' \\
&+\int_0^y\left(1-T'\left(y'\right)-y'T''\left(y'\right)\right)\bar{\gamma}\left(y',y^*\right)dy',
\end{aligned}$$

and hence

$$\begin{aligned}
h_2\left(y\left(\theta\right)\right) = -\frac{\tilde{\varepsilon}_{l,1-\tau}\left(\theta^*\right)}{1-T'\left(y\left(\theta^*\right)\right)}\Bigg\{&\left(1-T'\left(y^*\right)-y^*T''\left(y^*\right)\right)\bar{\bar{\gamma}}\left(y^*,y^*\right)\mathbb{I}_{\{y\geq y^*\}} \\
&+\left(\frac{dy\left(\theta^*\right)}{d\theta}\right)^{-1}\int_0^y\left(1-T'\left(y'\right)-y'T''\left(y'\right)\right)\bar{\gamma}\left(y',y^*\right)dy'\Bigg\}.
\end{aligned}$$

(Note that we could have obtained this expression by integrating over $\theta\in\Theta$ rather than $y\in\mathbb{R}_+$, the expression

$$\frac{dh_2\left(y\left(\theta\right)\right)}{d\theta} = -\frac{\tilde{\varepsilon}_{l,1-\tau}\left(\theta^*\right)}{1-T'\left(y\left(\theta^*\right)\right)}\left(\frac{dy\left(\theta^*\right)}{d\theta}\right)^{-1}\left(1-T'\left(y\left(\theta\right)\right)-y\left(\theta\right)T''\left(y\left(\theta\right)\right)\right)$$
$$\times\left(\bar{\gamma}\left(\theta,\theta^*\right)+\bar{\bar{\gamma}}\left(\theta^*,\theta^*\right)\delta_{\theta^*}\left(\theta\right)\right)\times\left(\frac{dy\left(\theta\right)}{d\theta}\right),$$

and and using a change of variables in the integral.) This concludes the proof.

□

We now derive the effects of the combination of perturbations h_1+h_2 on social welfare.

B.4.3 Proof of Proposition 5

Proof. We start by deriving the effect of the perturbation $h \equiv h_1 + h_2$ on the tax liability of any individual θ. We have, to a first order as $\mu \to 0$,

$$\mathrm{d}_h T\left(w\left(\theta\right)l\left(\theta\right)\right) \underset{\mu\to 0}{\sim} \mu^{-1}\left\{T\left(\tilde{w}\left(\theta\right)\left(l\left(\theta\right)+\mu \mathrm{d}l\left(\theta,h\right)\right)\right)-T\left(w\left(\theta\right)l\left(\theta\right)\right)\right\}+h_1\left(\tilde{w}\left(\theta\right)\left(l\left(\theta\right)+\mu dl_\theta\right)\right)$$
$$=\left\{\int_\Theta \gamma\left(\theta,\theta'\right)\mathrm{d}\hat{l}\left(\theta',h\right)d\theta'+\mathrm{d}\hat{l}\left(\theta,h\right)\right\}y\left(\theta\right)T'\left(y\left(\theta\right)\right)+h_1\left(y\left(\theta\right)\right)+\mu h_2\left(y\left(\theta\right)\right).$$

This can be rewritten as

$$\mathrm{d}_h T\left(y\left(\theta\right)\right) = \mathbb{I}_{\{y(\theta)\geq y^*\}} - \frac{\tilde{\varepsilon}_{l,1-\tau}\left(\theta^*\right)}{1-T'\left(y^*\right)}\left\{\int_\Theta \gamma\left(\theta,\theta'\right)\delta_{y^*}\left(y\left(\theta'\right)\right)d\theta'+\delta_{y^*}\left(y\left(\theta\right)\right)\right\}y\left(\theta\right)T'\left(y\left(\theta\right)\right)$$
$$-\frac{\tilde{\varepsilon}_{l,1-\tau}\left(\theta^*\right)}{1-T'\left(y\left(\theta^*\right)\right)}\left\{\left(1-T'\left(y^*\right)-y^*T''\left(y^*\right)\right)\bar{\bar{\gamma}}\left(y^*,y^*\right)\mathbb{I}_{\{y(\theta)\geq y^*\}}\right.$$
$$\left.+\left(\frac{dy\left(\theta^*\right)}{d\theta}\right)^{-1}\int_0^y\left(1-T'\left(y'\right)-y'T''\left(y'\right)\right)\bar{\gamma}\left(y',y^*\right)dy'\right\},$$

i.e.,

$$\mathrm{d}_h T\left(y\left(\theta\right)\right) = \mathbb{I}_{\{y(\theta)\geq y^*\}} - \frac{T'\left(y\left(\theta\right)\right)}{1-T'\left(y^*\right)}\tilde{\varepsilon}_{l,1-\tau}\left(\theta^*\right)y\left(\theta\right)\delta_{y^*}\left(y\left(\theta\right)\right)$$
$$-\frac{\tilde{\varepsilon}_{l,1-\tau}\left(\theta^*\right)}{1-T'\left(y\left(\theta^*\right)\right)}\left\{\gamma\left(\theta,\theta^*\right)\left(\frac{dy\left(\theta^*\right)}{d\theta}\right)^{-1}y\left(\theta\right)T'\left(y\left(\theta\right)\right)\ldots\right.$$
$$+\left(1-T'\left(y^*\right)-y^*T''\left(y^*\right)\right)\bar{\bar{\gamma}}\left(y^*,y^*\right)\mathbb{I}_{\{y(\theta)\geq y^*\}}\left(\frac{dy\left(\theta^*\right)}{d\theta}\right)$$
$$\left.+\int_0^y\left(1-T'\left(y'\right)-y'T''\left(y'\right)\right)\bar{\gamma}\left(y',y^*\right)dy'\right\},$$

i.e.,

$$\begin{aligned}\mathrm{d}_h T\left(y\left(\theta\right)\right)=&\mathbb{I}_{\{y(\theta)\geq y^*\}}-\frac{T'\left(y\left(\theta\right)\right)}{1-T'\left(y^*\right)}\tilde{\varepsilon}_{l,1-\tau}\left(\theta^*\right)y\left(\theta\right)\delta_{y^*}\left(y\left(\theta\right)\right)\\&-\frac{\tilde{\varepsilon}_{l,1-\tau}\left(\theta^*\right)}{1-T'\left(y\left(\theta^*\right)\right)}\left(\frac{dy\left(\theta^*\right)}{d\theta}\right)^{-1}\left\{\gamma\left(\theta,\theta^*\right)y\left(\theta\right)T'\left(y\left(\theta\right)\right)\ldots\right.\\&\left.+\int_0^y\left(1-T'\left(y'\right)-y'T''\left(y'\right)\right)\gamma\left(y',y^*\right)dy'\right\}.\end{aligned}$$

Next, we derive the effect of the perturbation h_1+h_2 on the aggregate tax liability. We have, to a first order as $\mu\to 0$,

$$\begin{aligned}&\mathrm{d}_h\left[\int_{\mathbb{R}_+}T\left(y\right)f_y\left(y\right)dy\right]\\=&\int_{\mathbb{R}_+}\left\{\mathbb{I}_{\{y\geq y^*\}}-\frac{T'\left(y\right)}{1-T'\left(y^*\right)}\tilde{\varepsilon}_{l,1-\tau}\left(y^*\right)y\delta_{y^*}\left(y\right)\right\}f_y\left(y\right)dy-\frac{\tilde{\varepsilon}_{l,1-\tau}\left(\theta^*\right)}{1-T'\left(y\left(\theta^*\right)\right)}\left(\frac{dy\left(\theta^*\right)}{d\theta}\right)^{-1}\ldots\\&\times\int_{\mathbb{R}_+}\left\{T'\left(y\right)\gamma\left(y,y^*\right)y+\int_0^y\left(1-T'\left(y'\right)-y'T''\left(y'\right)\right)\gamma\left(y',y^*\right)dy'\right\}f_y\left(y\right)dy\\=&1-F_y\left(y^*\right)-\frac{T'\left(y^*\right)}{1-T'\left(y^*\right)}\tilde{\varepsilon}_{l,1-\tau}\left(y^*\right)y^*f_y\left(y^*\right)\\&-\frac{\tilde{\varepsilon}_{l,1-\tau}\left(\theta^*\right)}{1-T'\left(y\left(\theta^*\right)\right)}\left(\frac{dy\left(\theta^*\right)}{d\theta}\right)^{-1}\int_{\mathbb{R}_+}T'\left(y\right)\gamma\left(y,y^*\right)yf_y\left(y\right)dy\\&-\frac{\tilde{\varepsilon}_{l,1-\tau}\left(\theta^*\right)}{1-T'\left(y\left(\theta^*\right)\right)}\left(\frac{dy\left(\theta^*\right)}{d\theta}\right)^{-1}\int_{y'=0}^{\infty}\int_{y=y'}^{\infty}\left(1-T'\left(y'\right)-y'T''\left(y'\right)\right)\gamma\left(y',y^*\right)f_y\left(y\right)dydy',\end{aligned}$$

where we obtained the expression in the last line by switching the two integrals. Hence we get

$$\begin{aligned}&\mathrm{d}\mathscr{R}\left(T,h\right)\\=&1-F_y\left(y^*\right)-\frac{T'\left(y^*\right)}{1-T'\left(y^*\right)}\tilde{\varepsilon}_{l,1-\tau}\left(y^*\right)y^*f_y\left(y^*\right)\\&-\frac{\tilde{\varepsilon}_{l,1-\tau}\left(\theta^*\right)}{1-T'\left(y\left(\theta^*\right)\right)}\left(\frac{dy\left(\theta^*\right)}{d\theta}\right)^{-1}\int_{\mathbb{R}_+}T'\left(y\right)\gamma\left(y,y^*\right)yf_y\left(y\right)dy\\&-\frac{\tilde{\varepsilon}_{l,1-\tau}\left(\theta^*\right)}{1-T'\left(y\left(\theta^*\right)\right)}\left(\frac{dy\left(\theta^*\right)}{d\theta}\right)^{-1}\int_{y'=0}^{\infty}\left(1-T'\left(y'\right)-y'T''\left(y'\right)\right)\gamma\left(y',y^*\right)\left(1-F_y\left(y'\right)\right)dy'\\=&1-F_y\left(y^*\right)-\frac{T'\left(y^*\right)}{1-T'\left(y^*\right)}\tilde{\varepsilon}_{l,1-\tau}\left(y^*\right)y^*f_y\left(y^*\right)-\frac{\tilde{\varepsilon}_{l,1-\tau}\left(\theta^*\right)}{1-T'\left(y\left(\theta^*\right)\right)}\left(\frac{dy\left(\theta^*\right)}{d\theta}\right)^{-1}\ldots\\&\times\int_0^{\infty}\left\{\left(1-T'\left(y\right)-yT''\left(y\right)\right)\left(1-F_y\left(y\right)\right)+T'\left(y\right)yf_y\left(y\right)\right\}\gamma\left(y,y^*\right)dy.\end{aligned}$$

But by Euler's homogeneous function theorem, we have

$$-\frac{\tilde{\varepsilon}_{l,1-\tau}\left(\theta^*\right)}{1-T'\left(y\left(\theta^*\right)\right)}\left(\frac{dy\left(\theta^*\right)}{d\theta}\right)^{-1}\int_0^{\infty}\gamma\left(y,y^*\right)yf_y\left(y\right)dy=0,$$

so that substracting this (zero) term from the previous equation, we can write

$$\begin{aligned} d\mathscr{R}(T,h) =& 1-F_y(y^*)-\frac{T'(y^*)}{1-T'(y^*)}\tilde{\varepsilon}_{l,1-\tau}(y^*)\,y^*f_y(y^*)-\frac{\tilde{\varepsilon}_{l,1-\tau}(\theta^*)}{1-T'(y(\theta^*))}\left(\frac{dy(\theta^*)}{d\theta}\right)^{-1}\ldots \\ &\times\int_0^\infty\left\{(1-T'(y)-yT''(y))(1-F_y(y))-(1-T'(y))\,yf_y(y)\right\}\gamma(y,y^*)\,dy. \end{aligned}$$

Now note that

$$(1-T'(y)-yT''(y))(1-F_y(y))-(1-T'(y))\,yf_y(y)=\left[(1-T'(y))\,y\,(1-F_y(y))\right]',$$

so that we finally obtain

$$\begin{aligned} d\mathscr{R}(T,h) =& 1-F_y(y^*)-\frac{T'(y^*)}{1-T'(y^*)}\tilde{\varepsilon}_{l,1-\tau}(y^*)\,y^*f_y(y^*) \\ &-\frac{\tilde{\varepsilon}_{l,1-\tau}(\theta^*)}{1-T'(y(\theta^*))}\left(\frac{dy(\theta^*)}{d\theta}\right)^{-1}\int_{\mathbb{R}_+}\left[(1-T'(y))\,y\,(1-F_y(y))\right]'\gamma(y,y^*)\,dy. \end{aligned}$$

Note that using (11), this equation can also be expressed as

$$\begin{aligned} d\mathscr{R}(T,h) =& 1-F_y(y^*)-\frac{T'(y^*)}{1-T'(y^*)}\tilde{\varepsilon}_{l,1-\tau}(y^*)\,y^*f_y(y^*) \\ &-\frac{\tilde{\varepsilon}_{l,1-\tau}(\theta^*)}{1-T'(y(\theta^*))}\left\{\psi'(y^*)\,\bar{\bar{\gamma}}(y^*,y^*)+\int_{\mathbb{R}_+}\psi'(y)\,\frac{\bar{\gamma}(y,y^*)}{y'(\theta^*)}dy\right\}, \end{aligned}$$

where we denote $\psi(y)\equiv(1-T'(y))\,y\,(1-F_y(y))$ and $y'(\theta)\equiv\frac{dy(\theta)}{d\theta}$.

Next, we derive the effect of the perturbation $h=h_1+h_2$ on the utility of any individual θ. First, the change in individual consumption due to the perturbation is

$$\begin{aligned} &d_h[y(\theta)-T(y(\theta))]=d_h[y(\theta)]-d_h[T(y(\theta))] \\ =&-\frac{\tilde{\varepsilon}_{l,1-\tau}(\theta^*)}{1-T'(y(\theta^*))}\left\{\int_\Theta\gamma(\theta,\theta')\,\delta_{y^*}(y(\theta'))\,d\theta'+\delta_{y^*}(y(\theta))\right\}y(\theta)\,(1-T'(y(\theta))) \\ &-h_1(y(\theta))-h_2(y(\theta)) \\ =&-\mathbb{I}_{\{y(\theta)\geq y^*\}}-\frac{1-T'(y(\theta))}{1-T'(y^*)}\tilde{\varepsilon}_{l,1-\tau}(\theta^*)\,y(\theta)\,\delta_{y^*}(y(\theta))-\frac{\tilde{\varepsilon}_{l,1-\tau}(\theta^*)}{1-T'(y(\theta^*))}\left(\frac{dy(\theta^*)}{d\theta}\right)^{-1}\ldots \\ &\times\left\{(1-T'(y(\theta)))\,y(\theta)\,\gamma(\theta,\theta^*)-\int_0^y(1-T'(y')-y'T''(y'))\,\gamma(y',y^*)\,dy'\right\}, \end{aligned}$$

where the last equality follows from the same steps as those of the derivation of $d_hT(y(\theta))$ above.

Thus the change in individual utility due to the perturbation is

$$\begin{aligned}
&\mathrm{d}_h \left[u\left(y\left(\theta\right)-T\left(y\left(\theta\right)\right)-v\left(l\left(\theta\right)\right)\right)\right] \\
=&\left(\mathrm{d}_h\left[y\left(\theta\right)\right]-\mathrm{d}_h\left[T\left(y\left(\theta\right)\right)\right]-v'\left(l\left(\theta\right)\right)\mathrm{d}_h l\left(\theta\right)\right)u'\left(y\left(\theta\right)-T\left(y\left(\theta\right)\right)-v\left(l\left(\theta\right)\right)\right) \\
=&\left\{-\mathbb{I}_{\{y(\theta)\geq y^*\}}-\frac{\tilde{\varepsilon}_{l,1-\tau}\left(\theta^*\right)}{1-T'\left(y^*\right)}\left[\left(1-T'\left(y\left(\theta\right)\right)\right)y\left(\theta\right)-l\left(\theta\right)v'\left(l\left(\theta\right)\right)\right]\delta_{y^*}\left(y\left(\theta\right)\right)\right\}u'\left(\theta\right) \\
&-\frac{\tilde{\varepsilon}_{l,1-\tau}\left(\theta^*\right)}{1-T'\left(y^*\right)}\left(\frac{dy\left(\theta^*\right)}{d\theta}\right)^{-1}u'\left(\theta\right)\left\{\left(1-T'\left(y\left(\theta\right)\right)\right)y\left(\theta\right)\gamma\left(\theta,\theta^*\right)\ldots\right. \\
&\qquad\qquad\left.-\int_0^y\left(1-T'\left(y'\right)-y'T''\left(y'\right)\right)\gamma\left(y',y^*\right)dy'\right\}.
\end{aligned}$$

Using the individual first-order condition (1), this can be rewritten as

$$\begin{aligned}
&\mathrm{d}_h \left[u\left(y\left(\theta\right)-T\left(y\left(\theta\right)\right)-v\left(l\left(\theta\right)\right)\right)\right] \\
=&-u'\left(\theta\right)\mathbb{I}_{\{y(\theta)\geq y^*\}}-\frac{\tilde{\varepsilon}_{l,1-\tau}\left(\theta^*\right)}{1-T'\left(y^*\right)}\left(\frac{dy\left(\theta^*\right)}{d\theta}\right)^{-1}\times\ldots \\
&\left\{\left(1-T'\left(y\left(\theta\right)\right)\right)y\left(\theta\right)\gamma\left(\theta,\theta^*\right)-\int_0^y\left(1-T'\left(y'\right)-y'T''\left(y'\right)\right)\gamma\left(y',y^*\right)dy'\right\}u'\left(\theta\right),
\end{aligned}$$

which is a manifestation of the envelope theorem. The first term in this expression is the PE term, the second is the GE welfare effect.

Finally we derive the effect of the perturbation $h_1 + h_2$ on the social welfare. Summing the previous expression over all individuals using the density function $\tilde{f}_\theta\left(\theta\right) = \tilde{f}_y\left(y\right)\frac{dy(\theta)}{d\theta}$, and defining the marginal social welfare weights as $g_y(y\left(\theta\right)) = \frac{u'(\theta)\tilde{f}_y(y(\theta))}{\lambda f_y(y(\theta))}$, we obtain that the change in the government objective due to the perturbation $h\left(\cdot\right) = h_1\left(\cdot\right) + h_2\left(\cdot\right)$ is

$$\begin{aligned}
&-\lambda^{-1}\mathrm{d}\mathscr{G}\left(T,h\right) = -\lambda^{-1}\mathrm{d}_h\left[\int_\Theta u\left(y\left(\theta\right)-T\left(y\left(\theta\right)\right)-v\left(l\left(\theta\right)\right)\right)\tilde{f}_\theta\left(\theta\right)d\theta\right] \\
=&\int_\Theta\left\{\mathbb{I}_{\{y(\theta)\geq y^*\}}+\frac{\tilde{\varepsilon}_{l,1-\tau}\left(\theta^*\right)}{1-T'\left(y^*\right)}\left(\frac{dy\left(\theta^*\right)}{d\theta}\right)^{-1}\left[\left(1-T'\left(y\left(\theta\right)\right)\right)y\left(\theta\right)\gamma\left(\theta,\theta^*\right)\ldots\right.\right. \\
&\quad\left.\left.-\int_{y'=0}^y\left(1-T'\left(y'\right)-y'T''\left(y'\right)\right)\gamma\left(y',y^*\right)dy'\right]\right\}\lambda^{-1}u'\left(\theta\right)\tilde{f}_\theta\left(\theta\right)d\theta.
\end{aligned}$$

Switching the two integrals in the last line of this expression implies that $-\lambda^{-1}\mathrm{d}\mathscr{G}(T,h)$ is equal to

$$
\begin{aligned}
&\int_{\mathbb{R}_+} \mathbb{I}_{\{y\geq y^*\}}\lambda^{-1}u'(y)\tilde{f}_y(y)\,dy \\
&+\frac{\tilde{\varepsilon}_{l,1-\tau}(y^*)}{1-T'(y^*)}\left(\frac{dy(\theta^*)}{d\theta}\right)^{-1}\left\{\int_{\mathbb{R}_+}(1-T'(y))\,y\gamma(y,y^*)\,\lambda^{-1}u'(y)\tilde{f}_y(y)\,dy\ldots\right. \\
&\left.-\int_{y'=0}^{\infty}\int_{y=y'}^{\infty}(1-T'(y')-y'T''(y'))\,\gamma(y',y^*)\,\lambda^{-1}u'(y)\tilde{f}_y(y)\,dydy'\right\} \\
=&\int_{y^*}^{\infty} g_y(y)\,f_y(y)\,dy+\frac{\tilde{\varepsilon}_{l,1-\tau}(y^*)}{1-T'(y^*)}\left(\frac{dy(\theta^*)}{d\theta}\right)^{-1}\ldots \\
&\times\left\{\int_{\mathbb{R}_+}(1-T'(y))\,y\gamma(y,y^*)\,g_y(y)\,f_y(y)\,dy\right. \\
&\left.-\int_{y'=0}^{\infty}(1-T'(y')-y'T''(y'))\,\gamma(y',y^*)\left[\int_{y=y'}^{\infty}g_y(y)\,f_y(y)\,dy\right]dy'\right\},
\end{aligned}
$$

But note that

$$
\begin{aligned}
&(1-T'(y))\,yg_y(y)\,f_y(y)-(1-T'(y)-yT''(y))\left(\int_y^{\infty}g_y(y')\,f_y(y')\,dy'\right) \\
=&-\left[(1-T'(y))\,y\left(\int_y^{\infty}g_y(y')\,f_y(y')\,dy'\right)\right]'.
\end{aligned}
$$

Thus, we obtain

$$
\begin{aligned}
-\lambda^{-1}\mathrm{d}\mathscr{G}(T,h)=&\int_{y^*}^{\infty}g_y(y)\,f_y(y)\,dy-\frac{\tilde{\varepsilon}_{l,1-\tau}(y^*)}{1-T'(y^*)}\left(\frac{dy(\theta^*)}{d\theta}\right)^{-1}\ldots \\
&\times\int_{\mathbb{R}_+}\left[(1-T'(y))\,y\left(\int_y^{\infty}g_y(y')\,f_y(y')\,dy'\right)\right]'\gamma(y,y^*)\,dy.
\end{aligned}
$$

Therefore, the normalized effect of the perturbation on social welfare is finally given by:

$$
\begin{aligned}
&\mathrm{d}\mathscr{W}(T,h)=\mathrm{d}\mathscr{R}(T,h)+\lambda^{-1}\mathrm{d}\mathscr{G}(T,h) \\
=&1-F_y(y^*)-\int_{y^*}^{\infty}g_y(y)\,f_y(y)\,dy-\frac{T'(y^*)}{1-T'(y^*)}\tilde{\varepsilon}_{l,1-\tau}(y^*)\,y^*f_y(y^*) \\
&-\frac{\tilde{\varepsilon}_{l,1-\tau}(\theta^*)}{1-T'(y(\theta^*))}\left(\frac{dy(\theta^*)}{d\theta}\right)^{-1}\int_{\mathbb{R}_+}\left[(1-T'(y))\,y\,(1-F_y(y))\right]'\gamma(y,y^*)\,dy \\
&+\frac{\tilde{\varepsilon}_{l,1-\tau}(y^*)}{1-T'(y^*)}\left(\frac{dy(\theta^*)}{d\theta}\right)^{-1}\int_{\mathbb{R}_+}\left[(1-T'(y))\,y\left(\int_y^{\infty}g_y(y')\,f_y(y')\,dy'\right)\right]'\gamma(y,y^*)\,dy.
\end{aligned}
$$

But note that

$$
\begin{aligned}
&\left[\left(1-T'(y)\right)y\left(1-F_y(y)\right)\right]' - \left[\left(1-T'(y)\right)y\left(\int_y^{\infty} g_y(y') f_y(y')\,dy'\right)\right]' \\
&= \left[\left(1-T'(y)\right)y\left(1-F_y(y)\right)\left(1-\int_y^{\infty} g_y(y')\frac{f_y(y')}{1-F_y(y)}dy'\right)\right]',
\end{aligned}
$$

so that, using the definition of $\bar{g}_y(y)$, we get

$$
\begin{aligned}
\mathrm{d}\mathscr{W}(T,h) = {} & 1-F_y(y^*) - \int_{y^*}^{\infty} g_y(y) f_y(y)\,dy - \frac{T'(y^*)}{1-T'(y^*)}\tilde{\varepsilon}_{l,1-\tau}(y^*)\,y^* f_y(y^*)\times \\
& -\frac{\tilde{\varepsilon}_{l,1-\tau}(\theta^*)}{1-T'(y(\theta^*))}\left(\frac{dy(\theta^*)}{d\theta}\right)^{-1}\int_{\mathbb{R}_+}\left[\left(1-T'(y)\right)y\left(1-F_y(y)\right)\left(1-\bar{g}_y(y)\right)\right]'\gamma(y,y^*)\,dy.
\end{aligned}
$$

Now, at the optimum we must have $\frac{\mathrm{d}\mathscr{W}(T,h)}{1-F(y^*)}=0$, therefore

$$
\begin{aligned}
0 = {} & 1-\bar{g}_y(y^*) - \frac{T'(y^*)}{1-T'(y^*)}\tilde{\varepsilon}_{l,1-\tau}(y^*)\frac{y^* f_y(y^*)}{1-F_y(y^*)} \\
& -\tilde{\varepsilon}_{l,1-\tau}(\theta^*)\int_{\mathbb{R}_+}\frac{\frac{d}{dy}\left[\left(1-\bar{g}_y(y)\right)\left(1-T'(y)\right)y\left(1-F_y(y)\right)\right]}{\left(1-T'(y(\theta^*))\right)\left(1-F_y(y^*)\right)y'(\theta^*)}\gamma(y,y^*)\,dy,
\end{aligned}
$$

which leads to the optimal tax formula (49).

□

We now prove that the optimal tax formula obtained by the variational approach coincides with the formula obtained by solving the mechanism design problem.

B.4.4 Proof of the equivalence of (42) and (49)

Proof. Substitute for $\mu(\theta)=\lambda\int_{\theta}^{\overline{\theta}}(1-g(x))dF_{\theta}(x)$ in the optimal tax formula (42) evaluated at θ^* to get:

$$
\begin{aligned}
&\frac{T'(y(\theta^*))}{1-T'(y(\theta^*))} \equiv \frac{\tau(\theta^*)}{1-\tau(\theta^*)} \\
&= \left(1+\frac{1}{\varepsilon_{l,1-\tau}(\theta^*)}\right)\frac{\int_{\theta^*}^{\overline{\theta}}(1-g_{\theta}(x))f_{\theta}(x)dx}{f_w(w(\theta^*))w(\theta^*)} - \frac{\int_{\Theta}\left[v'(l(x))l(x)\left(\int_x^{\overline{\theta}}(1-g_{\theta}(x'))f_{\theta}(x')dx'\right)\right]'\gamma(x,\theta^*)dx}{\left(1-\tau(\theta^*)\right)y(\theta^*)f_{\theta}(\theta^*)} \\
&= \left(1+\frac{1}{\varepsilon_{l,1-\tau}(\theta^*)}\right)\frac{1-F_{\theta}(\theta^*)}{f_w(w(\theta^*))w(\theta^*)}\left(\int_{\theta^*}^{\overline{\theta}}(1-g_{\theta}(x))\frac{f_{\theta}(x)}{1-F_{\theta}(\theta^*)}dx\right) \\
&\quad -\frac{\int_{\Theta}\left[\left(1-T'(y(x))\right)y(x)\left(1-F_{\theta}(x)\right)\left(\int_x^{\overline{\theta}}(1-g_{\theta}(x'))\frac{f_{\theta}(x')}{1-F_{\theta}(x)}dx'\right)\right]'\gamma(x,\theta^*)dx}{\left(1-T'(y(\theta^*))\right)y(\theta^*)f_{\theta}(\theta^*)},
\end{aligned}
$$

where the last equality uses individual x's first order condition (1). Using the definition of the average marginal welfare weight $\bar{g}_{\theta}(\theta)=\int_{\theta}^{\overline{\theta}}g_{\theta}(x))\frac{f_{\theta}(x)}{1-F_{\theta}(\theta)}dx$, and multiplying and dividing the first term on the right hand side by $w'(\theta^*)/w(\theta^*)$, we can rewrite this expression as

$$\frac{T'\left(y\left(\theta^*\right)\right)}{1-T'\left(y\left(\theta^*\right)\right)} = \left(1+\frac{1}{\varepsilon_{l,1-\tau}\left(\theta^*\right)}\right)\frac{w'(\theta^*)}{w(\theta^*)}\left(1-\bar{g}_\theta\left(\theta^*\right)\right)\frac{1-F_\theta(\theta^*)}{f_w(w(\theta^*))w'(\theta^*)}$$
$$-\frac{\int_\Theta\left[\left(1-T'\left(y\left(\theta\right)\right)\right)y(\theta)\left(1-F_\theta(\theta)\right)\left(1-\bar{g}_\theta\left(\theta\right)\right)\right]'\gamma(\theta,\theta^*)d\theta}{\left(1-T'\left(y\left(\theta^*\right)\right)\right)y(\theta^*)f_\theta(\theta^*)}.$$

We now change variables from types and wages to incomes in each of the terms of this equation. First, recall that $F_\theta(\theta^*) = F_w\left(w\left(\theta^*\right)\right) = F_y\left(y\left(\theta^*\right)\right)$, and

$$f_\theta\left(\theta^*\right) = f_y\left(y\left(\theta^*\right)\right)\times\left.\frac{dy\left(\theta\right)}{d\theta}\right|_{\theta=\theta^*}.$$

Second, we can rewrite the integral as

$$\begin{aligned}
&\int_\Theta \frac{d}{d\theta}\left[\left(1-T'\left(y\left(\theta\right)\right)\right)y(\theta)\left(1-F_\theta(\theta)\right)\left(1-\bar{g}_\theta\left(\theta\right)\right)\right]\times\gamma\left(\theta,\theta^*\right)d\theta \\
=&\int_\Theta\bigg[\left(1-T'\left(y\left(\theta\right)\right)-y\left(\theta\right)T''\left(y\left(\theta\right)\right)\right)\left(1-F_\theta(\theta)\right)\left(1-\bar{g}_\theta\left(\theta\right)\right)\frac{dy\left(\theta\right)}{d\theta} \\
&\quad-\left(1-T'\left(y\left(\theta\right)\right)\right)y\left(\theta\right)\left(1-g_\theta\left(\theta\right)\right)f_\theta\left(\theta\right)\bigg]\gamma\left(\theta,\theta^*\right)d\theta \\
=&\int_\Theta\bigg[\left(1-T'\left(y\left(\theta\right)\right)-y\left(\theta\right)T''\left(y\left(\theta\right)\right)\right)\left(1-F_\theta(\theta)\right)\left(1-\bar{g}_y\left(y\left(\theta\right)\right)\right) \\
&\quad-\left(1-T'\left(y\left(\theta\right)\right)\right)y\left(\theta\right)\left(1-g_y\left(y\left(\theta\right)\right)\right)f_y\left(y\left(\theta\right)\right)\bigg]\frac{dy\left(\theta\right)}{d\theta}\gamma\left(y\left(\theta\right),y\left(\theta^*\right)\right)d\theta \\
=&\int_{\mathbb{R}_+}\bigg[\left(1-T'\left(y\right)-yT''\left(y\right)\right)\left(1-F_y(y)\right)\left(1-\bar{g}_y\left(y\right)\right) \\
&\quad-\left(1-T'\left(y\right)\right)y\left(1-g_y\left(y\right)\right)f_y\left(y\right)\bigg]\gamma\left(y,y^*\right)dy \\
=&\int_{\mathbb{R}_+}\frac{d}{dy}\left[\left(1-T'\left(y\right)\right)y\left(1-F_y(y)\right)\left(1-\bar{g}_y\left(y\right)\right)\right]\times\gamma\left(y,y^*\right)dy,
\end{aligned}$$

where the second equality uses

$$g_y\left(y\left(\theta\right)\right) = \frac{1}{\lambda}u'\left(\theta\right)\frac{\tilde{f}_y\left(y\left(\theta\right)\right)}{f_y\left(y\left(\theta\right)\right)} = \frac{1}{\lambda}u'\left(\theta\right)\frac{\left(y'\left(\theta\right)\right)^{-1}\times\tilde{f}_\theta\left(\theta\right)}{\left(y'\left(\theta\right)\right)^{-1}\times f_\theta\left(\theta\right)} = g_\theta\left(\theta\right),$$

and $\gamma\left(\theta,\theta^*\right) = \gamma\left(y\left(\theta\right),y\left(\theta^*\right)\right)$, and the third equality follows from a change of variables in the integral. Third, we have

$$\frac{w'(\theta^*)}{w(\theta^*)}\frac{1-F_\theta(\theta^*)}{f_w(w(\theta^*))w'(\theta^*)} = \frac{w'(\theta^*)}{w(\theta^*)}\frac{1-F_\theta(\theta^*)}{f_\theta(\theta^*)} = \frac{\frac{w'(\theta^*)}{w(\theta^*)}}{\frac{y'(\theta^*)}{y(\theta^*)}}\frac{1-F_y\left(y(\theta^*)\right)}{y(\theta^*)f_y\left(y(\theta^*)\right)}.$$

To compute $\frac{y'(\theta)}{y(\theta)}/\frac{w'(\theta)}{w(\theta)}$, note that the first order condition $l(\theta)=w(\theta)^{\varepsilon}\left(1-T'(y(\theta))\right)^{\varepsilon}$ implies

$$\frac{l'(\theta)}{l(\theta)}=\varepsilon\left(\frac{w'(\theta)}{w(\theta)}-\frac{y'(\theta)T''(y(\theta))}{1-T'(y(\theta))}\right).$$

Using (8), we can write

$$\tilde{\varepsilon}_{l,w}(\theta)=\frac{l(\theta)^{\frac{1}{\varepsilon}}-w(\theta)y(\theta)T''\left(y(\theta)\right)}{l(\theta)^{\frac{1}{\varepsilon}}+\varepsilon w(\theta)y(\theta)T''\left(y(\theta)\right)}\varepsilon.$$

These two equations imply that $\frac{l'(\theta)}{l(\theta)}/\frac{w'(\theta)}{w(\theta)}$ is equal to

$$\begin{aligned}
&\left(1-\frac{w(\theta)}{w'(\theta)}\frac{y'(\theta)T''(y(\theta))}{1-T'(y(\theta))}\right)\varepsilon=\frac{\varepsilon}{l(\theta)^{\frac{1}{\varepsilon}}+\varepsilon w(\theta)y(\theta)T''\left(y(\theta)\right)}\left\{\left[l(\theta)^{\frac{1}{\varepsilon}}-w(\theta)y(\theta)T''\left(y(\theta)\right)\right]\ldots\right.\\
&\left.+\left[\left(1+\varepsilon\right)w(\theta)y(\theta)T''\left(y(\theta)\right)-\frac{w(\theta)}{w'(\theta)}\frac{y'(\theta)T''(y(\theta))}{1-T'(y(\theta))}\left(l(\theta)^{\frac{1}{\varepsilon}}+\varepsilon w(\theta)y(\theta)T''\left(y(\theta)\right)\right)\right]\right\}\\
=&\tilde{\varepsilon}_{l,w}(\theta)\left\{1+\frac{\left(1+\varepsilon\right)w(\theta)y(\theta)T''\left(y(\theta)\right)-\frac{w(\theta)}{w'(\theta)}\frac{y'(\theta)T''(y(\theta))}{1-T'(y(\theta))}\left(l(\theta)^{\frac{1}{\varepsilon}}+\varepsilon w(\theta)y(\theta)T''\left(y(\theta)\right)\right)}{l(\theta)^{\frac{1}{\varepsilon}}-w(\theta)y(\theta)T''\left(y(\theta)\right)}\right\}.
\end{aligned}$$

But the second term in the curly brackets is equal to zero; to see this, note that its numerator is proportional to

$$\begin{aligned}
&\left(1+\varepsilon\right)y(\theta)-\frac{y'(\theta)}{\left(1-T'(y(\theta))\right)w'(\theta)}\left(l(\theta)^{\frac{1}{\varepsilon}}+\varepsilon w(\theta)y(\theta)T''\left(y(\theta)\right)\right)\\
&\propto\left(1+\varepsilon\right)w'(\theta)l(\theta)-\left(1+\varepsilon y(\theta)\frac{T''\left(y(\theta)\right)}{1-T'(y(\theta))}\right)y'(\theta)=0,
\end{aligned}$$

where the last equality follows from the fact that $y'(\theta)=w'(\theta)l(\theta)+w(\theta)l'(\theta)$ and the expression above for $\frac{l'(\theta)}{l(\theta)}$. Therefore, we finally obtain

$$\frac{\frac{y'(\theta)}{y(\theta)}}{\frac{w'(\theta)}{w(\theta)}}=1+\frac{\frac{l'(\theta)}{l(\theta)}}{\frac{w'(\theta)}{w(\theta)}}=1+\left(\frac{d\ln w(\theta)}{d\theta}\right)^{-1}\frac{d\ln l(\theta)}{d\theta}=1+\tilde{\varepsilon}_{l,w}(\theta).$$

This implies

$$\frac{1-F_w\left(w(\theta^*)\right)}{w(\theta^*)f_w\left(w(\theta^*)\right)}=\frac{1}{1+\tilde{\varepsilon}_{l,w}(\theta^*)}\frac{1-F_y\left(y(\theta^*)\right)}{y(\theta^*)f_y\left(y(\theta^*)\right)}.$$

Finally, note that

$$\frac{1}{1+\tilde{\varepsilon}_{l,w}\left(\theta\right)}=\frac{\frac{1-T'(\theta)}{1-T'(\theta)+\varepsilon_{l,1-\tau}(\theta)y(\theta)T''(\theta)}\varepsilon_{l,1-\tau}\left(\theta\right)}{1+\frac{1-T'(\theta)-y(\theta)T''(\theta)}{1-T'(\theta)+\varepsilon_{l,1-\tau}(\theta)y(\theta)T''(\theta)}\varepsilon_{l,1-\tau}\left(\theta\right)}\frac{1}{\tilde{\varepsilon}_{l,1-\tau}\left(\theta\right)}=\frac{\varepsilon_{l,1-\tau}\left(\theta\right)}{1+\varepsilon_{l,1-\tau}\left(\theta\right)}\frac{1}{\tilde{\varepsilon}_{l,1-\tau}\left(\theta\right)}.$$

Collecting all the terms, we obtain

$$
\begin{aligned}
\frac{T'(y(\theta^*))}{1-T'(y(\theta^*))} =& \left(1+\frac{1}{\varepsilon_{l,1-\tau}(\theta^*)}\right)(1-\bar{g}_\theta(\theta^*))\left(\frac{1}{\tilde{\varepsilon}_{l,1-\tau}(\theta^*)}\frac{\varepsilon_{l,1-\tau}(\theta^*)}{1+\varepsilon_{l,1-\tau}(\theta^*)}\right)\frac{1-F_y(y(\theta^*))}{y(\theta^*)f_y(y(\theta^*))} \\
&+\frac{\int_{\mathbb{R}_+}\frac{d}{dy}\left[(1-T'(y))\,y\,(1-F_y(y))\,(1-\bar{g}_y(y))\right]\times\gamma(y,y^*)\,dy}{(1-T'(y(\theta^*)))\,y(\theta^*)\,f_y(y(\theta^*))\times y'(\theta^*)} \\
=&\frac{1}{\tilde{\varepsilon}_{l,1-\tau}(\theta^*)}(1-\bar{g}_\theta(\theta^*))\frac{1-F_y(y(\theta^*))}{y(\theta^*)f_y(y(\theta^*))} \\
&-\frac{\int_{\mathbb{R}_+}\frac{d}{dy}\left[(1-T'(y))\,y\,(1-F_y(y))\,(1-\bar{g}_y(y))\right]\times\gamma(y,y^*)\,dy}{(1-T'(y(\theta^*)))\,y(\theta^*)\,f_y(y(\theta^*))\times y'(\theta^*)},
\end{aligned}
$$

which is exactly formula (49).

□

B.4.5 Optimum Characterization and Proof of Corollary 4

Here we show how to rewrite formula (49) as an integral equation.

Proof. The idea is to integrate by parts the last term in equation (49). Let

$$
\psi(y)=\frac{(1-\bar{g}_y(y))\,(1-T'(y))\,y\,(1-F_y(y))}{(1-T'(y^*))\,(1-F(y^*))},
$$

so that optimal taxes satisfy

$$
0=1-\bar{g}_y(y^*)-\frac{T'(y^*)}{1-T'(y^*)}\tilde{\varepsilon}_{l,1-\tau}(y^*)\frac{y^*f_y(y^*)}{1-F_y(y^*)}-\frac{\tilde{\varepsilon}_{l,1-\tau}(\theta^*)}{y'(\theta^*)}\int_{\mathbb{R}_+}\psi'(y)\,\gamma(y,y^*)\,dy.
$$

Disentangle the Dirac and the smooth part of $\gamma(y,y^*)$, i.e., the own- and cross-wage elasticities. We get:

$$
\begin{aligned}
0=&1-\bar{g}_y(y^*)-\frac{T'(y^*)}{1-T'(y^*)}\tilde{\varepsilon}_{l,1-\tau}(y^*)\frac{y^*f_y(y^*)}{1-F_y(y^*)} \\
&-\frac{\tilde{\varepsilon}_{l,1-\tau}(\theta^*)}{y'(\theta^*)}\int_{\mathbb{R}_+}\psi'(y)\left[\bar{\gamma}(y,y^*)+\bar{\bar{\gamma}}(y^*,y^*)\,y'(\theta^*)\,\delta_{y^*}(y)\right]dy \\
=&1-\bar{g}_y(y^*)-\frac{T'(y^*)}{1-T'(y^*)}\tilde{\varepsilon}_{l,1-\tau}(y^*)\frac{y^*f_y(y^*)}{1-F_y(y^*)} \\
&-\tilde{\varepsilon}_{l,1-\tau}(\theta^*)\,\psi'(y^*)\,\bar{\bar{\gamma}}(y^*,y^*)-\frac{\tilde{\varepsilon}_{l,1-\tau}(\theta^*)}{y'(\theta^*)}\int_{\mathbb{R}_+}\psi'(y)\,\bar{\gamma}(y,y^*)\,dy.
\end{aligned}
$$

Assuming that $y\mapsto\bar{\gamma}(y,y^*)$ is continuously differentiable for each y^*, we can integrate by parts the last term of this equation. Using $\psi(0)=0$ and $\psi(\bar{y})=0$ yields

$$
\begin{aligned}
0=&1-\bar{g}_y(y^*)-\frac{T'(y^*)}{1-T'(y^*)}\tilde{\varepsilon}_{l,1-\tau}(y^*)\frac{y^*f_y(y^*)}{1-F_y(y^*)}-\tilde{\varepsilon}_{l,1-\tau}(\theta^*)\,\psi'(y^*)\,\bar{\bar{\gamma}}(y^*,y^*) \\
&+\tilde{\varepsilon}_{l,1-\tau}(\theta^*)\int_{\mathbb{R}_+}\psi(y)\left[\left(\frac{dy(\theta^*)}{d\theta}\right)^{-1}\frac{d\bar{\gamma}(y,y^*)}{dy}\right]dy.
\end{aligned}
\tag{85}
$$

Now, since the marginal social welfare weights $g_y(y)$ sum to 1, we have

$$\begin{aligned}(1-\bar{g}_y(y))(1-F_y(y)) =& 1-F_y(y)-\int_y^{\infty} g_y(y') f_y(y') dy' \\ =& \int_0^y g_y(y') f_y(y') dy' - F_y(y) \equiv G_y(y) - F_y(y),\end{aligned}$$

so that $[(1-\bar{g}_y(y))(1-F_y(y))]' = (g_y(y)-1) f_y(y)$ and thus

$$\psi'(y) = \frac{(1-T'(y)) y (g_y(y)-1) f_y(y) + (1-T'(y)-yT''(y))(1-\bar{g}_y(y))(1-F_y(y))}{(1-T'(y^*))(1-F(y^*))}.$$

Hence (85) can be rewritten as

$$\begin{aligned}\frac{T'(y^*)}{1-T'(y^*)} =& \frac{1}{\tilde{\varepsilon}_{l,1-\tau}(y^*)} \frac{1-F_y(y^*)}{y^* f_y(y^*)}(1-\bar{g}_y(y^*))\ldots \\ &-\frac{1-F_y(y^*)}{y^* f_y(y^*)} \frac{(1-T'(y^*)-y^*T''(y^*))(1-\bar{g}_y(y^*))(1-F_y(y^*))}{(1-T'(y^*))(1-F(y^*))} \bar{\bar{\gamma}}(y^*,y^*) \\ &-\frac{1-F_y(y^*)}{y^* f_y(y^*)} \frac{(1-T'(y^*)) y^* (g_y(y^*)-1) f_y(y^*)}{(1-T'(y^*))(1-F(y^*))} \bar{\bar{\gamma}}(y^*,y^*) \\ &+\frac{1-F_y(y^*)}{y^* f_y(y^*)} \int_{\mathbb{R}_+} \frac{(1-\bar{g}_y(y))(1-T'(y)) y (1-F_y(y))}{(1-T'(y^*))(1-F(y^*))} \left[\left(\frac{dy(\theta^*)}{d\theta}\right)^{-1} \frac{d\bar{\gamma}(y,y^*)}{dy}\right] dy.\end{aligned}$$

The right hand side of this expression is equal to

$$\begin{aligned}&\frac{1-F_y(y^*)}{y^* f_y(y^*)}(1-\bar{g}_y(y^*)) \frac{1}{\tilde{\varepsilon}_{l,1-\tau}(y^*)}\left[1-\frac{(1-T'(y^*)-y^*T''(y^*))\tilde{\varepsilon}_{l,1-\tau}(y^*)}{1-T'(y^*)}\bar{\bar{\gamma}}(y^*,y^*)\right] \\ &-(g_y(y^*)-1)\bar{\bar{\gamma}}(y^*,y^*) + \int_{\mathbb{R}_+} \frac{(1-\bar{g}_y(y))(1-T'(y)) y (1-F_y(y))}{(1-T'(y^*)) y^* f_y(y^*)} \left(\frac{dy(\theta^*)}{d\theta}\right)^{-1} \frac{d\bar{\gamma}(y,y^*)}{dy} dy \\ =&\frac{1-F_y(y^*)}{y^* f_y(y^*)}(1-\bar{g}_y(y^*)) \frac{1-\tilde{\varepsilon}_{l,w}(y^*)\bar{\bar{\gamma}}(y^*,y^*)}{\tilde{\varepsilon}_{l,1-\tau}(y^*)} + (1-g_y(y^*))\bar{\bar{\gamma}}(y^*,y^*) \\ &+\int_{\mathbb{R}_+}(1-\bar{g}_y(y)) \frac{(1-T'(y)) y}{(1-T'(y^*)) y^*} \frac{1-F_y(y)}{f_y(y^*)} \left(\frac{dy(\theta^*)}{d\theta}\right)^{-1} \frac{d\bar{\gamma}(y,y^*)}{dy} dy.\end{aligned}$$

We thus get

$$\begin{aligned}\frac{T'(y^*)}{1-T'(y^*)} =& \frac{1}{\tilde{E}_{l,1-\tau}(y^*)}(1-\bar{g}_y(y^*))\left(\frac{1-F_y(y^*)}{y^* f_y(y^*)}\right) - (g(y^*)-1)\bar{\bar{\gamma}}(y^*,y^*) \\ &+\int_{\mathbb{R}_+}(1-\bar{g}_y(y))\left(\frac{1-F_y(y)}{y^* f_y(y^*)}\right)\left(\frac{1-T'(y)}{1-T'(y^*)}\right) y \frac{\bar{\gamma}'_y(y,y^*)}{y'(\theta^*)} dy.\end{aligned}$$

Denoting by $\frac{\bar{T}'(y^*)}{1-\bar{T}'(y^*)}$ the first line in the right hand side of the previous equation, and changing variables from incomes y to types θ (using the identities $f_y(y^*)\frac{dy(\theta^*)}{d\theta} = f_\theta(\theta^*)$, $F_y(y(\theta)) = F_\theta(\theta)$, and $\frac{d\bar{\gamma}(y,y^*)}{dy} = \frac{d\bar{\gamma}(\theta,\theta^*)}{d\theta}\left(\frac{dy(\theta)}{d\theta}\right)^{-1}$), we obtain

$$\frac{T'(y(\theta^*))}{1-T'(y(\theta^*))} = \frac{\bar{T}'(y(\theta^*))}{1-\bar{T}'(y(\theta^*))} + \int_\Theta (1-\bar{g}_\theta(\theta)) \frac{(1-T'(y(\theta)))\, y(\theta)}{(1-T'(y(\theta^*)))\, y(\theta^*)} \frac{1-F_\theta(\theta)}{f_\theta(\theta^*)} \frac{d\bar{\gamma}(\theta,\theta^*)}{d\theta} d\theta.$$

Equation (52) follows immediately from this expression since $\frac{d\bar{\gamma}(y,y^*)}{dy} = 0$ when the production function is CES.

Multiplying both sides of the previous equation by $1-T'(y(\theta^*))$, we then get

$$\begin{aligned} T'(y(\theta^*)) =& \frac{\bar{T}'(y(\theta^*))}{1-\bar{T}'(y(\theta^*))}(1-T'(y(\theta^*))) \\ &+ \int_\Theta (1-T'(y(\theta)))\, y(\theta) \frac{(1-\bar{g}_\theta(\theta))(1-F_\theta(\theta))}{y(\theta^*) f_\theta(\theta^*)} \frac{d\bar{\gamma}(\theta,\theta^*)}{d\theta} d\theta, \end{aligned}$$

which leads to the following formula for the optimal net-of-tax rate $1-\tau(\theta^*) \equiv 1-T'(y(\theta^*))$:

$$\begin{aligned} (1-\tau(\theta^*)) =& \frac{1}{1+\frac{\bar{\tau}(\theta^*)}{1-\bar{\tau}(\theta^*)}} \left[1 - \int_\Theta (1-\tau(\theta)) \frac{y(\theta)(1-\bar{g}_\theta(\theta))(1-F_\theta(\theta))\bar{\gamma}'(\theta,\theta^*)}{y(\theta^*) f_\theta(\theta^*)} d\theta\right] \\ =& (1-\bar{\tau}(\theta^*)) \left\{1 - \int_\Theta \left[(1-\bar{g}_\theta(\theta)) \left(\frac{1-F_\theta(\theta)}{y(\theta^*) f_\theta(\theta^*)}\right) y(\theta)\bar{\gamma}'(\theta,\theta^*)\right](1-\tau(\theta))\, d\theta\right\}. \end{aligned}$$

This is now a well-defined integral equation in $(1-T'(y(\theta)))$, so that we can use the mathematical apparatus introduced in Section 2 to characterize its solution, i.e., the optimal tax schedule. Similar to the integral equation (21), it can be solved in closed form if $\bar{\gamma}'(\theta,\theta^*)$ is the sum of multiplicatively separable terms. We can use the same techniques as in Section xx in the Translog case to get a separable kernel.

□

B.4.6 Proof of Corollary 5

We now derive the formula for the optimal top tax rate when the production function is CES.

Proof. Assume that in the data (i.e., given the current tax schedule with a constant top tax rate, assuming that the aggregate production function is CES), the income distribution has a Pareto tail, so that the (observed) hazard rate $\frac{1-F_y^d(y^*)}{y f_y^d(y^*)}$ converges to a constant $1/\alpha$. We show that under these assumption, the income distribution at the optimum tax schedule is also Pareto distributed at the tail with the same Pareto coefficient $1/\alpha$. That is, the hazard rate of the income distribution at the top is independent of the level of the top tax rate. At the optimum, we have

$$\frac{1-F_y(y(\theta))}{y(\theta) f_y(\theta)} = \frac{1-F_\theta(\theta)}{\frac{y(\theta)}{y'(\theta)} f_\theta(\theta)} = \frac{1-F_\theta(\theta)}{\theta f_\theta(\theta)} \frac{\theta y'(\theta)}{y(\theta)} = \frac{1-F_\theta(\theta)}{\theta f_\theta(\theta)} \varepsilon_{y,\theta}, \tag{86}$$

where we define the income elasticity $\varepsilon_{y,\theta} \equiv d\ln y(\theta)/d\ln\theta$. To compute this elasticity, use the individual first order condition (1) with isoelastic disutility of labor to get $l(\theta) = (1-\tau(\theta))^\varepsilon\, w(\theta)^\varepsilon$.

Thus we have

$$\varepsilon_{l,\theta} \equiv \frac{d\ln l\left(\theta\right)}{d\ln\theta} = \varepsilon\frac{d\ln\left(1-\tau(\theta)\right)}{d\ln\theta} + \varepsilon\frac{d\ln w(\theta)}{d\ln\theta} = \varepsilon\left(\frac{\theta w'(\theta)}{w(\theta)} - \frac{\theta\tau'(\theta)}{1-\tau(\theta)}\right).$$

But since the production function is CES, we have, from equation (14),

$$\begin{aligned}\frac{d\ln w(\theta)}{d\ln\theta} =& \frac{d\ln a\left(\theta\right)}{d\ln\theta} + \left(\rho-1\right)\frac{d\ln L\left(\theta\right)}{d\ln\theta}\\ =& \frac{d\ln a\left(\theta\right)}{d\ln\theta} - \frac{1}{\sigma}\frac{d\ln l\left(\theta\right)}{d\ln\theta} - \frac{1}{\sigma}\frac{d\ln f_\theta\left(\theta\right)}{d\ln\theta} = \frac{\theta a'\left(\theta\right)}{a\left(\theta\right)} - \frac{1}{\sigma}\varepsilon_{l,\theta} - \frac{1}{\sigma}\frac{\theta f_\theta'\left(\theta\right)}{f_\theta\left(\theta\right)}.\end{aligned}$$

Thus, substituting this expression for $\frac{\theta w'(\theta)}{w(\theta)}$ in the previous equation, we obtain

$$\varepsilon_{l,\theta} = \varepsilon\left(\frac{\theta a'\left(\theta\right)}{a\left(\theta\right)} - \frac{1}{\sigma}\varepsilon_{l,\theta} - \frac{1}{\sigma}\frac{\theta f_\theta'\left(\theta\right)}{f_\theta\left(\theta\right)} - \frac{\theta\tau'(\theta)}{1-\tau(\theta)}\right).$$

Moreover, since we assume that the second derivative of the optimal marginal tax rate, $T''\left(y\right)$, converges to zero for high incomes, we have $\lim_{\theta\to\infty}\tau'(\theta)$. Therefore, the previous equation yields

$$\lim_{\theta\to\infty}\varepsilon_{l,\theta} = \frac{\varepsilon}{1+\frac{\varepsilon}{\sigma}}\left(\lim_{\theta\to\infty}\frac{\theta a'\left(\theta\right)}{a\left(\theta\right)} - \frac{1}{\sigma}\lim_{\theta\to\infty}\frac{\theta f_\theta'\left(\theta\right)}{f_\theta\left(\theta\right)}\right).$$

Note that the variables $\frac{\theta a'(\theta)}{a(\theta)}$ and $\frac{\theta f_\theta'(\theta)}{f_\theta(\theta)}$ are primitive parameters that do not depend on the tax rate. Assuming that they converge to constants as $\theta\to\infty$, we obtain that $\lim_{\theta\to\infty}\varepsilon_{l,\theta}$ is a constant independent of the tax rates, and hence

$$\varepsilon_{y,\theta} = \varepsilon_{l,\theta} + \varepsilon_{w,\theta} = \left(1+\frac{1}{\varepsilon}\right)\varepsilon_{l,\theta}$$

converges to a constant as $\theta\to\infty$. Therefore, the hazard rate of the income distribution at the optimum tax schedule, given by (86), converges to the same constant $1/\alpha$ as the hazard rate of incomes observed in the data.

Now let $y^*\to\infty$ in equation (52), to obtain an expression for the optimal top tax rate $\tau_{top} = \lim_{y^*\to\infty}T'\left(y^*\right)$. Since the production function is CES with parameter σ, and the disutility of labor is isoelastic with parameter ε, we have

$$\lim_{y^*\to\infty}\tilde{E}_{l,1-\tau}\left(y^*\right) = \frac{\varepsilon}{1+\varepsilon/\sigma}.$$

Furthermore assume that $\lim_{y^*\to\infty}g_y\left(y^*\right) = \bar{g}$, so that $\lim_{y^*\to\infty}\bar{g}_y\left(y^*\right) = \bar{g}$. Therefore (52) implies

$$\frac{\tau_{top}}{1-\tau_{top}} = \frac{1+\varepsilon/\sigma}{\varepsilon}\left(1-\bar{g}\right)\frac{1}{\alpha} + \frac{\bar{g}-1}{\sigma} = \frac{1-\bar{g}}{\alpha\varepsilon} + \frac{1-\bar{g}}{\alpha\sigma} + \frac{\bar{g}-1}{\sigma}.$$

This concludes the proof.

□

B.4.7 General equilibrium wedge accounting

We finally propose a decomposition into several components of the difference between the partial and general equilibrium optimal taxes, respectively given by formulas (54) and (52).

Proposition 6. *The optimal marginal tax rate of type θ in general equilibrium can be expressed as a function of $\tau_{PE}(\theta)$ and three additional terms:*

$$\begin{aligned}\frac{\tau(\theta)}{1-\tau(\theta)} =& \frac{\tau_{PE}(\theta)}{1-\tau_{PE}(\theta)} + \frac{g_\theta(\theta)-1}{\sigma} \\ &+ \frac{1-F_w(w(\theta))}{f_w(w(\theta))w(\theta)}\left(1+\frac{1}{\varepsilon}\right)(1-\bar{g}_\theta(\theta))\left(\frac{\tilde{\varepsilon}_{l,1-\tau}(\theta)}{\tilde{E}_{l,1-\tau}(\theta)}-1\right) \\ &+ \left(1+\frac{1}{\varepsilon}\right)(1-\bar{g}_\theta(\theta))\left(\frac{1-F_w(w(\theta))}{f_w(w(\theta))w(\theta)} - \frac{1-F_w^d(w_d(\theta))}{f_w^d(w_d(\theta))w_d(\theta)}\right). \end{aligned} \tag{87}$$

Proof. Adding and substracting from equation (52) the partial equilibrium tax $\frac{\tau_{PE}(\theta)}{1-\tau_{PE}(\theta)}$ constructed in (54), we find

$$\begin{aligned}\frac{\tau(\theta)}{1-\tau(\theta)} =& \frac{\tau_{PE}(\theta)}{1-\tau_{PE}(\theta)} + \frac{g_\theta(\theta)-1}{\sigma} \\ &+ \frac{1}{\tilde{E}_{l,1-\tau}(\theta^*)}(1-\bar{g}(\theta^*))\left(\frac{1-F_y(y(\theta^*))}{y(\theta^*))f_y(y(\theta^*))}\right) - \frac{\tau_{PE}(\theta)}{1-\tau_{PE}(\theta)}.\end{aligned}$$

Substituting for $\frac{\tau_{PE}(\theta)}{1-\tau_{PE}(\theta)}$ in the second line of this expression using the definition (54), we obtain

$$\begin{aligned}&\frac{\tau(\theta)}{1-\tau(\theta)} - \left[\frac{\tau_{PE}(\theta)}{1-\tau_{PE}(\theta)} + \frac{g_\theta(\theta)-1}{\sigma}\right] \\ =& \frac{1}{\tilde{E}_{l,1-\tau}(\theta^*)}(1-\bar{g}(\theta^*))\left(\frac{1-F_y(y(\theta^*))}{y(\theta^*))f_y(y(\theta^*))}\right) - \left(1+\frac{1}{\varepsilon}\right)\frac{1-F_w^d(w_d(\theta))}{f_w^d(w_d(\theta))w_d(\theta)}(1-\bar{g}(\theta)) \\ =& \frac{1}{\tilde{E}_{l,1-\tau}(\theta^*)}(1-\bar{g}(\theta^*))(1+\tilde{\varepsilon}_{l,w}(\theta^*))\left(\frac{1-F_w(w(\theta^*))}{w(\theta^*))f_w(w(\theta^*))}\right) - \left(1+\frac{1}{\varepsilon}\right)\frac{1-F_w^d(w_d(\theta))}{f_w^d(w_d(\theta))w_d(\theta)}(1-\bar{g}(\theta)),\end{aligned}$$

where the second equality follows from a change of variables from incomes to wages in the hazard rate, and the fact that

$$\begin{aligned}\frac{1-F_y(y(\theta^*))}{y(\theta^*))f_y(y(\theta^*))} =& \frac{y'(\theta^*))}{y(\theta^*))}\frac{1-F_y(y(\theta^*))}{y'(\theta^*))f_y(y(\theta^*))} = \frac{y'(\theta^*))}{y(\theta^*))}\frac{1-F_w(w(\theta^*))}{w'(\theta^*))f_w(w(\theta^*))} \\ =& \frac{\frac{y'(\theta^*))}{y(\theta^*))}}{\frac{w'(\theta^*))}{w(\theta^*))}}\frac{1-F_w(w(\theta^*))}{w(\theta^*))f_w(w(\theta^*))} = (1+\tilde{\varepsilon}_{l,w}(\theta))\frac{1-F_w(w(\theta^*))}{w(\theta^*))f_w(w(\theta^*))}\end{aligned}$$

(we showed the last equality above). Now, note that

$$\frac{1+\tilde{\varepsilon}_{l,w}(\theta)}{\tilde{\varepsilon}_{l,1-\tau}(\theta)} = \frac{1+\frac{1-\tau(\theta)-y(\theta)\tau'(\theta)}{1-\tau(\theta)+\varepsilon y(\theta)\tau'(\theta)}\varepsilon}{\frac{1-\tau(\theta)}{1-\tau(\theta)+\varepsilon y(\theta)\tau'(\theta)}\varepsilon} = \frac{1+\varepsilon}{\varepsilon},$$

so that

$$\frac{\tau(\theta)}{1-\tau(\theta)} - \frac{\tau_{PE}(\theta)}{1-\tau_{PE}(\theta)} + \frac{g_\theta(\theta)-1}{\sigma}$$
$$= \frac{\tilde{\varepsilon}_{l,1-\tau}(\theta)}{\tilde{E}_{l,1-\tau}(\theta^*)}\left(1-\bar{g}(\theta^*)\right)\left(1+\frac{1}{\varepsilon}\right)\left(\frac{1-F_w(w(\theta^*))}{w(\theta^*))f_w(w(\theta^*))}\right) - \left(1+\frac{1}{\varepsilon}\right)\frac{1-F_w^d(w_d(\theta))}{f_w^d(w_d(\theta))w_d(\theta)}\left(1-\bar{g}(\theta)\right).$$

Adding and subtracting

$$\frac{1-F_w(w(\theta))}{f_w(w(\theta))w(\theta)}\left(1+\frac{1}{\varepsilon}\right)\left(1-\bar{g}(\theta)\right)$$

yields the result.

□

The first correction in formula (87) is the cross-wage effect $(g(y^*)-1)/\sigma$ (see (52)). It is always negative for a Rawlsian planner, and hence pushes in the direction of lower tax rates. The second correction is due to the own-wage effect, and is captured by the adjusted elasticity $\tilde{E}_{l,1-\tau}$ vs. $\tilde{\varepsilon}_{l,1-\tau}$. It is always positive, and hence pushes in the direction of higher tax rates. Finally the third correction is due to the fact that (54) and (52) are evaluated at different wage distributions. In (87), this is accounted for by the difference between the hazard rate of the wage distribution at the general equilibrium optimum, $f_w(w(\theta))$, and that of the wage distribution inferred from the data, $f_w^d(w_d(\theta))$.[56] Figure 10 decomposes quantitatively the relative importance of each of these three forces.

C Numerical Simulations: Details and Robustness

C.1 Details on Calibration of Income Distribution

We assume that incomes are log-normally distributed apart from the top, where we append a Pareto distribution for incomes above \$150,000. To obtain a smooth resulting hazard ratio $\frac{1-F_y(y)}{yf_y(y)}$, we decrease the thinness parameter of the Pareto distribution linearly between \$150,000 and \$350,000 and let it be constant at 1.5 afterwards (Diamond and Saez, 2011). In the last step we use a standard kernel smoother to ensure differentiability of the hazard ratios at \$150,000 and \$350,000. We set the mean and variance of the lognormal distribution at 10 and 0.95, respectively. The mean parameter is chosen such that the resulting income distribution has a mean of \$64,000, i.e., approximately the average US yearly earnings. The variance parameter was chosen such that the hazard ratio at level \$150,000 is equal to that reported by Diamond and Saez (2011, Fig.2). The resulting hazard ratio is illustrated in Figure 6.

[56] With endogenous marginal social welfare weights, there would be an additional correction term to account for the fact that the welfare weights are endogenous to the tax schedule.

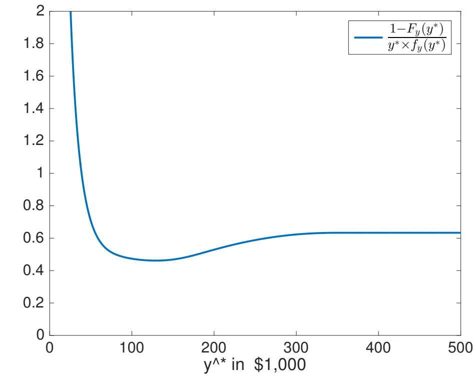


Figure 6: Calibrated Hazard Ratios of the Income Distribution

C.2 Additional Graphs for Benchmark Specification

Figure 7 illustrates optimal marginal tax rates as a function of income in the optimal allocation. Marginal tax rates in this graph reflect the policy recommendations of the optimal tax exercise which is to set marginal tax rates at each income (rather than unobservable productivity) level. A general pattern is that the marginal tax rate schedule is shifted to the left because individuals work less for optimal taxes than current taxes. This is visible most clearly for the top bracket and the bottom of the U that start earlier.

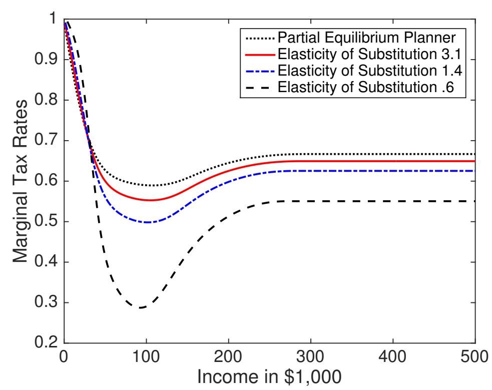
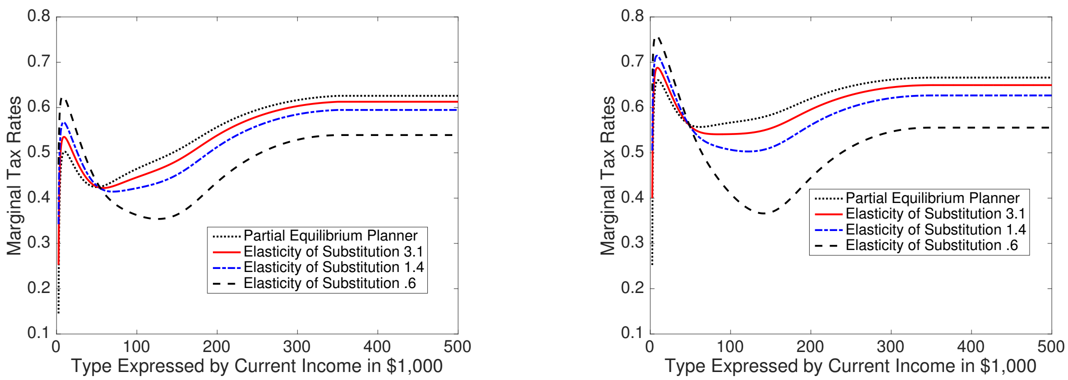


Figure 7: Optimal Marginal Tax Rates as a Function of Income

C.3 Utilitarian Welfare Function

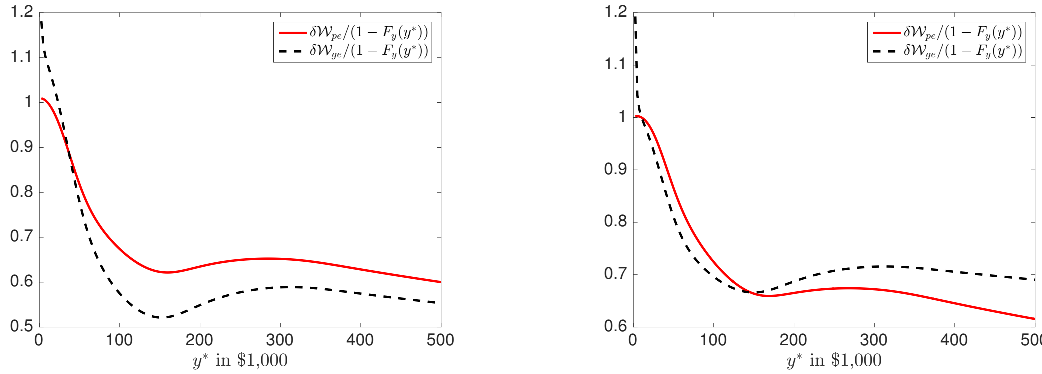
We here consider another often used social welfare function, namely the utilitarian welfare function. This implies we set $\tilde{f}(\theta) = f(\theta)$. To obtain a desire for redistribution, we assume the utility function to be $\frac{1}{1-\kappa}\left(c - l^{1+\frac{1}{\varepsilon}} / \left(1 + \frac{1}{\varepsilon}\right)\right)^{1-\kappa}$. Thus, κ determines the concavity of utility and therefore the desire for redistribution. Figure 8 illustrates optimal Utilitarian marginal tax rates for two values of κ (1 and 3). As in the Rawlsian case, the optimal U-shape of marginal tax rates is reinforced. Given that low income levels now also have positive welfare weights, the cross wage effect here is a force for higher marginal tax rates for low income levels. Thus, the result that marginal tax rates should be higher for low income levels is stronger than in the Rawlsian case in two ways: (i) the size of these effects is larger and (ii) it holds for a broader range (up to $50,000).

Figure 8: Optimal Utilitarian Marginal Tax Rates ($CRRA = 1$ in left panel and $CRRA = 3$ in right panel)



Next we ask how the results about tax incidence are differ in the Utilitarian case. In contrast to the Rawlsian case, the policy implications of the optimal tax schedule are not necessarily overturned. For a relatively low desire to redistribute ($\kappa = 1$, see the left panel of Figure 9), the welfare gains of raising tax rates on high incomes are muted due to general equilibrium. For a stronger desire to redistribute ($\kappa = 3$, see the right panel of Figure 9), general equilibrium effects make raising top tax rates more desirable. How can that be explained? General equilibrium effects make rising top tax rates more desirable because the tax revenue increase is higher. At the same time the implied wage decreases for the working poor make them worse of. In case of very strong redistributive tastes, the tax revenue get a stronger weight (as they are used for lump-sum redistribution at the margin). In case where relatively richer workers (for whom the lump-transfer is less important relative to the very poor) still have significant welfare weights, the wage effects dominates.

Figure 9: Tax Incidence Utilitarian for $\sigma = 1.4$ ($CRRA = 1$ in left panel and $CRRA = 3$ in right panel)



C.4 General Equilibrium Wedge Accounting

Figure 10 decomposes quantitatively the relative importance of each of the three forces highlighted in Proposition 6, for $\varepsilon = 0.33$ and $\sigma = 1.4$. The partial (resp., general) equilibrium optimum is represented by the black dashed (resp., red bold) curve. The black dotted, blue dashed-dotted, and diamond-marked curves illustrate suboptimal tax schedules where each of the three elements of the decomposition (87) (respectively, the cross-wage term, the elasticity correction term, and the hazard rate correction term) are ignored. This graph shows that the hazard rate correction term (iii) has a minor quantitative importance.

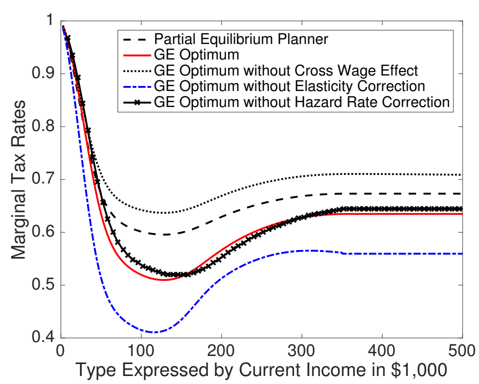


Figure 10: General Equilibrium Wedge Accounting

C.5 Translog Production Function

First we again look at tax incidence but change the parameterization of the Translog production function such that $\bar{\bar{\gamma}}(y^*, y^*) = -1$ for $y^* = \$80,000$ instead of \$250,000.

Figure 11: $\bar{\gamma}(\theta, \theta^*)$ with $y(\theta^*) = \$250,000$ (left panel) and the implied tax incidence (right panel)

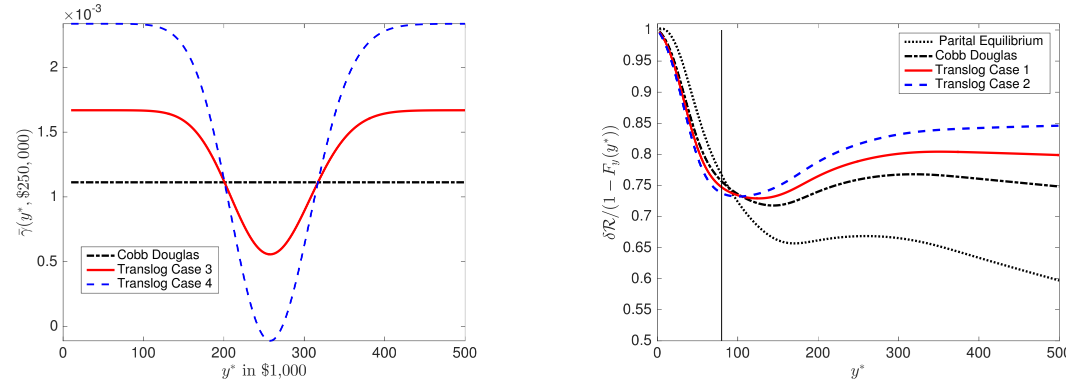


Figure 11 illustrates the distance dependent wage effects for these Cases 3 and 4. The tax incidence results are illustrated in the right panel. Here the best comparison level for y^* to understand the effects of distance dependence is \$80,000. In line with the results in the main body of the text, the general equilibrium effects on tax incidence are increased in magnitude through distance dependence.

Figure 12: Optimal Marginal Tax Rates for Translog Production Functions

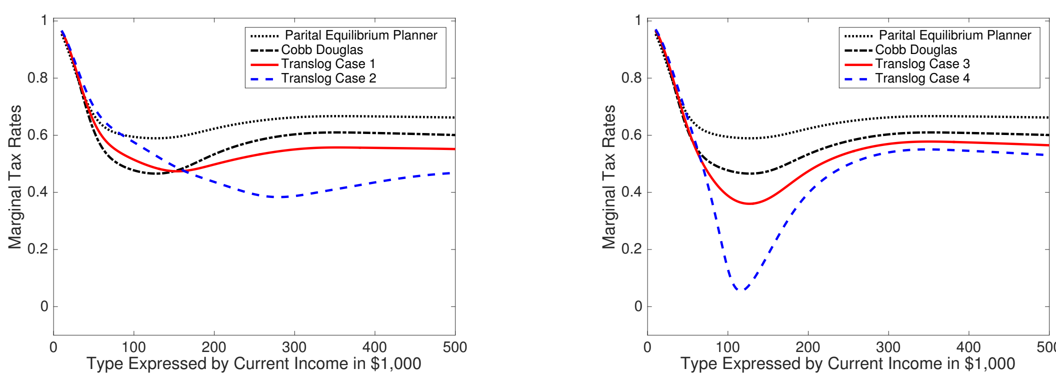


Figure 12 illustrates the resulting optimal marginal tax rates for all 4 cases. As cases 1 and 3 are more similar to Cobb Douglas than cases 2 and 4, not surprisingly, marginal tax rates are closer to the Cobb Douglas counterpart. These results can be best interpreted by looking at Figure 13 where the own-wage effects are illustrated.[57] For Cases 1 and 3, $\bar{\bar{\gamma}}(y, y)$ is relatively flat and close to -1 (as in Cobb Douglas). For Case 4, the wage effects are generally a bit larger in magnitude and still relatively flat (magnitude varies by a factor 2), which explains why the CES effects are mainly increased in magnitude. For Case 2, the magnitude of the wage effects strongly increases and varies

[57]They are illustrated for the current tax system but do look very similar for the respective optimal tax system.

by a factor of up to 6, which explains why the shape of marginal tax rates is more different from CES than in the other 3 cases. Finally, welfare gains in consumption equivalents are 1.09%, 0.31%, 1.85% and 2.53% respectively for these cases.

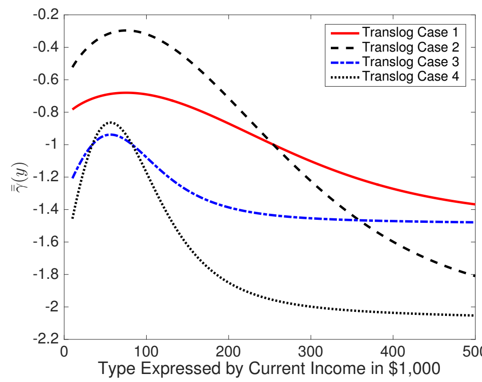


Figure 13: Illustration of the own-wage effect for current policies